Blind Source Separation Based Vibration Mode Identification

Wenliang Zhou and David Chelidze
Department of Mechanical Engineering and Applied Mechanics
203 Wales Hall, 92 Upper College Road
University of Rhode Island, Kingston, Rhode Island 02881

Abstract

In this paper, a novel method for linear normal mode identification based on Blind Source Separation (BSS) is introduced. Modal coordinates are considered as a specific case of sources that have certain time structure. This structure can be identified by many BSS algorithms. However, algorithms based on second order statistics are particularly suited for the linear normal mode identification. Two well-known BSS algorithms are considered. First, Algorithm for Multiple Unknown Signals Extraction (AMUSE) is used to illustrate the similarity with Ibrahim Time Domain (ITD) modal identification method. Secondly, Second Order Blind Identification (SOBI) is used to demonstrate noise robustness of BSS-based mode shape extraction. Numerical results from these BSS algorithms and ITD method under different noise environments are provided. The Monte Carlo numerical simulations for the noisy cases are specifically used to show the robustness of BSS methods.

1 Introduction

Blind Source Separation (BSS) methods, or closely related algorithms called Independent Component Analysis (ICA), deal with recovering a set of underlying sources from observations without knowing mixing process and sources. These methods were originally developed to address signal processing problems [1] and have been shown to have tremendous applications in engineering fields. Other applications include image processing [2], biomedical data analysis [3], telecommunications [4] and stock analysis [5]. In the past several years, an increasing number of BSS or ICA applications in structural dynamics have been found in technical publications. Some examples of these are: signal separation from convoluted mixing sources for bearing diagnosis [6], robust extraction of rotating machinery signals in noisy environments [7], compression of experimental data for further damage identification [8], identification of particular machine signals from complex machinery sensor measurements [9], etc. The purpose of this paper is to study the application of BSS in the extraction of linear normal modes of a vibration system. Natural frequencies and damping ratios can be further obtained by examining each separated source.

Conventional linear modal analysis decomposes displacement matrix into mode shape matrix and modal coordinate matrix. Modal coordinates express separated fundamental oscillations, while mode shapes mathematically describe the participation of each oscillation in the output (displacement matrix). Current time domain modal parameter identification methods [10, 11, 12, 13] are based on closed-form solutions to an idealized, deterministic, linear vibratory system. However, the accuracy of results greatly depends on the validity of mathematical model of data and deteriorates drastically when noise is present. Here an alternative approach is advocated, which is to formulate the modal identification as a BSS problem, and to employ multivariate analysis to separate each independent oscillation. As a result the mixing matrix will provide mode shape information and the separated oscillations will contain the associated natural frequencies and damping ratios.

The paper is organized as follows: the general theory of BSS is briefly described in Section 2. Section 3 formulates mode shape extraction from BSS viewpoint. One of BSS algorithms, Algorithm for Multiple Unknown Signals Extraction (AMUSE), is examined in detail and compared to a well-known time domain modal analysis method, Ibrahim Time Domain (ITD) method. We show the intrinsic relationship between two different problems. Another
BSS algorithm, Second Order Blind Identification (SOBI), is used to demonstrate the noise robustness of the BSS-based mode shape extraction. In Section 4, results from numerical experiments using both ITD method and BSS algorithms are provided. Effects of noise on the quality of results are also illustrated. At the end, in Section 5, several concerns regarding practical applications are discussed and concluding remarks are given.

2 A Brief Overview of the BSS Theory

2.1 A Problem Statement

Blind Source Separation (BSS) is an emerging technique for signal processing and data analysis. Given a series of observed signals, BSS aims at recovering the underlying sources by exploiting the assumption of their mutual independence. The adjective "blind" means that very little, if anything, is known about the mixing process or sources. Depending upon the type of mixing, BSS can be classified as linear [14, 15] or nonlinear [16], where the mixtures are linear or nonlinear combinations, respectively, of the sources. Additional classifications include linear simultaneous mixing [14, 15] or convolutive mixing [17, 18], which can be further subclassified as blind deconvolution or blind equalization. BSS model considered in this paper is a linear simultaneous mixture formulated as:

\[ x = As + n, \]  

(1)

where \( x = [x_1, x_2, \ldots, x_m]^T \in \mathbb{R}^m \) is a vector containing measured scalar signals \( x_i \), \( s = [s_1, s_2, \ldots, s_n]^T \in \mathbb{R}^n \) is a vector containing original sources \( (m \geq n) \), \( A \in \mathbb{R}^{m \times n} \) is an unknown mixing matrix with full column rank and \( n \in \mathbb{R}^m \) represents addictive measurement noise. For simplicity, the discussion here is restricted to the case of \( m = n \). The goal of BSS is to find a demixing matrix \( A^{-1} \), such that \( A^{-1}x \) recover the underlying sources \( s \) using measured \( x \). A schematic illustration of BSS process is given in Fig. 1.

![Figure 1: The framework of BSS process](image)

2.2 Basic Assumptions

Due to the blindness of the problem, certain assumptions about sources are needed to proceed with the analysis. The most general assumption in BSS or ICA is to assume that sources are mutually independent. In another words, the joint probability density of the sources should be factorizable into the product of their marginal densities:

\[ p_{s_1, s_2, \ldots, s_n}(s_1, s_2, \ldots, s_n) = p_{s_1}(s_1)p_{s_2}(s_2) \cdots p_{s_n}(s_n), \]  

(2)

where \( p_{s_1, s_2, \ldots, s_n}(s_1, s_2, \ldots, s_n) \) is a joint probability density and \( p_{s_i} \) represents each marginal density. This is a theoretically strong but practically natural assumption for many applications. Secondly, sources are generally...
assumed to be non-Gaussian or to have at most one Gaussian signal. Thirdly, the mixing matrix $A$ is assumed to be full column rank but is otherwise unknown. However, these general assumptions can sometimes be relaxed when dealing with signals of specific characteristics. For example, if the sources are time signals the mutual independence can be relaxed to mutual uncorrelation. In addition, for time signals, the Gaussian distribution is also not required. Vibration modal analysis deals exactly with this type of sources.

### 2.3 Indeterminacy of the BSS

With the above assumptions BSS is solvable with two inherent ambiguities. Firstly, the order of the estimated sources is not identifiable, as any permutation of these sources is also a solution. Secondly, we can not determine the original variance of sources since in Eq. (1) $x$ is the only known variable and any scaling of sources can be canceled by inverse scaling of the associated components of matrix $A$. Therefore, generally, all sources are assumed to have unit variances.

### 2.4 Solution Strategy

ICA or BSS is essentially an optimization problem, which can be solved if appropriate objective function, constraint (e.g., unit variance of sources) and numerical algorithm are employed. The objective functions are usually chosen as some measures of non-Gaussianity or independence of the sources, such as the Kurtosis or fourth-order cumulant [19] statistics, Negentropy [20], likelihood [21], etc. The optimization methods range from the classical gradient descent method or Newton-like algorithms [22] to fast-point algorithm [20] or joint diagonalization [19, 15]. Most ICA or BSS algorithms start with preprocessing steps, which usually include centering (removal of mean values from measurements) and whitening (basically, Principal Component Analysis). They are intended to reduce noise level and avoid over-learning as well as to improve convergence properties.

When sources represent time signals we can make use of the time structure to achieve the desired source separation. To this end various methods have been developed, such as Algorithm for Multiple Unknown Signals Extraction (AMUSE) [14], Second Order Blind Identification (SOBI) [15], Temporal Decorrelation Separation (TDSEP) [23] or Time-Frequency Blind Source Separation (TFBSS) [24]. Among these methods, AMUSE, SOBI and TFBSS are quite similar in basic theory, since all of them exploit the information contained in auto- and cross-covariance matrices. TDSEP, in contrast, examines the nonstationarity of the signals and uses the auto- and cross-time-frequency distribution matrix to extract independent sources. All these four methods employ the technique of joint-diagonalization of matrices. In this paper, measured signals are assumed to be stationary, thus TDSEP is not discussed here. Since AMUSE and SOBI can be closely related with current time domain modal analysis methods and TFBSS is essentially the same as SOBI when dealing with time signals, we will concentrate on AMUSE and SOBI algorithms in the following discussion.

### 3 Mode Shape Extraction and the BSS

Modal parameter identification consists of extracting a set of natural frequencies, damping factors and mode shapes of a structure. Among these parameters, mode shapes provide mathematical description of deflection patterns of vibration when the system vibrates at one of the natural frequencies. Viewed from another point, mode shapes describe the participation of each independent oscillation in the output response.

Considering a classical linear modal analysis, governing equations of motion for a $n$-degree-of-freedom free linear vibrating system can be written as:

$$M\ddot{x} + C\dot{x} + Kx = 0,$$

where, $M$, $C$ and $K \in \mathbb{R}^{n \times n}$ are mass, damping and stiffness matrices, respectively, $\ddot{x}$, $\dot{x}$ and $x \in \mathbb{R}^{n}$ are acceleration, velocity and displacement vectors, respectively. The free decay (or transient) oscillations for proportionally or lightly damped system can be described as:

$$x(t) = \sum_{i=1}^{n} w_i a_i \exp(-\zeta_i t) \cos(\beta_i t + \varphi_i),$$

(4)
\( \zeta_i, \beta_i, \varphi_i \) represent damping ratio, natural frequency and phase angle, respectively, \( w_i, a_i \) are constants, and \( t \) is time. Eq. (4) can be written in a matrix form:

\[
x(t) = W \phi(t),
\]

where \( x(t) \) is the output vibration displacement, \( W \in \mathbb{R}^{n \times n} \) is the mode shape matrix composed of mode shapes \( w_i \), and \( \phi(t) \in \mathbb{R}^n \) is vector containing modal coordinates \( a_i \exp(-\zeta_i t) \cos(\beta_i t + \psi_i) \). Time domain modal parameter identification involves extracting mode shape matrix \( W \) and natural frequencies \( \beta_i \) and damping ratios \( \zeta_i \) contained in \( \phi(t) \) by using only the output signal \( x(t) \). Assuming the output signals are sampled every \( \Delta t \) time period and a total of \( m \) data points are recorded for each component of \( x \), then Eq. (5) can be written as:

\[
X = W \Phi,
\]

where \( X \in \mathbb{R}^{n \times m} \) is the trajectory matrix composed of the sampled components of \( x \), and \( \Phi \in \mathbb{R}^{n \times m} \) is a matrix of corresponding modal coordinates.

Similarities between the time domain modal analysis and BSS can be seen as: first, both deal with estimating the underlying components by using output data only. The modal coordinates \( \phi(t) \) are a special case of general sources \( s \) with time structure. Secondly, when each modal coordinate has different and distinct oscillation frequencies they automatically meet the requirement of uncorrelation of sources in BSS. Most BSS algorithms focus on finding the demixing matrix \( A^{-1} \) and many sophisticated algorithms robust to noise have been proposed. These algorithms could also be implemented to extract the mode shape information of a vibration system in noisy environments. Fig. 2 shows the schematic illustration of mode extraction process based on BSS. As mentioned earlier, AMUSE, SOBI and TFBSS use second order statistics by constructing the auto- and cross-covariance matrices and simultaneously diagonalizing these matrices to estimate the underlying mixing matrix. This analytical procedure is quite similar to current time domain modal analysis methods, for example Ibrahim Time Domain (ITD) method can be described as a generalized eigenvalue decomposition of measured signals’ auto- and cross-covariance matrices [25].

3.1 AMUSE algorithm and Mode Shape Extraction

Algorithm for Multiple Unknown Signals Extraction (AMUSE) was introduced by Tong, et. al in 1991 [14]. It was originally developed for overcoming the shortcoming of EFOBI [26], which can not handle Gaussian sources. AMUSE exploits the second order statistics of the mixed signals and performs an eigenvalue decomposition to the time-lagged covariance matrix. The specific procedures are:

- Centering and whitening of the mixed signal matrix to get \( Z_t, Z_t = PX_t, P \) is the whitening matrix, and subscript \( t \) is used to describe a particular time sequence.

- Calculating the eigenvalue decomposition of \( \mathbf{R}_z = \frac{1}{2}[\mathbf{R}_z + \mathbf{R}_z^T], \) where \( \mathbf{R}_z = E[Z_tZ_{t+\tau}^T] \) is a time-lagged covariance matrix, \( E \) represents expectation value, and \( \tau \) is some time delay.

- The estimated mixing matrix is obtained as \( \hat{A} = P^{-1}U \), where \( U \) is the eigenvector matrix of \( \mathbf{R}_z \) from the second step.

The whitening is usually done by performing Principal Component Analysis of the original mixed signals, \( X_t \) for instance. If the eigenvalue decomposition of the covariance matrix of \( X_t \) is \( \mathbf{R}_x = \mathbf{V} \mathbf{D} \mathbf{V}^T \), \( \mathbf{D} \) is the eigenvalue matrix.
and $V$ is the eigenvector matrix, then the whitening matrix is $P = D^{-\frac{1}{2}}V^T$ and the estimated mixing matrix is $\hat{A} = VD^{\frac{1}{2}}U$.

Looking at the procedure of AMUSE, one obvious difference between BSS methods and time domain modal analysis algorithms is that the latter do not have the centering and whitening step, while AMUSE or most BSS algorithms do. However, we say these steps are actually implicitly combined in all time domain modal analysis.

Let us examine AMUSE a little further. After centering and whitening, signal matrix becomes:

$$Z_t = D^{-\frac{1}{2}}V^T X_t$$

and

$$\hat{R}_x = \frac{1}{2}[R_x + R_x^T]$$

$$= \frac{1}{2}D^{-\frac{1}{2}}V^T[X_tX_{t+\tau}^T + X_{t+\tau}X_t^T]VD^{-\frac{1}{2}}$$

or

$$\hat{R}_x = D^{-\frac{1}{2}}V^T\hat{R}_x VD^{-\frac{1}{2}}, \quad \text{where} \quad \hat{R}_x = \frac{1}{2}(X_tX_{t+\tau}^T + X_{t+\tau}X_t^T)$$

The eigenvalue decomposition to the matrix $\hat{R}_x$ is:

$$\hat{R}_x = U\Lambda U^T,$$

Substituting Eq. (9) into Eq. (10):

$$D^{-\frac{1}{2}}V^T\hat{R}_x VD^{-\frac{1}{2}} = U\Lambda U^T,$$

Now, post-multiply both sides by $D^{-\frac{1}{2}}V^T$:

$$D^{-\frac{1}{2}}V^T\hat{R}_x VD^{-\frac{1}{2}}V^T = U\Lambda U^T D^{-\frac{1}{2}}V^T,$$

Pre-multiply both sides by $U^T$:

$$U^TD^{-\frac{1}{2}}V^T\hat{R}_x VD^{-\frac{1}{2}}V^T = \Lambda U^TD^{-\frac{1}{2}}V^T,$$

Letting $\Psi^T = U^TD^{-\frac{1}{2}}V^T$ and noticing that $VD^{-\frac{1}{2}}V^T = R_x^{-1}$, Eq. (13) becomes:

$$\Psi^T\hat{R}_x = \Lambda \Psi^T R_x,$$

or

$$\hat{R}_x\Psi = R_x\Psi\Lambda^{-1}.$$  \hspace{1cm} (15)

Now we can see $\Psi$ is actually the generalized eigenvector matrix of Eq. (15). Since $\Psi^{-T} = VD^{\frac{1}{2}}U = \hat{A}$, we say the inverse and transpose of the generalized eigenvector matrix of matrix pair $(\hat{R}_x, R_x)$ provides the mixing matrix.

This result corresponds well with ITD method formatted as a generalized eigenvalue decomposition. In fact, many well-know time domain modal analysis methods, such as Least Square Complex Exponent (LSCE) method [10], Eigenvalue Realization Algorithm (ERA) [12] and Subspace Stochastic Identification methods [27] can be formatted as a generalized eigenvalue decomposition of different matrix pairs and the generalized eigenvector matrix provides modal shape information [25].

The above demonstrates that the whitening step can be skipped by performing a generalized eigenvalue decomposition, as Eq. (15) in AMUSE. And Eq. (15) is just another form of ITD method [25]. Now we can see AMUSE and ITD method are quite similar, except that there is no centering step in ITD method or other time domain methods and the symmetric form of the time-lagged covariance matrix $R_x$ used in AMUSE and other BSS algorithms. However, we say ITD method implicitly makes the centering assumption since time signals are assumed to have a form of Eq. (4), where the means of the signals are automatically zeros. In fact, if the means of the signals are not zeros due to the noise or other effects, ITD method gives misleading results.

As for the symmetry of the cross-covariance matrix $\hat{R}_x$, this is associated with the number of the dimensions for the reconstructed signal matrix. In ITD method the analytic matrix needs to be reconstructed into $2n$ dimensions (where $n$ is the number of degrees-of-freedom) to account for the complex conjugate modal parameters. This is because ITD aims to find exact solution by taking advantage of mathematical form of the output signals. AMUSE, however, is based on statistical analysis and works with whatever the real dimension of the mixtures is. The technique
of simultaneous diagonalization the auto- and cross-covariance matrices is the essence in AMUSE or other BSS algorithms. If the cross-covariance matrix is not symmetric, not only the extracted mixing matrices are sometimes complex but also the desired simultaneous diagonalization of matrix pair by a single matrix is not possible. Essentially, for a nonsymmetric matrix pair case we have to use both the right and left generalized eigenvector matrices to realize simultaneous diagonalization. The use of symmetric format in BSS algorithms provides a possibility of using a single real diagonalization matrix to best approximate both the left and right eigenvector matrices.

3.2 SOBI and Mode shape Extraction

Second Order Blind Identification (SOBI) algorithm was introduced by Belouchrani et al in 1997 [15] and is an extension of AMUSE. It is intended to overcome the shortcoming of AMUSE when the time lag \( \tau \) is unfortunately chosen to result in two similar eigenvalues and leads to the unidentifiability of sources. Compared with AMUSE, SOBI proposes to simultaneously diagonalize several time-lagged covariance matrices with different time lags \( \tau \). Using off function to describe a diagonalization of matrix \( B \):

\[
\text{off}(B) = \sum_{1 \leq i \neq j \leq n} |B_{ij}|^2,
\]

the simultaneous diagonalization of \( p \) matrices becomes an optimization problem with respect to a matrix \( Q \) such that the sum of all the off-diagonal terms in \( \text{off}(B_i), (i = 1, \ldots, p) \) is minimum:

\[
\min_{Q} \sum_{i=1}^{p} \text{off}(Q^T B_i Q)
\]

Numerical algorithm based on Jacobi rotation technique was provided to implement the joint diagonalization in the original paper [15]. The procedures are the same as AMUSE, except the step of eigenvalue decomposition of time-lagged covariance \( \hat{R}_z \) is replaced by joint diagonalization of several time-lagged covariance matrices with different time lags \( \tau \). In addition to the reduction of the possibility of unidentifiability of \( \Psi \), SOBI appears to be much more robust to noise than AMUSE. Thus, when applied to linear normal mode identification, SOBI has similar physical explanation as AMUSE but the results are expected to be more stable and robust.

4 Numerical Examples

In this section, we apply both AMUSE and SOBI algorithms to the mode shape extraction of a linear vibration system. A three-degree-of-freedom vibration system with and without damping in the noise-free and noisy environments are examined. The noisy environment is especially emphasized to show the superiority of the BSS-based methods over the conventional time domain modal analysis method. We also consider the effect of the signal preprocessing which is used in current time domain methods.

4.1 Noise-free Vibration Systems

4.1.1 Undamped Free Vibration Case

The first example is an undamped three-degree-of-freedom discrete-parameter system vibrating under initial excitations shown in Fig. 3. The differential equation of motion for the system is:

\[
M\ddot{x} + Kx = 0, \quad \text{where} \quad M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}
\]

(18)

The initial displacements are \( x(0) = [1, 0, 0]^T \) and the initial velocities are \( v(0) = [0, 0, 0]^T \). The sampling time in the simulation is 0.1s and a total of 2000 points are used. The linear normal vibrational modes from the vibration theory are:

\[
W = \begin{bmatrix} 0.3602 & 0.7071 & 0.2338 \\ 0.5928 & 0.0000 & -0.8524 \\ 0.7204 & -0.7071 & 0.4676 \end{bmatrix},
\]

(19)
Figure 3: A three-degree-of-freedom undamped free vibration system

Figure 4: Separated sources from AMUSE and SOBI. Left column represents results from AMUSE and right column gives results from SOBI

where each column represents a modal vector which has been normalized to have unitary norm. The error of mode shape approximation is described by Euclidian distance of two mode shape vectors $E_r$, and the Modal Assurance Criterion (MAC). MAC, with a range between 0 and 1, is used to measure how close two mode shapes are. A value of 1 means two mode shapes perfectly match, while lower values indicate less similarity of mode shapes.

$$E_r = \|w_i - \bar{w}_i\|_2 \quad \text{and} \quad \text{MAC}_i = \frac{(w_i^T \bar{w}_i)^2}{(w_i^T w_i)(\bar{w}_i^T \bar{w}_i)}$$

$w_i$ and $\bar{w}_i$ represent theoretical and estimated mode vectors respectively.

In Fig. 4 we provide the separated sources or modal coordinates from AMUSE and SOBI by using simulated displacements from Eq. (18). Table 1 lists the mode shape extraction from AMUSE, SOBI, and ITD method.

<table>
<thead>
<tr>
<th></th>
<th>$E_r(MAC_1)$</th>
<th>$E_r(MAC_2)$</th>
<th>$E_r(MAC_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMUSE</td>
<td>0.0120 (0.99)</td>
<td>0.0009 (1.00)</td>
<td>0.0112 (0.99)</td>
</tr>
<tr>
<td>SOBI</td>
<td>0.0023 (1.00)</td>
<td>0.0039 (1.00)</td>
<td>0.0045 (1.00)</td>
</tr>
<tr>
<td>ITD</td>
<td>0.0000 (1.00)</td>
<td>0.0000 (1.00)</td>
<td>0.0000 (1.00)</td>
</tr>
</tbody>
</table>

Table 1: Errors for the mode shape extraction
For SOBI algorithm, we use a simultaneous diagonalization of 50 time-lagged covariance matrices with delay time from 1 sampling unit to 50 sampling units.

As expected, ITD method provides exact solution while AMUSE and SOBI give approximated results from statistical analysis. However, these approximations are acceptably good.

Theoretical natural frequencies are: 0.4209, 1.0000, 1.6801 rad/s. Estimated natural frequencies from AMUSE and SOBI are: 0.4295, 1.0124, 1.6874 rad/s. These results are from reading the peaks of Power Spectrum Density of sources.

4.1.2 Damped Free Vibration Case

When damping is present the mode shapes are generally composed of complex conjugate vectors which include both the amplitude and phase angle information. The BSS based mode shape identification, however, gives us the amplitude information since we consider the linear simultaneous mixing case. Again we use the same vibration system with the same simulation parameters as the previous case but with damping ratios $\zeta = [0.12, 0.05, 0.03]$ added as shown in Fig. 5.

![Figure 5: Three-degrees-of-freedom damped free vibration system](image)

We list the separated sources in Fig. 6. The results of mode shape extraction are given in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>$Er_1(MAC_1)$</th>
<th>$Er_2(MAC_2)$</th>
<th>$Er_3(MAC_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMUSE</td>
<td>0.0911 (0.99)</td>
<td>0.0493 (0.99)</td>
<td>0.1311 (0.98)</td>
</tr>
<tr>
<td>SOBI</td>
<td>0.0046 (1.00)</td>
<td>0.0306 (0.99)</td>
<td>0.0515 (0.99)</td>
</tr>
<tr>
<td>ITD</td>
<td>0.0001 (1.00)</td>
<td>0.0001 (1.00)</td>
<td>0.0000 (1.00)</td>
</tr>
</tbody>
</table>

Again the estimated frequencies are: 0.4295, 1.0124, 1.6874 rad/s from both BSS algorithms. The modal damping ratios are estimated as 0.1141, 0.0484, 0.029 by examining the decay rates of each separated sources.

The added damping ratios affect the performance of the BSS algorithms. AMUSE results suffer the most while SOBI looks more robust than AMUSE. ITD again provides exact solutions.

4.2 Noisy Vibration System

4.2.1 Robustness Test on Noisy Vibration Data

In this section, we apply the aforementioned algorithms to a noisy vibration data without any signal preprocessing. The output displacement responses are contaminated by Gaussian white noise. We use the same simulation setup and parameters as before. The Signal-to-Noise-Ratio (SNR) considered are 11.8323, 8.9503, 12.6611 dB for each of output channel. We run the Monte Carlo simulation 50 times and plot the result in Fig. 7. The left column represents the undamped case and the right column are for the results from damped environment. X-axis shows the simulation index and Y-axis gives the values of the errors. Table 3 gives the average error for each mode shape of these methods in the 50 Monte Carlo simulations.
Figure 6: Separated damped sources from AMUSE and SOBI. Left column represents results from AMUSE and right column gives results from SOBI.

<table>
<thead>
<tr>
<th>Method</th>
<th>$E_r(MAC_1)$</th>
<th>$E_r(MAC_2)$</th>
<th>$E_r(MAC_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamped</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AMUSE</td>
<td>0.4875 (0.81)</td>
<td>0.3069 (0.91)</td>
<td>0.1246 (0.98)</td>
</tr>
<tr>
<td>SOBI</td>
<td>0.0030 (1.00)</td>
<td>0.0043 (1.00)</td>
<td>0.0052 (1.00)</td>
</tr>
<tr>
<td>Ibrahim</td>
<td>0.8230 (0.43)</td>
<td>0.7288 (0.52)</td>
<td>0.8173 (0.44)</td>
</tr>
<tr>
<td>Damped</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AMUSE</td>
<td>0.4525 (0.83)</td>
<td>0.1674 (0.97)</td>
<td>0.2703 (0.93)</td>
</tr>
<tr>
<td>SOBI</td>
<td>0.0117 (0.99)</td>
<td>0.0346 (0.99)</td>
<td>0.0685 (0.99)</td>
</tr>
<tr>
<td>Ibrahim</td>
<td>1.2001 (0.21)</td>
<td>0.8797 (0.40)</td>
<td>1.0601 (0.31)</td>
</tr>
</tbody>
</table>

As can be seen from the results, the performance of AMUSE and ITD deteriorate obviously. While SOBI turns out to be robust to noise. The results show that ITD method seems to be the worst in the noisy simulations. This is probably because ITD relies on the exact mathematical format of output signals. When there is noise contained in the data, the mathematical model of the data does not hold any more.

Estimations of natural frequencies and damping ratios from separated sources are not listed here due to unacceptable results. This problem will be addressed by applying Maximum Likelihood Estimation (MLE) to the separated sources in a later paper.

4.2.2 Noise Reduction on the Performance of the Algorithms

Most of current time domain modal analysis methods have a common signal preprocessing step which is based on PCA. The effect of this step is to identify noisy subspaces in the signal space and project them out of the signal. Quite similarly, in many BSS algorithms when the number of the mixed signals is greater than the underlying sources PCA is also performed to identify the number of sources as well as to reduce noise. In this section, we add this PCA preprocessing step to the above BSS algorithms and ITD method for mode shape extraction. Instead of extracting mode shapes from the trajectory matrix directly, we first reconstruct a higher dimensional matrix $\tilde{X}$ by assembling
Figure 7: Comparison of the mode shape extraction in the noisy environment. Left column contains the results for the undamped case. Right column is composed of the results from the damped case. The solid lines represent the results from AMUSE. The dotted lines are from the ITD method. Stars (⋆) are for SOBI.

Then PCA is used to examine the noise floor and we project $$\tilde{X}$$ to a lower dimensional matrix $$\hat{X}$$ according to the noise floor [28]. The reconstructed dimension of the matrix $$\hat{X}$$ is chosen as 18 in the following analysis. For ITD method, we project $$\tilde{X}$$ to a dimension of 6, while for the BSS algorithms we extract 3 independent sources from this reconstructed matrix.

Fig. 8 shows the results of mode shape extraction. The left column contains the results from AMUSE and ITD method, while the right column represents results from SOBI. Numerical results are listed in Table 4.

<table>
<thead>
<tr>
<th></th>
<th>Method</th>
<th>$$E_{r_1}(MAC_1)$$</th>
<th>$$E_{r_2}(MAC_2)$$</th>
<th>$$E_{r_3}(MAC_3)$$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>AMUSE</td>
<td>0.4525 (0.83)</td>
<td>0.1674 (0.97)</td>
<td>0.2703 (0.93)</td>
</tr>
<tr>
<td></td>
<td>SOBI</td>
<td>0.0117 (0.99)</td>
<td>0.0346 (0.99)</td>
<td>0.0685 (0.99)</td>
</tr>
<tr>
<td></td>
<td>ITD</td>
<td>1.2001 (0.21)</td>
<td>0.8797 (0.40)</td>
<td>1.0601 (0.31)</td>
</tr>
<tr>
<td>Preprocessed</td>
<td>AMUSE</td>
<td>0.0439 (0.99)</td>
<td>0.0646 (0.99)</td>
<td>0.0780 (0.98)</td>
</tr>
<tr>
<td></td>
<td>SOBI</td>
<td>0.0133 (0.99)</td>
<td>0.0265 (0.99)</td>
<td>0.0251 (0.99)</td>
</tr>
<tr>
<td></td>
<td>ITD</td>
<td>0.0526 (0.99)</td>
<td>0.5043 (0.78)</td>
<td>0.2160 (0.94)</td>
</tr>
</tbody>
</table>
Figure 8: Comparison of the mode shape extraction before and after preprocessing. Left column are for AMUSE and ITD. The solid lines represent the results from AMUSE, the dotted lines are from the ITD method, circles (◦) and pluses (+) are from PCA processed data respectively. Right column is for the case of SOBI. The dotted lines represent the results from SOBI. Circles (◦) are for the results of PCA processed data.

5 Discussion and Conclusion

Applications of BSS to mode shape extraction of vibration systems was presented in this paper. BSS, as an emerging signal processing technique, has been widely used in various fields, such as speech signal processing, biomedical data analysis, machine fault diagnosis etc. The striking feature of BSS is that it can identify the underlying components of a set of mixed data without knowing the mixing process and the information on the sources. Viewing from the BSS point of view, the conventional time domain modal analysis problem can be addressed by extracting independent oscillations and mixing matrix from the output signals. The mixing matrix contains mode shape information and natural frequencies and damping ratios can be further identified from each independent oscillation. This paper focuses on the mode shape extraction from vibration systems. *Maximum Likelihood Estimation* based natural frequencies and damping ratios extraction from separated sources will be examined later. The BSS-based mode shape extraction by using observed displacement signals was investigated. In practical test, acceleration signals are more often measured. However, this should not affect the results of this study, since the acceleration signal introduces only the phase and amplitude changes in displacement signal. We can use acceleration signals to perform the BSS-based mode shape extraction directly.

Time domain modal analysis methods have certain advantages over frequency domain modal analysis methods. They use only the output response and need not to know the input, which is more desirable in the practical experiments. Moreover, no frequency analysis is performed and therefore the leakage problem associated with frequency domain methods is avoided. However, current time domain modal analysis methods rely heavily on the explicit mathematical model and are sensitive to noise. The BSS-based mode shape extraction aims at estimating the mixing matrix from statistical point of view. Compared with the conventional time domain mode shape extraction, the BSS-based method is more robust to noise.
6 Acknowledgements

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REFERENCES


