STRUCTURAL REAL TIME CONTROL USING FLUID MAGNETORHEOLOGIC DAMPER

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Abstract: In the last decades there was a great development in the study of control systems to attenuate the harmful effect of natural events in great structures, as buildings and bridges. Magnetorheological fluid (MR), that is an intelligent material, has been considered in many proposals of project for these controllers. This work presents the controller design using feedback of states through LMI (Linear Matrix Inequalities) approach. The experimental test were carried out in a structure with two degrees of freedom with a connected shock absorber MR. Experimental tests were realized in order to specify the features of this semi-active controller. In this case, there exist states that are not measurable, so the feedback of the states involves the project of an estimator. The coupling of the MR damper causes a variation in dynamics properties, so an identification methods, based on experimental input/output signal was used to compare with the numerical application. The identification method of Prediction Error Methods - (PEM) was used to find the physical characteristics of the system through realization in modal space of states. This proposal allows the project of a semi-active control, where the main characteristic is the possibility of the variation of the damping coefficient.

Keywords: Magnetorheological damper, semi-active controller, identification method, Prediction Error Methods(PEM), Seismic input, Linear Matrix Inequalities (LMI)

1. INTRODUCTION

In the last decades there was a great development in the study of control systems to attenuate the harmful effect of natural events in great structures, as buildings and bridges. Two approaches can be taken to help buildings withstand seismic excitations. The first involves designing the structure with sufficient strength, stiffness and inelastic deformation capacity to withstand an earthquake (MCEER Information Service, 2005).

This can be done by combining structural components such as shear walls, braced frames, moment resisting frames, diaphragms, and horizontal trusses, in order to form lateral load resisting systems. This approach also considers the shape of the building. The choice of material used in construction is also important, since ductile materials, such as steel, were found to perform better than brittle ones, such as brick. The soil beneath the structure is yet another factor that greatly influences structural vibration characteristics and amount of damage sustained. Because this approach relies on the inherent strength of the structure to dissipate the seismic energy generated, a certain level of deformation and damage has to be accepted.

The second approach consists in using control devices to reduce the forces acting on the structure. The objective is to reduce all structural responses, that is, floor accelerations, velocities and displacements. They are categorized according to their energy consumption as: passive, active, and semi-active. Several very comprehensive reviews have been made about different types of control devices currently used or developed. These include passive control
devices (Kasai et al., 1998), active control (Housner et al., 1994), and semi-active systems (Symans and Constantinou, 1999).

2. STRUCTURAL MODELING

A linear differential inclusion (LDI) system, in modal state-space form, considering the matrices with appropriate dimensions and assumed to be known is given by:

\[
\dot{x} = A(t)x + B_1w + B_2u, \quad A(t) \in \Omega
\]

\[
y = Cx
\]

where \( \Omega \) is a polytope that is described by a list of vertexes in a convex space, \( A(t) \) is the dynamic matrix that is represented as time function to emphasize the uncertainties in the parameters, \( B_1 \) is the matrix of disturbance, \( B_2 \) is the matrix of control input, \( C \) is the output matrix, \( w \) is the vector of disturbance input, \( u \) is the vector of control input, and \( y \) is the output vector.

The state vector \( x \) of the modal coordinates system consists of \( n \) independent components, \( x_i \), that represent a state of each mode. The \( x_i \) (ith state component), related to Eq. (3), is defined as (Gawronski, 1998):

\[
x_i = \left\{ \begin{array}{c} q_{mi} \\ q_{m0i} \end{array} \right\}, \quad \text{where} \quad q_{m0i} = \frac{\zeta_i q_{mi} + \dot{q}_{mi}}{\omega_i}
\]

The modal state-space realization is characterized by the block-diagonal dynamic matrix and the related input and output matrices, Gawronski, 1998

\[
A(t) = \text{diag}(A_{mi}(t)), \quad B_1 = \begin{bmatrix} B_{1m1} \\ \vdots \\ B_{1mn} \end{bmatrix}, \quad B_2 = \begin{bmatrix} B_{2m1} \\ \vdots \\ B_{2mn} \end{bmatrix}, \quad C = [C_{m1} \ C_{m2} \ \cdots \ C_{mn}]
\]

where \( i = 1, 2, \ldots, n \), \( A_{mi}, B_{1mi}, B_{2mi} \) and \( C_{mi} \) are \( 2 \times 2 \), \( 2 \times k \), \( 2 \times s \) and \( r \times 2 \) blocks, respectively; \( k \) is the number of disturbances; \( s \) is the number of control inputs; and \( r \) is the number of outputs. These blocks can be obtained by several different forms and also it is possible to convert it in another realization through a linear transformation. One possible form to block \( A_{mi}(t) \) is

\[
A_{mi} = \begin{bmatrix} \zeta_i \omega_i & \omega_i \\ -\omega_i (\zeta_i^2 - 1) & -\zeta_i \omega_i \end{bmatrix}
\]

The parameters of the structure were identified by subspace identification method using experimental data. In general, there are noises on input and output signals, so, it is very probable that the dynamic matrix contains uncertainties in some modal parameters. This problem can be minimized realizing the identification tests several times, in order to quantify a range of variation. In this way, it is possible to implement a robust control design considering all dynamic matrices identified. These uncertainties are described by a polytopic LDI (PLDI):

\[
A_e(t) \hat{\Omega}, \quad \Omega = \text{Co}\{A_{e1}, L, A_{eN}\}
\]
where \( A_c \) is relative to controlled modes; \( \Omega \) is a polytope described by a list of vertexes in a convex space \( C_0 \) (Boyd et al. 1994a), and \( v \) is the number of vertexes of the polytopic system.

For controller design a reduced-order model is obtained by truncating the states. Let \( x \) and the state \((A, B_1, B_2, C)\) be partitioned considering the canonical modal decomposition. From the Jordan canonical form can be obtained:

\[
\begin{bmatrix}
  x_c \\
  x_r
\end{bmatrix} =
\begin{bmatrix}
  A_c(t) & 0 \\
  0 & A_r(t)
\end{bmatrix}
\begin{bmatrix}
  x_c \\
  x_r
\end{bmatrix} +
\begin{bmatrix}
  B_{1c} \\
  B_{1r}
\end{bmatrix} w +
\begin{bmatrix}
  B_{2c} \\
  B_{2r}
\end{bmatrix} u, \quad y = \begin{bmatrix}
  C_c & C_r
\end{bmatrix}
\begin{bmatrix}
  x_c \\
  x_r
\end{bmatrix}
\tag{6}
\]

where \( A_c(t) \) is given by Eq. (4) and the subscript \( r \) is relative to the residual modes.

3. PARAMETERS IDENTIFICATION

System identification is about building mathematical models of dynamical systems using measured input-output data. This can be done using a number of different techniques. In this paper, it is used Prediction Error Methods (PEMs), that is a broad family of parameter estimation methods that can be applied to quite arbitrary model parameterizations. This method has a close kinship with the maximum likelihood method (Ljung; 2002). In this section, the properties of this method are described.

The output at time \( t \) is \( y(t) \), and similarly the input is \( u(t) \). These signals may be vectors of arbitrary (finite) dimension. The case of no input \( (u = 0) \) corresponds to a time series or signal model. Let \( Z^N = \{u(1), y(1), u(2), y(2), \ldots u(N), y(N)\} \) collect all past data up to time \( N \). For the measured data, we always assume that they have been sampled at discrete time points (here just enumerated for simplicity). However, we may use continuous-time models. The basic idea behind the prediction error approach is very simple. Describe the model as a predictor of the next output:

\[
\hat{y}_m(t/t-1) = f(Z^{t-1})
\tag{7}
\]

where \( \hat{y}_m(t/t-1) \) denotes the one-step ahead prediction of the output, and \( f \) is an arbitrary function of past, observed data. Parameterize the predictor in terms of a finite-dimensional parameter vector \( \theta \):

\[
\hat{y}(t/\theta) = f(Z^{t-1}, \theta)
\tag{8}
\]

Some regularity conditions may be imposed on the parameterization. Determine an estimate of \( \theta \) (denoted \( \hat{\theta}_N \)) from the model parameterization and the observed data set \( Z^N \), so that the distance between \( \hat{y}(1/\theta), \ldots, \hat{y}(N/\theta) \) and \( y(1), \ldots, y(N) \) is minimized in a suitable norm. If the above-mentioned norm is chosen in a particular way to match the assumed probability density functions, the estimate \( \hat{\theta}_N \) will coincide with the maximum likelihood estimate.

PEM has a number of advantages:

- It can be applied to a wide spectrum of model parameterizations.
- It gives models with excellent asymptotic properties, due to its kinship with maximum likelihood.
- It can handle systems that operate in closed loop (the input is partly determined as output feedback, when the data are collected) without any special tricks and techniques.
It also has some drawbacks:

- It requires an explicit parameterization of the model. To estimate, say, an arbitrary linear, fifth-order model, some kind of parameterization, covering all fifth-order models, must be introduced.
- The search for the parameters that gives the best output prediction fit may be laborious and involve search surfaces that have many local minima.

More details can be found in Ljung (2002).

4. LINEAR QUADRATIC REGULATOR BY LMI APPROACH

In the following, one presents the procedure for the LQR – LMI approach.

The first step in LQR control design process is the definition of a performance index, which can be defined by a quadratic cost function in state and control variables. This index can be written as

\[
J = \int_0^\infty (u^T R u + x^T Q x) dx
\]  

where \( Q \) is a symmetric and positive semi-definite weighting matrix on the states and \( R \) is a symmetric and positive definite weighting matrix on the controller outputs.

Considering linear time-invariant system in state-space form, the object of the regulator design is to find a linear control law of the form

\[
u(t) = -K x(t)
\]

which minimizes \( J \). If the regulator design is restricted to time-invariant control laws, \( K \) will be a constant coefficient matrix and \( u(t) \) will be a linear combination of the states. It is assumed in the regulator design that all states are measured. It can be shown that the gain matrix \( K \), which minimizes the performance index, is given by (Anderson and Moore, 1990)

\[
K = R^{-1} B^T P
\]

where \( P \) is a symmetric, positive definite, constant coefficient matrix obtained from the solution of the algebraic Riccati equation

\[
0 = PA + A^T P - PB R^{-1} P + Q
\]

Using LMI technique, the linear quadratic regulator can be written by (Erkus and Lee, 2004;)

\[
\min_{Y,P,X} \text{Tr}(QP) + \text{Tr}(X) + \text{Tr}(YN) + \text{Tr}(N^T Y^T)
\]

subject to

\[
A P - B_2 Y + P A^T - Y^T B_1^T + B_1 B_1^T < 0,
\]

where

\[
\begin{bmatrix}
X & R^{\frac{1}{2}} Y \\
Y^T R^{\frac{1}{2}} & P
\end{bmatrix} > 0
\]

where \( \text{Tr}(\cdot) \) is the trace of the matrix, \( N \) is the noise vector; \( P, X \) and \( Y \) are the solution of the LMI. In the following application \( N \) was considered null. Solving equation (12), the LQR optimal gain of feedback is given by

\[
K = Y P^{-1}
\]
5. EXPERIMENTAL APPLICATION

The proposed methodology for vibrations attenuation was applied in a steel structure that can represent a building of two floors, as shown in Figure (1a). The structure is constituted of two steel plates, square shaped that are considered as base and ceiling of the floors. Also, in each one of the floors, four pillars were built with thin beams. With this, the structure of two floors can vibrate easily in direction x, shown in Figure (1b). It can be noticed that the structural vibrations in directions y and z are much smaller when compared with the vibrations in direction x. However, this methodology can be applied without losing of generalization for more complex cases. The physics and geometric properties of the structure are shown in the Table 1.

![Figure 1: Structure of two floors used – (a) photo, (b) schematical illustration, (c) structure with connected MR damper absorber](image)

**Table 1 – Physical and geometric properties of the structure of two floors.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 (m)</td>
<td>0.25</td>
</tr>
<tr>
<td>L2 (m)</td>
<td>0.25</td>
</tr>
<tr>
<td>L3 (m)</td>
<td>0.15</td>
</tr>
<tr>
<td>L4 (m)</td>
<td>0.20</td>
</tr>
<tr>
<td>Módulus of Elasticity (GPa)</td>
<td>210</td>
</tr>
<tr>
<td>Density (kg.m⁻³)</td>
<td>7800</td>
</tr>
</tbody>
</table>

The acquisition of the signals of input and output was carried out using the software SignalCalc® Ace II. The structure was excited with a random signal through a electro-mechanic shaker. The input signal was measured using a head impedance device and the output signals using an accelerometer PCB model 352C22 Piezotronics®. The sample frequency was 31.25Hz, considering 100 points. Frequency response function (FRF) of the structure was obtained considering 50% of overlap and Hanning window. Figure (2) shows the FRF, of the identified original structure, i.e., for the case without damper. The first and second vibration modes are 3.7Hz and 8.7Hz, respectively.
The objective of the control system is to attenuate the two first vibration modes of the structure, using semi-active MR damper, as shown the Figure (1c). The FRF of this new system was obtained considering the same acquisition features. Figure 3 shows a schematics picture of the experimental setup. It was used the MR damper RD 1097-01 friction damper, supplied by Lord Corporation. With this, it was possible to verify that the coupling of the actuator (MR damper) generated a passive attenuation in the first vibration mode, as shown in Figure 4. The FRF shown in this figure was obtained through signals of input and output in the time domain, considering Hanning window and time of sampling 0,0125s.
Evaluating the FRF of the structure with MR damper, one can verify that it appears only the frequency of vibration of approximately 6.8 Hz. The variation of the dynamics properties was caused due the introduction of the MR damper and the support setting. The experimental identification was made in this new structure using the signals of input and output after the coupling of the MR shock absorber. The matrices were identified in modal space of states, through method PEM. The order adopted for the model was 3 and, with this, a representative model was obtained. It must be noticed that the choice of the order of the model is a very important stage in the project of the semi-active controller, since it must generate models that represent the structural dynamics adequately. The robust controller was designed considering parametric uncertainties. It was considered that the natural frequency and the damping factor of the identified model have uncertainties of ±15% and ±20%, respectively. With this, it was possible to define the V1, V2, V3 and V4 vertices of convex space defined by the polytope shown in Figure 5.
Figure 5: Polytope of representation of the uncertainties defining the convex space in the project of the semi-active controller.

Considering the identified model and all uncertainties shown in Figura 5, the controller with feedback of states LQR through LMI was designed. Figure 6 shows the signal of input used in the simulation, which represent a seismic excitation (earthquake).

![Figure 6: Signal of Seismic input](image)

Using LMI Toolbox of the Matlab® 6.5, the LQR controller gain were obtained solving the Eq (13) and (14). Figure (7) shows the response in time domain of the system with and without control. One can see the significative attenuation for the closed loop.
The MR shock absorber modifies its characteristics of damping in function of the variation of the applied magnetic field, and consequently change the viscosity of the fluid so, it is needed to calculate the electric current to generate such variation in the magnetic field. Although MR absorber allows the application of great forces in the structure, it possess a limitation in the magnitude of the electric current that can be submitted. In particular, if 1A is submitted for more than 30 seconds, its piston and other components can be damaged by the temperature of the bobbin of the magnetic field. To convert the control force into electric current, it was used the chart supplied for the manufacturer. The relation between the applied current and the damping force is shown in Figure 8.

The inclusion of the magnetorheologic fluid in the modeling can cause variation in the dynamic matrix. This consideration implies in a great increase of the complexity the control
problem, and it demands the use of non-linear approaches. In the present work, the force applied for the MR damper is considered as external damping force, as well as in the modeling presented by Bhaskar (1995). The external force (or control force) can be calculated in function of the electric current as

\[ F_a = 191.55i_c - 570.82i_c^3 + 476.08i_c^5 + 23.731i_c + 10.737 \]  

(5.2)

where \( F_a \) is the damping force and \( i_c \) is the current. Even for the case without applied electric current (or \( i_c = 0 \)), the damper applies a damping force. This initial force acts in the system during the acquisition process so, it can be said that in the system with connected MR shock absorber a pre-force of damping exists. Thus, to find the control current, the following equation was considered;

\[ F_a = 191.55i_c^2 - 570.82i_c^4 + 476.08i_c^6 + 23.731i_c \]  

(5.3)

Figure (9) shows the control force and the electric current for the system submitted to an earthquake excitation, as shows in Figure (6). The vibration attenuation was shows in figure (7).

Special attention must be taken on eventual "peaks" of electric current during the action of the control. In this project was considered robustness in relation to parametric uncertainties, however, it is important to formulate controller that consider restrictions and limit the control signal, thus, guarantying the performance of the MR damper.

Figure 9: Electric current and force control for excitation shows in figure (6) and vibration attenuation shows in figure (7).
6. FINAL REMARKS

In this work a methodology for implementation of a robust controller was presented using damping through MR fluid. The project involved a MR damper manufactured by Lord Corporation, and the formulation of an LQR controller solve by LMI. The project considered uncertainties in modal parameters of the structure. The presented semi-active controller can operate with low electric current then, evidently, it can be used with auxiliary system of batteries. This is a great advantage in the case of the controller to be installed in building with the possibility of earthquake.

It is important to notice that, for structures with a big number of vibration modes (as real buildings), the coupling of MR damper modifies the structural dynamics and can generate passive damping in some modes. The project of the controller, generally does not involve all the modes. Thus, they would remain as residual modes in the control design. In these cases, special attention must be taken on eventual effect for spillover.

Another important point is the experimental identification of civil mechanical structures, as buildings. Frequently, the cost is very high for impact drivers. In these structures, the surrounding excitation as wind, traffic or small tremors can be used as excitation source. In this case, it is more interesting to use a method of random identification, or either, one method of identification based only in the structural output, (Nunes Jr., 2006).

REFERENCES


http://mceer.buffalo.edu/infoService/faqs/bsdesign.asp


