Stochastic Change Detection in Uncertain Nonlinear Systems Using Data-Driven System Identification Method

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ABSTRACT
A stochastic change detection methodology for reliable monitoring complex nonlinear dynamic systems is proposed. For a semi-active magneto-rheological (MR) damper, the non-parametric, data-driven restoring force method was used to identify the nonlinear dynamic damping device. Both supervised and unsupervised statistical pattern recognition techniques were used to detect the changes in the physical characteristics of the MR damper with different input currents. The classification errors were analyzed to find the optimal strategy for designing change detection classifiers for reliable structural health monitoring (SHM) applications.

1. INTRODUCTION
For the last three decades, a number of SHM methodologies have been developed to detect changes in various types of structures in the fields of civil, mechanical and aerospace engineering. However, most of these methodologies have significant limitations to be practical in real-world applications. A reliable change detection methodology should satisfy the following aspects: (1) the detectibility of “small” changes in complicated nonlinear systems, (2) feasibility of physical interpretation of the detected changes, and (3) capability of uncertainty quantification of detected changes.

In this paper, a stochastic change detection methodology is proposed and investigated to satisfy above requirements of SHM. An experimental study was performed using a complicated nonlinear device as shown in Section 2; the data-driven system identification results are discussed in Section 3; the change detection results using supervised support vector classification method are discussed in Section 4; the change detection results using unsupervised k-means clustering are shown in Section 5; and the error analysis of detected changes is discussed in Section 6.

2. EXPERIMENTAL SETUP
Having a precise control of the damper’s characteristics is critical for a successful experimental study. Consequently, in order to evaluate the developed stochastic change detection methodology, a semi-active magneto-rheological (MR) damper, whose physical characteristics are adjustable with the magnitude of the input current, was used in this study rather than a passive viscous damper, whose physical characteristics are fixed to a constant value. Figure 1 (a) shows the MR damper test apparatus used in this study. The tested MR damper was a complex nonlinear device with hysteretic nonlinearity due to the viscous action of the MR fluid combined with a dead-space nonlinearity due to a mechanical gap near the damper’s neutral axis as shown in Figure 1 (b).

A series of tests were performed with eight different sets of mean ($\mu_I$) and standard deviation ($\sigma_I$) of the MR damper input current: $\mu_I = 1.0$ A, 0.8 A, 0.6 A and 0.4 A and $\sigma_I = 0.1$ A and 0.15 A. Therefore, with the MR damper input current, the effective (nominal) damper characteristics can be precisely controlled through $\mu_I$, and the uncertainty of the damper characteristics through $\sigma_I$. For each set of tests, 500 tests were performed. Consequently, a total of 4000 tests were conducted. The MR damper was subjected to broadband random excitation with the cutoff frequencies of 0.1 ~3.0 Hz.
3. DATA-DRIVEN IDENTIFICATION

The Restoring Force Method (RFM) is a non-parametric, data-driven system identification method for nonlinear dynamic systems [1, 2]. Using two-dimensional normalized Chebyshev polynomial series expansion, the restoring force of a single-degree-of-freedom nonlinear system can be expressed as

\[ r(x, \dot{x}) = \sum_{i=0}^{P} \sum_{j=0}^{Q} \tilde{C}_{ij} T_i(\tilde{x}) T_j(\tilde{\dot{x}}) \]  

(1)

where \( r(x, \dot{x}) \) is the nonlinear restoring force, \( \tilde{C}_{ij} \) is the normalized Chebyshev coefficient, \( T_i(\bullet) \) is the \( k^{\text{th}} \) order Chebyshev polynomial, \( \tilde{x} \) and \( \tilde{\dot{x}} \) are the normalized displacement and velocity, respectively, in the range of \([-1, 1]\).

Using the relationships of \[ T_{k+1}(y) = 2yT_k(y) - T_{k-1}(y) \] [3], Equation (1) can be converted into two-dimensional normalized power series expansion

\[ r(x, \dot{x}) = \sum_{i=0}^{P} \sum_{j=0}^{Q} \tilde{a}_{ij} x^i \dot{x}^j \]  

(2)

where \( \tilde{a}_{ij} \) is the normalized power series coefficient. Consequently, \( \tilde{a}_{ij} \) can be different from the corresponding coefficient directly identified using polynomial basis functions.

Using the RFM, the MR damper was identified with different series expansion orders, and some sample identification results are shown in Figure 2. The figure illustrates term-wise identification results for the series expansion orders of 5 and 20 are used (O(5) and O(20), respectively). Note that the term-wise identification results of O(5) and O(20) are identical. This is due to the orthogonal property of normalized Chebyshev polynomials [4]. Consequently, the identification results for a specific order are not influenced by the model complexity in the RFM identification.

4. CHANGE DETECTION USING SUPERVISED SUPPORT VECTOR CLASSIFICATION

Once the MR damper was identified, different pattern recognition techniques were used for the stochastic change detection in the physical characteristics of the MR damper. Detailed description of various pattern recognition techniques can be found in [5]. First, the supervised support vector classification (SVC) was performed [6-10]. Figure 3 shows the precision of the supervised SVC for different numbers of features. In this classification, the identified Chebyshev and power series coefficients were used as the features of classification. Consequently, as the number of features increases, the number of series expansion terms included in the classification increases.
Figure 2. Sample term-wise RFM identification results with the series expansion orders of 5 and 20 ($O(5)$ and $O(20)$, respectively). The dashed lines are the measured MR damper response, and the solid lines identified.

Figure 3 shows that the classification precision increases as the number of features increases for both Chebyshev and power series coefficients. The classification precision with Chebyshev coefficients, however, is always greater than that with power series coefficients for the same number of features. Consequently, using the orthogonal coefficients is more advantageous for change detection in nonlinear systems due to better classification precision as well as no influence of model complexity.

Figure 3. Precisions of supervised support vector classification for different numbers of features.
5. CHANGE DETECTION USING UNSUPERVISED $k$-MEANS CLUSTERING

The second pattern recognition technique for change detection in nonlinear systems investigated in this study was unsupervised $k$-means clustering. The description of $k$-means clustering can be found in [11]. Figure 4 illustrates that the precision of $k$-means clustering with the orthogonal Chebyshev coefficients is higher than that with the non-orthogonal power series coefficients.

6. ERROR ANALYSIS

Once the change detection was performed using pattern recognition techniques, analyses were conducted on the classification results of SVC to find the optimal strategy of designing classifiers to detect changes in nonlinear systems. Using the detection theory [12, 13], the error analysis was performed with the following null and alternative hypotheses:

$$H_0 : \text{The MR damper does NOT belong to this class} \quad (3)$$
$$H_a : \text{The MR damper belongs to this class} \quad (4)$$

In the MR damper change detection, two types of classification error can be considered:

- **Type-I error**: $H_0$ is rejected when $H_0$ is true (“false alarm”) (5)
- **Type-II error**: $H_0$ is accepted when $H_0$ is false (“missed”) (6)

Figure 5 shows that both Type-I and Type-II errors decrease with a larger number of features in the classification. For a given number of features, in general, there is a “trade-off” between Type-I and Type-II errors (i.e., if Type-I error increases, Type-II error decreases, and *vice versa*). For the purpose of the SHM, minimizing Type-II errors would be more appropriate since the probability of “missing” significant damages can be reduced, and increasing “false-alarm” errors is more conservative in practical damage monitoring applications.

7. CONCLUSION

A stochastic change detection methodology for complex nonlinear system was studied using a semi-active magneto-rheological damper. Using a non-parametric, data-driven restoring force method, the complicated nonlinear magneto-rheological damper was accurately identified without *a priori* knowledge of the identified system. Both supervised and unsupervised pattern recognition techniques were successfully applied to detect changes in the physical characteristics of the MR damper. The analysis results of change detection errors showed that the change detection classifiers should be designed on the basis minimizing Type-II error (“missed” error) for reliable damage monitoring applications.
Figure 5. Type-I and Type-II errors of the support vector classification for different numbers of the normalized Chebyshev coefficients (features). In the classification, four classes of data are classified, including $\mu_I = 1.0 \text{ A}$ ($\circ$), $0.8 \text{ A} (\triangle)$, $0.6 \text{ A} (\square)$ and $0.4 \text{ A} (X)$ with $\sigma_I = 0.1 \text{ A}$.

REFERENCES


