Optimal Actuator and Sensor Placement for Active Sensing

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NOMENCLATURE

- \( r \): Feature vector
- \( \mathbf{m} = \{ m_0, m_1 \} \): Set of possible local damage states = \{ undamaged, damaged \}
- \( d_{kj} \): Decide \( m_j \) is the local damage state in region \( k \)
- \( h_{kj} \): \( m_j \) is the true local damage state in region \( k \)
- \( P(d_{kj}; h_{kj}) \) or \( P(d_{kj} | h_{kj}) \): Probability of \( d_{kj} \) given \( h_{kj} \) (under Classical or Bayesian view)
- \( \Gamma \): Global performance
- \( \mathbf{x}_{ai} \): Spatial coordinate vector of actuator \( i \)
- \( \mathbf{x}_{sj} \): Spatial coordinate vector of sensor \( j \)
- \( \mathbf{x}_k \): Spatial coordinate vector of point of interest (POI) \( k \)
- \( Q(\gamma) \): Right tail probability of zero mean, unit variance Gaussian random variable

ABSTRACT

In active sensing, actuators are used to impart user-defined energy into a structure from which resulting sensor responses may be mined for useful information regarding the structure’s health. Optimal actuator-sensor placement works to maximize the observability of this information given two primary constraints: the number of actuators and sensors available for the network and the power available for interrogation. We present a generalized detection theory-based methodology for optimal actuator and/or sensor placement. Placements are chosen to maximize one of three presented performance measures, one based on classical probability and two on Bayesian. Generalization is achieved by assuming the critical detection region is a subset of the space spanned by an arbitrary set of extracted SHM features. The methodology is applied to the particular case of active sensing by applying the statistics of the application-specific feature set. In this study, we considered two sensing schemes: pitch-catch, where the actuator and sensor are distinct, and pulse-echo, where the actuator and sensor are one in the same. The method has the capability of optimizing about non-uniform damage probability distributions, concave structure geometry, and potential sensor failure. The optimization space was searched using a genetic algorithm specifically tailored for sensor placement. We provide five example actuator-sensor placement scenarios and the

1 INTRODUCTION

Structural Health Monitoring (SHM) involves a paradigm of actuation, sensing, data feature extraction, and data feature classification as a means of appropriately assessing the performance condition of a structure. One of the most useful SHM implementation modes is that of “active sensing,” whereby actuators impart user-defined energy into the structure from which resulting sensor responses may be mined for useful information. Given such an active sensing approach, two primary constraints exist: the number of actuators and sensors available for the
network and the power available for interrogation. Optimal sensor-actuator placement strategies work to maximize the ability to detect and discriminate relevant data features given these limited resources.

In ultrasonic Lamb wave-based active sensing, actuators impart mechanical waves in a plate-like structure of interest. After propagating through the structure, the waves are detected by a set of receiving sensors. It is assumed that any damage to the structure will manifest itself in the received waveforms through mechanisms such as reflection and attenuation. The presence of damage is then determined by comparing these waveforms to a baseline that is representative of the undamaged structure. A baseline can take the form of waveforms acquired when the structure was healthy or modeling assumptions such as linearity [1] or correlation among sensor-actuator pairs [2]. The final, reduced order decision variable for determining the presence of damage at a given location is referred to as the feature.

While actuators and sensors used in active sensing come in many forms, this paper considers the use of piezoelectric patches [3]. For the most part, however, the optimization method we present here is independent of the instrumentation type used. With piezoelectric patches, each patch can serve as either an actuator or sensor. With the proper controlling circuitry, two actuation-sensing schemes are possible. The first, known as pulse-echo, involves a single patch actuating a waveform and then detecting its reflections. The second, referred to as pitch-catch, involves one patch actuating and a second, remote patch sensing.

While there have been a number of studies on optimal sensor placement, there have only been a few, limited studies on sensor placement for ultrasonic based structural health monitoring. Worden et al. [4] tackled the case of passive monitoring with the objective of detecting excitations from damage inducing impacts on the structure. Lee et al. consider the placement for sensors for a given fixed single actuator placement and known damage location [5]. Gao et al. [6] discusses active sensing actuator/sensor placement but limits their study to only the pulse-echo actuation scheme. In the present study, we develop an optimal placement strategy for both actuators and sensors which supports pulse-echo and pitch-catch actuation.

2 APPROACH

In this paper, we consider a structure divided into $K$ discrete regions. Each of these regions is small enough in size that relevant functions over its space can be approximated as constant. Through active sensing, for a given sensor arrangement, we can extract a vector of features, $\mathbf{r}$, for the entire structure. An example element of a feature vector could be the change in the received mechanical waveform at a particular instance in time as actuated and sensed by a particular actuator-sensor pair. With respect to each region and for a given actuator/sensor arrangement, we treat the feature vector as a random variable which can take on one of two known distributions depending on whether or not that region of the structure is damaged. The feature vector then has a pair of known distributions corresponding to each individual region of the structure. If we assume that the feature is extracted by means of sufficient averaging, then by the central limit theorem we say that the feature vector is normally distributed according to

$$
\mathbf{r} \sim \begin{cases} 
N(0, \mathbf{C}_k) & \text{under } h_{k0} \text{ (no damage in } k) \\
N(\mathbf{s}_k, \mathbf{C}_k) & \text{under } h_{k1} \text{ (damage in } k) 
\end{cases}
$$

(1)

Here, $\mathbf{s}_k$ is the expected feature vector under damage, and $\mathbf{C}_k$ is the covariance matrix. If the original feature vector is not expected to be zero under $h_{k0}$ then the original expected value under $h_{k0}$ should be subtracted off such that the final vector is expected to be zero. The statistics of the feature vector for each region depends on the placement of actuators and sensors and is calculated using a model of the feature extraction process. Our goal, then, is to maximize some measure of detectability across all regions in the structure through implementation of an optimal detector and optimal sensor arrangement.

3 FEATURE EXTRACTION MODEL

Here we derive a model of the feature extraction process in order to determine the statistics of the feature vector. Active-sensing involves imparting energy into a structure via an actuator and detecting the response via a sensor. Generally this involves multiple pairs of actuators and sensors which we wish to optimally place in order to maximize the global performance measures.
We consider the use of isotropic piezo-ceramic devices which can serve as both actuators and sensors. Since each device serves dual roles, we refer to the general device as just “sensor”. While the examples presented later are limited to two-dimensional plates, the derivations in this section are general to one-dimensional and three-dimensional wave-guides as well.

3.1 Actuator/Sensor Definitions

We assume each actuator-sensor pair produces one feature element for each region on the structure. With \( N \) pairs, the length of the full feature vector is \( KN \). For notational simplicity, we say the full feature vector \( \mathbf{r} \) is constructed of \( K \) local vectors \( \mathbf{r}_k \) of length \( N \) which are relevant to each region so that

\[
\mathbf{r} = [\mathbf{r}_1^T, \mathbf{r}_2^T, \ldots, \mathbf{r}_K^T]^T.\tag{2}
\]

Without specifying the details of a particular feature extraction process, we say that the features are representative of a change in the measured waveform due to reflection or attenuation from damage. Geometric computations involving a discrete region are carried out according to its centroid, which we call a point of interest (POI). Given that there are \( N_a \) actuators and \( N_s \) sensors, and we implement all possible actuator sensor pair combinations, then actuator \( a_i \) and sensor \( s_j \) construct feature vector element \( r_k[n] \) by the assignment

\[
i = \left\lfloor \frac{n-1}{N_a} \right\rfloor + 1, \quad j = (n-1) \text{mod} N_s + 1.
\]

In other words,

\[
(a_{i_1}, s_{j_1}) \rightarrow r_k[1], \quad (a_{i_2}, s_{j_2}) \rightarrow r_k[2], \quad \ldots, \quad (a_{N_a}, s_{N_s}) \rightarrow r_k[N].\tag{3}
\]

For the case where all devices serve as both actuators and sensors and the actuators can be self-sensing, the total number of actuators and sensors are equal, and \( a_i \) and \( s_j \) refer to the same device for \( i = j \).

3.2 Expected Value

We first derive the form of the expected value, \( s_k \), of the local feature vector. Assuming that the expected values under undamaged conditions have already been subtracted off, we say that the expected values are approximated by a product of a system efficiency term, a wave propagation term, and a defect scattering term such that

\[
\mathbf{r}_k[n] = A \mathbf{P}_{ij}[k] D_{ij}[k].\tag{4}
\]

The efficiency term, \( A \), captures the base efficiency of the feature extraction process and is driven primarily by actuator amplitude, sensor sensitivity, and the signal processing effectiveness. We consider this term independent of the positioning of the actuator, sensor, and POI. The wave propagation term, \( P_{ij}[k] \), accounts for the attenuation in the wave traveling from actuator to POI to sensor. The amplitude of a wave radiating from a near-point source approximately has a falloff of order \( d^{-1} \), where \( d \) is the dimension of the waveguide. Therefore,

\[
P_{ij}[k] = \left( \frac{\eta_p}{\eta_p + \|\mathbf{x}_{a_i} - \mathbf{x}_k\| + \|\mathbf{x}_k - \mathbf{x}_{s_j}\|} \right)^{d-1} P_{\text{LOS}},\tag{5}
\]
where \( \eta_P \) is the corner distance of the falloff. In the case of Lamb waves propagating in a plate, the dimension \( d \) is equal to two. The line of site term, \( P_{LOS} \), nulls out the signal path if a line of site is not present from actuator to POI and from POI to sensor. Line of site can be blocked by either a lack of physical path, or the lack of ability in the feature extraction process to resolve the path if it is too complicated.

The scattering term, \( D_{ij}[k] \), accounts for a change in the feature amplitude due to the relative positioning of the POI with respect to the actuator and sensor. As a wave interacts with a defect, a portion of the wave is scattered in one or more directions. For geometric reference, consider the POI at the origin and the wave propagating from the left. We say that the downstream change in amplitude as seen by the sensor located at angle \( \phi_{ij}[k] \) takes the form

\[
D_{ij} = \frac{a_0 \eta_0}{\eta_0 + |\phi_{ij}[k] - 0|} + \frac{a_\eta \eta_\eta}{\eta_\eta + |\phi_{ij}[k] - \pi|} + c_D,
\]

where \( a_0 \) and \( \eta_0 \) are the peak-amplitude and corner-angle for their respective angles and \( c_D \) is the baseline scattering level. This formula places emphasis on the forward (0°) and the reverse (180°) directions. The forward direction is favored since regardless of scattering direction, the forward path sees the full reduction in amplitude. The reverse direction is favored in accordance with wave propagation theory, which prefers scattering at 180° [7].

For arbitrary actuator, sensor, and POI positioning, \( \phi_{ij}[k] \) is the angle between the vector formed by the actuator and POI and the vector formed by the POI and sensor, or

\[
\phi_{ij}[k] = \cos^{-1}\left(\frac{(x_k - x_{a_i}) \cdot (x_{a_i} - x_k)}{\|x_k - x_{a_i}\| \|x_{a_i} - x_k\|}\right)
\]

### 3.3 Covariance Matrix

Next, we derive the second necessary statistic of the feature vector, the covariance matrix. For this example, we assume the noise on each feature are independent of each other so that

\[
C_k = \text{diag}(\sigma_k[1] \ldots \sigma_k[N]).
\]

We consider two contributors to the noise process. The first is due to irregularities or changes in the propagation path and is scaled by the total wave propagation distance. Examples of this are changes in temperature or boundary conditions. The second is not path-dependent and is an accumulation of sensor, electrical, and mechanical noise. The noise variances then take the form

\[
\sigma_k[n] = \sigma_p L_{ij}[k] + \sigma_m
\]

\[
L_{ij}[k] = \left(\|x_{a_i} - x_k\| + \|x_k - x_{a_i}\|\right).
\]

With a diagonal covariance matrix, the deflection can be expressed as the summation

\[
d^2[k] = s_k^T C_k^{-1} s_k = \sum_{n=1}^{N} \frac{s_n^2[n]}{\sigma_k^2[n]} = \sum_{i=1}^{N_{a}} \sum_{j=1}^{N_{s}} \left(\frac{A_{P_{ij}}[k]}{\sigma_p L_{ij}[k]} + \sigma_m\right).
\]

### 4 OPTIMAL DETECTOR

Here we define the optimal local detector for each region. Following derivations outlined in Kay [7], the optimal local detector is either the likelihood ratio (Classical) or conditional likelihood ratio (Bayesian), which can be expressed as
\[T[k] = \frac{p(r|h_{k1})}{p(r|h_{k0})} > \gamma[k] \quad \text{or} \quad T[k] = \frac{p(r|h_{k1})}{p(r|h_{k0})} > \gamma'[k].\] (11)

For a Gaussian feature vector whose distribution is given by (1), the optimal detector is reduced to the matched filter

\[T[k] = r^T C_k^{-1} s_k > \gamma'[k],\] (12)

where the non-data terms are group with the new threshold \(\gamma'[k]\). The detector is normally distributed according to

\[T \sim \begin{cases} \mathcal{N}(0, d^2[k]) & \text{under } h_{k0}, \\ \mathcal{N}(d^2[k], d^2[k]) & \text{under } h_{k1}, \end{cases}\] (13)

where the local deflection coefficient \(d^2[k]\) is defined as

\[d^2[k] = s_k^T C_k^{-1} s_k\] (14)

The threshold value will depend on the optimality criteria chosen, which we discuss next.

5 GLOBAL OPTIMALITY CRITERIA

In order to search for the optimal sensor arrangement, we must select a method by which to combine the local detector performances of each region into a single global performance measure. The optimal sensor arrangement will be that which maximizes the global performance. We introduce three global measure of performance, one based on a classical probability view, and two on a Bayesian view.

5.1 Maximize Neyman-Pearson Poorest Local Performance

The first global optimality criterion we discuss is the Neyman-Pearson performance of the poorest performing region on the structure. This criteria is particularly useful in situations where there is no a prior knowledge regarding the rate of damage. To determine the performance, we first choose a uniform constraint on either the local detection rate \(P_D[k]\) or local false alarm rate \(P_{FA}[k]\):

\[P_{FA}[k] = P(d_{k1};h_{k0}) = Q\left(\frac{\gamma'[k]}{\sqrt{d^2[k]}}\right) = \alpha_{FA} \quad \text{or} \quad P_D[k] = P(d_{k1};h_{k1}) = Q\left(\frac{\gamma'[k] - d^2[k]}{\sqrt{d^2[k]}}\right) = \alpha_D.\] (15)

Next, we use the deflection coefficient to calculate the threshold in each region needed to achieve the chosen local detection or false alarm rate constraint:

\[\gamma'[k] = Q^{-1}(\alpha_D)\sqrt{d^2[k]} + d^2[k] \quad \text{or} \quad \gamma'[k] = Q^{-1}(\alpha_{FA})\sqrt{d^2[k]},\] (16)

Finally, we calculated the opposite local rate for each region and determine either the maximum (for false alarm rate) or minimum (for detection rate) across all the regions.

\[GP = P_{FA} = \max_k [P_{FA}[k]] \quad \text{or} \quad GP = P_D = \min_k [P_D[k]]\] (17)

From (17) it is clear that an equivalent performance criterion is the maximization of the deflection coefficient. However, as will be seen in the case of sensor failure, if the true detector statistics are different than those used to calculate the threshold values, then the performance measures as expressed in equation (17) should be used.
5.2 Minimize Total Bayes Risk

The second global criterion follows a Bayesian view, where each region has an associated a priori probability of damage as well as a set costs for the possible outcomes of the local detection. The total Bayes Risk of the entire structure is then the sum of the expected costs of detection across all regions, which is expressed as

\[
GP = \bar{c} = \sum_{k=1}^{K} \sum_{j=0}^{1} c_{ij} [k] P(d_{kj} | h_{kj}) P(h_{kj})
\]

(18)

The detection and false alarm rate for each region are calculated in the same manner as in (15). The optimal set of thresholds for the detector (12) that minimizes Bayes risk is then

\[
\gamma'[k] = \log \left( \frac{c_{10}[k] - c_{00}[k]}{c_{01}[k] - c_{11}[k]} \right) \frac{P(h_{k1})}{P(h_{k0})} + \frac{1}{2} d^2[k]
\]

(19)

The cost \( c_{10} \) is incurred when a region is incorrectly identified as damaged and is related to cost of manual inspection. The cost \( c_{01} \), usually the highest of the costs, is incurred when the region is incorrectly identified as not damaged and is associated with the cost of structural failure. The cost \( c_{00} \) is usually zero as it is incurred when a region is correctly identified as not damaged, which generally does not result in any course of action. The cost \( c_{11} \), which is associated with correctly detecting damage, may be greater than zero incorporate cost of inspection and repair, however is expected to be much less than \( c_{01} \) (otherwise there is no sense in monitoring for damage to begin with).

5.3 Maximize/Minimize Global Detection/False Alarm Rate

The final global performance measure is also based on a Bayesian philosophy. We refer to the two versions of this performance as the global detection rate and the global false alarm rate. The global detection rate is the expected proportion of the structure’s damaged regions that will be correctly identified as damaged, that is

\[
\bar{P}_D = \sum_{k=1}^{K} \frac{P(d_{k1} | h_{k1}) P(h_{k1})}{\sum_{k=1}^{K} P(h_{k1})}.
\]

(20)

Similarly, the global false alarm rate is the expected proportion of the structure’s undamaged regions that will be incorrectly identified as damaged. It is defined as

\[
\bar{P}_{FA} = \sum_{k=1}^{K} \frac{P(d_{k1} | h_{k0}) P(h_{k0})}{\sum_{k=1}^{K} P(h_{k0})}.
\]

(21)

To implement, we optimize one of these performance measures with the other fixed at some requirement value. In other words,

\[
GP = \bar{P}_D \quad \text{with} \quad \bar{P}_{FA} = \bar{\alpha}_{FA} \quad \text{or} \quad GP = \bar{P}_{FA} \quad \text{with} \quad \bar{P}_D = \bar{\alpha}_D.
\]

(22)

Using Langrange multipliers, we can determine the optimal threshold to be equal to

\[
\gamma'[k] = \lambda + \ln \left( \frac{P(h_{k0})}{P(h_{k1})} \right) + \frac{1}{2} d^2[k],
\]

(23)

where the scalar multiplier \( \lambda \) is the solution to one of the constraint equations in (22). For a fixed global false alarm rate, the full constraint equation is
$$\tilde{\alpha}_{FA} = \sum_{k=1}^{N} Q \frac{\lambda + \ln \left( \frac{P(h_{k0})}{P(h_{k1})} \right) + \frac{1}{2} d_{k}^{2}}{\sqrt{\sigma_{k}^{2}}} \frac{P(h_{k0})}{\sum_{k=1}^{N} P(h_{k0})},$$  \hspace{1cm} (24)$$

and for a fixed global detection rate, the constraint equation is

$$\tilde{\alpha}_{D} = \sum_{k=1}^{N} Q \frac{\lambda + \ln \left( \frac{P(h_{k0})}{P(h_{k1})} \right) - \frac{1}{2} d_{k}^{2}}{\sqrt{\sigma_{k}^{2}}} \frac{P(h_{k1})}{\sum_{k=1}^{N} P(h_{k1})}.$$  \hspace{1cm} (25)$$

6 SENSOR FAILURE

We now consider what happens if a sensor fails. If the system is able to recognize the failure, then it will simply remove the sensor’s data from the computation of the features and adjust the detector threshold accordingly. However, as it is often the case, the system may be unaware of the failure. In this case, the true probability distribution function of the detector will be different than that used to calculate the optimal threshold, altering the detector’s performance. We may then write the actual feature vector as the original feature vector plus some modification due to the failed sensor, or

$$r_{a} = r + \tilde{r}.$$  \hspace{1cm} (26)$$

We then need to calculate the corresponding actual detector statistics in order to determine the true performances. Starting with the expected value, we have

$$E[T_{a}] = E[r_{a}^{T} C_{k}^{-1} s_{k}] = E[r^{T} C_{k}^{-1} s_{k}] + E[\tilde{r}^{T} C_{k}^{-1} s_{k}]$$  \hspace{1cm} (27)$$

Let $\tilde{N}_{kq}$ be the set of indices which correspond to the nonzero elements of $\tilde{r}$ and define masked versions of the feature vector and expected signal vector as

$$r_{q}'[n], s_{kq}'[n] = \begin{cases} r[n], s_{k}[n] & \text{for } n \in N_{q} \\ 0 & \text{for } n \notin N_{q} \end{cases}.$$  \hspace{1cm} (28)$$

Now we express the expected value of the detector as applied to the failed sensor feature vector as a fraction (positive or negative) of the expected value of the detector applied to the masked version of the original feature vector so that

$$E[\tilde{r}^{T} C_{k}^{-1} s_{k}] = v_{iq}[k] E[r_{q}^{T} C_{k}^{-1} s_{k}] = v_{iq}[k] s_{kq}'[k] C_{k}^{-1} s_{k} = v_{iq}[k] d_{q}^{2}[k] \text{ under } h_{kq},$$  \hspace{1cm} (29)$$

where $d_{q}^{2}[k]$ is the deflection coefficient computed from just the elements affected by the failed sensor. Here we have modified the local state condition such that $h_{kq}$ implies that the true state is $m_{i}$ in region $k$ with failed sensor $q$. Next, looking at the variance of the detector with a failed sensor, we have

$$\text{var}[T_{a}] = \text{var}[r_{a}^{T} C_{k}^{-1} s_{k}] = s_{k}^{T} C_{k}^{-1} \left( E[ww^{T}] + 2E[ww^{T}] + E[ww^{T}] \right) C_{k}^{-1} s_{k}.$$  \hspace{1cm} (30)$$

We assume that the additional noise associated with the failed sensor is uncorrelated with the original feature noise, or
\[ E[ww^T] = 0 . \] (31)

Following a similar procedure as with the expected value computation, we assume the covariance matrix of the failed sensor noise is some fraction of the original covariance matrix masked to the relevant elements, so that
\[ E[ww^T] = \mu_{iq}[k]E[w_qw_q^T] = \mu_{iq}[k]C_k \quad \text{under } h_{kq}. \] (32)

The failed sensor variance contribution is then
\[ s_k^T C_k^{-1} \mu_{iq}[k] C_k C_k^{-1} s_k = \mu_{iq}[k] s_k^T C_k^{-1} s_k = \mu_{iq}[k] d_q^2[k]. \] (33)

With the expected value and variance computed, and assuming that the failed sensor noise contribution is Gaussian, we can express the actual detector statistics as
\[ T_a \sim \begin{cases} N(\nu_{0q}[k]d_q^2[k], d^2[k] + \mu_{0q}[k]d_q^2[k]) & \text{under } h_{k0q} \\ N(d^2[k] + \nu_{1q}[k]d_q^2[k], d^2[k] + \mu_{1q}[k]d_q^2[k]) & \text{under } h_{k1q} \end{cases} \] (34)

The modified local detection rate and false alarm rate with the failed sensor are now
\[ P(d_{k1}|h_{k1q}) = Q\left(\frac{\gamma'[k] - \nu_{1q}[k]d_q^2[k]}{\sqrt{d^2[k] + \mu_{1q}[k]d_q^2[k]}}\right) \] (35)

and
\[ P(d_{k1}|h_{k0q}) = Q\left(\frac{\gamma'[k] - \nu_{0q}[k]d_q^2[k]}{\sqrt{d^2[k] + \mu_{0q}[k]d_q^2[k]}}\right) \] (36)

The form of the failed sensor constants \( \nu \) and \( \mu \) will depend on the type of failure. The case that we will consider is that in which the contribution from the failed sensor is the same regardless of the damage condition, where the constants will be related by
\[ \nu_{1q} = \nu_{0q} - 1 \]
\[ \mu_{1q} = \mu_{0q} \] (37)

Further, if we assume features are extracted through signal comparison to a baseline acquired when the sensor was functional, then we’d expect the erratic measurements from a failed sensor to result in a positive shift in the expected value under no damage, so that \( \nu_{1q} > 0 \). The direction of shift in the PDF under damage will depend on how much the failed sensor feature vector elements resemble those which are a result of damage. Normally, we’d expected the failed sensor to diminish the ability to detect damage, shifting the PDF under damage to the left. However, if the failed sensor feature vector elements “look” more like damage than actual damage, then when there actually is damage, the failed sensor will appear as if it is actually doing a better job than when it was working, shifting the damage PDF curve to the right. This corresponds to \( \nu_{0q} \) being greater than one.

We incorporate robustness to a failed sensor in the optimal sensor placement strategy by optimizing the worst-case scenario. We first determine the optimal map of detector thresholds based on an assumed full set of working sensors. Then, for each sensor, we calculate the performance of the arrangement as if that sensor had failed. The final fitness value is then the lowest of the failed sensor performances.
7 GENETIC ALGORITHM

Genetic algorithms are effective and simple to implement in finding optimal or near-optimal solutions to complicated, non-linear, discontinuous search spaces. This makes them especially popular for optimal sensor placement. Genetic algorithms mimic the process of natural selection, crossover, and mutation to intelligently search the solution space. We only briefly outline the genetic algorithm here. For a more thorough description, see Mitchell [9].

The first “generation” in the genetic algorithm is initialized with a random population of sensor arrangements. The detection performances are calculated for each arrangement, and the arrangements with the highest performances are selected. The sensor locations between the fittest arrangements (i.e., those with the highest fitness values as evaluated by the objective function) are randomly combined through crossover to produce a new population. The single fittest arrangement is also included, unaltered, in the new population. For a fraction of this new population, the sensor locations are randomly shifted, or mutated, according to a Gaussian distribution. The detection performances are recalculated for the new generation, and the process is repeated. For each subsequent generation, the variance of the Gaussian mutation is linearly reduced in order to converge to the precise optimal arrangement. The algorithm continues to loop until some convergence criteria are met. This entire process is usually repeated with different initial conditions in order to verify that the optimum is global.

One important consideration for genetic algorithms is the encoding of the optimization variables in the member genes. For our sensor placement problem, we coded each gene as a pair of single precision variables representing the coordinates of a single sensor. As such, during crossover, coordinate pairs are swapped together.

Every genetic algorithm requires a fitness function, which provides the relative performance, or fitness, of a given member. For this study, the fitness function is one of the two global performance measures: either the global detection rate or the global false alarm rate. As we have shown, these performance measures are a function of the feature statistics, which are in turn a function of the sensor locations.

8 EXAMPLES

We now present sensor placement scenarios to demonstrate each of the three global performance criteria. In all the examples, we used the following feature extraction model parameters:

\[
A = 120, \quad \eta_p = 20, \quad \sigma_p = 0.1, \quad \sigma_m = 4, \\
\alpha = c_D = 0.1, \quad a_0 = 0.9, \quad \eta_0 = \eta_x = 0.1
\]  

We chose parameter values based on experience and for demonstration purposes only, as the ultimate contribution of this paper is the presentation of this problem within the context of detection theory. Field application of these methods would include formal quantitative model development and verification. We reserve this for future work.

All spatial units are in centimeters and all densities are per square centimeter. The discrete region sizes ranged from 0.25 cm\(^2\) to 1.0 cm\(^2\) depending on the size of the plate. We selected these sizes by evaluating the convergence of the performance measures for each scenario.

8.1 Maximize Neyman-Pearson Poorest Local Performance

The first example involves the optimal placement of six sensors on a 125 cm by 78 cm rectangular plate. We fixed the local detection rate uniformly at 0.90 with the goal of minimizing the largest local false alarm rate. Figure 1 shows the resulting optimal arrangement along with maps of the deflection coefficient (left) and local false alarm rate (right). The highest false alarm rate for the optimal arrangement was 0.19. When maximizing to the lowest local performer, the algorithm has the tendency to spread the arrangement to the boundaries of the plate so that every region has at least some degree of coverage.
In the second example, we tasked the algorithm with optimally placing seven sensors on a delta shaped plate 150 cm in length with the criteria of minimizing the total Bayes Risk. The cost of missed detection was uniform over the plate and set to 1000 per region. The cost of false alarm was proportional to the square of the distance to the nearest edge, with an average cost per square centimeter of 1. The other two costs types were set to zero. The damage rate was uniform over the plate and normalized such that the expected number of damaged regions was equal to one. This normalized form of the costs and damage rates made the optimal arrangement and resulting Bayes Risk independent of region size.

The optimal arrangement along with a map of the Bayes Risk density (per square cm) is shown in Figure 2 (left). The resulting Bayes Risk was 33.7. A lack of symmetry wins over symmetry in this arrangement since it produces fewer overlapping sensor-pair coverage regions, improving the overall coverage of the plate. For example, had the three sensors in the middle of the plate lined up vertically there would have been an unbeneﬁcial redundancy in the coverage.

We then computed the optimal sensor placement for the same plate with the added criteria of robustness to sensor failure. We characterized the failure with the parameters

\[ \mu_0 q = \frac{1}{2}, \quad \mu_1 q = -\frac{1}{2}, \quad \nu_0 q = \nu_1 q = 0. \]  

The optimal arrangement is given on the right of Figure 2. With the inclusion of sensor failure, redundancy is much more important. The sensors line up such that no single sensor on its own is responsible for providing coverage of a significant area. Every pair therefore has a backup pair in case of sensor failure. The cost of this robustness is a higher total Bayes Risk of 62.0. As expected, both arrangements work to maximize the coverage of the inner region, which has the highest cost of manual inspection.
8.3 Maximize/Minimize Global Detection/False Alarm Rate

In the third example, we looked to optimally place 15 sensors on a 200 cm square plate in order minimize the global false alarm rate given a fixed global detection rate of 0.90. The damage rate of the plate was proportional to the Von Mises stress of one of the low order modes of the plate. Again, the damage rate was normalized such that the expected number of damaged regions was equal to one. Figure 3 shows a map of the damage rate density (left) and thresholds (right) along with the optimal arrangement. The threshold map is shown using a nonlinear color map in order to emphasize details. Figure 4 gives maps the local detection rate (left) and false alarm rate (right). The resulting global false alarm rate for the optimal arrangement was 0.00099. The arrangement places higher concentrations of sensors about regions most likely to experience damage.

![Figure 3](image3.png)

**Figure 3: Optimal fifteen-sensor arrangement for global detection rate criteria on map of damage rate density (left) and thresholds (right)**

![Figure 4](image4.png)

**Figure 4: Optimal fifteen-sensor arrangement for global detection rate criteria with map of local detection rate (left) and false alarm rate (right)**

In the next example, we looked to optimally place 20 sensors on a plate than was significantly concave in shape, limiting sensor line of site. The plate was 200 cm square with a T-shape section removed. The global false alarm rate was fixed at 0.01 with the goal of maximizing the global detection rate. The damage rate was uniform over the plate. Figure 5 gives the optimal arrangement as well as maps of the local detection and false alarm rate. The resulting global detection rate was 0.97. The symmetry across the two disjoint regions in the plate highlights the effectiveness of the genetic algorithm. Notice, also, the presence of coverage shadows around the edges of the 'T' due to line-of-sight limitations.

![Figure 5](image5.png)
In the final scenario, we took the same T-notched plate, but instead applied the much tighter global false alarm rate constraint of one in one million. The resulting optimal arrangement and the local detection and false alarm rate maps are shown in Figure 6. The global detection rate for this arrangement was 0.61. With less room for error, the algorithm produced a more conservative arrangement, preferring effective coverage of a smaller area over thin coverage of a larger area. This arrangement also reveals that the optimal local false alarm rate map is not necessarily a monotonic function of the local detection rate. The local false alarm rate is highest in the transition regions where the sensor coverage is only average, while at the outskirts of the detection region, the low optimal threshold results in no attempt at detection, resulting in a low local false alarm rate.

9 CONCLUSIONS
We presented a flexible sensor placement strategy within a detection theory framework. This strategy supports the optimal placement of actuators and sensors under one of three global performance criteria. The method was specialized to the case of active sensing through the development of a basic model of the active sensing process. However, by focusing on the observability of a general form of extracted features, the strategy could be applied to a variety of structural health monitoring systems. We also presented an approach to include robustness to sensor failure in the optimization process. We proposed five representative sensor placement scenarios and provided the results of the optimization algorithm for each case.
We showed that a lack of symmetry can improve coverage, except in the case of sensor failure, where symmetry and redundancy are preferred. We demonstrated that the placement of actuators and sensors is specific to the performance constraints. As such, it is important for the user to accurately define the application-specific constraints and costs before instrumentation and subsequent optimizations are undertaken. Since the sensor arrangement and threshold map are simultaneously optimized, the OSP scheme also provides for optimal system management for the given performance constraints. The optimal arrangement remains optimal so long as the matching threshold map is used in the data classification step of the structural health monitoring process.

Through the test cases, we found that the smoothness of the optimization surface was dependent on the optimization criteria. The Neyman-Pearson criteria, robustness to sensor failure, and plate concavity produced the most irregular optimization surfaces. The genetic algorithms therefore took significantly more time to search the optimization space for solutions to problems involving these criteria.

Future work will include a more formal model development and verification for the active sensing feature extraction process. The most important additional consideration would be the inclusion of correlation between actuator-sensor pairs based on their relative positions. Highly-correlated actuator-sensor pairs would diminish the detector performance. Other considerations may include support for heterogeneous networks of sensors and sensor which are non-isotropic.

REFERENCES


