ARMAX Modal Parameter Identification for Structures Excited with Piezoceramic Actuators

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Nomenclature

\( A(q) \) Autoregressive (AR) matrix polynomial
\( A_b(q) \) Backwards estimate of AR matrix polynomial
\( a_i(q) \) Element of AR matrix polynomial
\( B(q) \) Exogenous (X) matrix polynomial
\( B_b(q) \) Backwards estimate of X matrix polynomial
\( b_i(q) \) Element of AR matrix polynomial
\( C(q) \) Moving Average (MA) matrix polynomial
\( C \) Viscous damping matrix

\( F_x[t] \) Filtered excitation vector
\( F_s \) Sampling frequency
\( f(t) \) Continuous-time excitation vector
\( f[t] \) Sampled excitation vector
\( \Delta f \) FRF line spacing
\( H_x(q) \) X matrix from stage 1 ARX model
\( H_y(q) \) AR matrix from stage 1 ARX model
\( h_k \) Parameter vector from stage 1 ARX model
\( I_s \) \( s \times s \) identity matrix
\( K \) Stiffness matrix
\( M \) Mass matrix

\( m \) Number of excitation points
\( N \) Number of samples in excitation and response vectors

\( na \) Order of AR matrix polynomial

\( nb \) Order of X matrix polynomial
\( nc \) Order of MA matrix polynomial
\( p \) Order of stage 1 AR matrix polynomial
\( q \) Backshift operator
\( R_i^{(k)} \) kth residue of the \( i \)th transfer function
\( s \) Number of response measurement points
\( T_s \) Sampling period
\( U_F \) Regression vector from stage 3 ARX model

\( v(t) \) Continuous-time displacement vector
\( w[t] \) Unmeasured disturbance (white noise)
\( y[t] \) Sampled response vector
\( z_r \) Complex pole

\( \varepsilon_1[t] \) Stage 1 ARX model error
\( \varepsilon_2[t] \) Stage 3 ARX model error

\( \zeta_r \) Modal damping for \( r \)th mode
\( \phi_k \) Normalised mode shape vector

\( \theta^{(k)} \) Parameter vector for stage 3 ARX model

\( \omega^{*}_{nr} \) Complex conjugate

\( \hat{\omega}^{(superscript)}_{na} \) Estimate
Abstract

Experimental methods used to determine the dynamic behaviour of helicopter structures can be broadly categorised as either ground testing methods or in-flight testing methods. A major limitation of ground testing is that dynamic properties determined from a grounded helicopter structure are usually significantly influenced by the boundary conditions. Therefore, it is desirable to carry out testing while the helicopter is in flight. A requirement of many testing methods is that the structure be excited with a known force, and in this study the use of multiple piezoceramic actuators to excite a structure is investigated. A multiple-input multiple-output autoregressive moving average with exogenous excitation (ARMAX) model is used for modal parameter estimation for the case where some excitation sources are not measured. The performance of this estimation technique is demonstrated with an experimental study involving a cantilever aluminium beam. Modal parameter estimates obtained from the ARMAX model are compared with results obtained from single-input single-output modal analysis with curve fitting of frequency response functions. The ARMAX estimation algorithm accurately estimates modal parameters; however, the effectiveness of piezoceramic actuators is dependent on the modal deformation in the actuator contact area. Accurate estimates of modal parameters are obtained for cases where over 200% unmeasured periodic excitations are present.

1. Introduction

Accurately determining the dynamic properties of a helicopter structure is critical for the effective maintenance of the aircraft; however, there are significant technical difficulties associated with applying existing analysis techniques. Modal analysis can be carried out on grounded helicopters with various boundary conditions, for example, suspended by the rotor-head [1-7], supported on the undercarriage [1, 2], or supported by an airmat [1]. A number of studies has shown that vibration data obtained while the helicopter is in flight are quite different from those obtained from ground tests; hence the importance of boundary conditions [7-9]. Applying a sufficiently powerful excitation force is difficult and has limited the application of input-output modal analysis methods. Another issue is the presence of significant unmeasured sources of excitation. A number of signal processing techniques has been able to identify natural frequency and damping information [9] and, to limited extent, mode shape information [7, 8] from vibration response data. The ambient vibrations of a helicopter in flight include broadband components and significant periodic components due to the main and tail rotors and drive train. The presence of periodic excitations does not satisfy the assumptions of white, or almost white ambient excitation required for many operational modal analysis techniques [10, 11]. Another arguably less important issue is that operational modal analysis techniques do not yield scaled mode shapes.

The purpose of this study was to investigate the use of system identification techniques and the use of piezoceramic actuators to overcome the difficulties discussed above. An autoregressive moving average with exogenous excitation (ARMAX) model is assumed to represent the dynamic behaviour of the helicopter structure, measurement noise, and the unmeasured periodic excitation. A multistage estimation algorithm including a method to distinguish between spurious numerical modes and vibration modes and a simple model selection criterion was used to estimate the parameters of the ARMAX model, which can then be transformed into modal parameters describing the dynamic behaviour of a structure. Estimation of the ARMAX model requires knowledge of excitation and response data and the performance of the algorithm was assessed using experimental data obtained from a cantilever aluminium beam. Significant unmeasured sources of excitation were simulated to model conditions similar to those for a helicopter in flight.

The accuracy of the ARMAX estimation algorithm is assessed using electromagnetic shakers to excite the cantilever aluminium beam and results are compared to those obtained from frequency domain curve-fitting. The use of multiple piezoceramic plates for structural excitation is also investigated.

2. ARMAX Estimation Algorithm

The dynamic behaviour of a helicopter structure is assumed to be governed by the vector differential equation [12]
\[
  \mathbf{M} \cdot \ddot{\mathbf{v}}(t) + \mathbf{C} \cdot \dot{\mathbf{v}}(t) + \mathbf{K} \cdot \mathbf{v}(t) = \mathbf{f}(t),
\]

which is a function of the displacement vector, \( \mathbf{v}(t) \) and its time derivatives, and the force vector \( \mathbf{f}(t) \). \( \mathbf{M}, \mathbf{C}, \) and \( \mathbf{K} \) are the mass, damping and stiffness matrices, respectively. The response vectors are assumed to be corrupted with measurement noise and some components of \( \mathbf{f}(t) \) are assumed to be unmeasured. The ARMAX model structure in equations (2) – (5) is used to model the behaviour of the structure, including the effects of measurement noise and unmeasured sources of excitation.

\[
  \mathbf{A}(q) \cdot \mathbf{y}[t] = \mathbf{B}(q) \cdot \mathbf{f}[t] + \mathbf{C}(q) \cdot \mathbf{w}[t],
\]

where

\[
  \mathbf{A}(q) \equiv \mathbf{I}_s + \mathbf{A}_1 \cdot q + \cdots + \mathbf{A}_{na} \cdot q^{na}, \quad [s \times s]
\]

\[
  \mathbf{B}(q) \equiv \mathbf{B}_0 + \mathbf{B}_1 \cdot q + \cdots + \mathbf{B}_{nb} \cdot q^{nb}, \quad [s \times m]
\]

\[
  \mathbf{C}(q) \equiv \mathbf{I}_s + \mathbf{C}_1 \cdot q + \cdots + \mathbf{C}_{nc} \cdot q^{nc}. \quad [s \times s]
\]

\( \mathbf{y}[t] \) is an \( s \) dimensional displacement, velocity, or acceleration response vector assumed to be corrupted with zero-mean white noise. \( \mathbf{f}[t] \) is the \( m \) dimensional measured excitation vector and both the excitation and response are assumed to be sampled at discrete times: \( t = k.T_s, k = 0, \ldots, N - 1 \), where \( T_s \) is the sampling period. The vibration response of the structure is assumed to be positively damped, linear, causal and time-invariant. The measured excitation is assumed to be persistently exciting over the chosen analysis frequency range. \( \mathbf{w}[t] \) is an \( s \) dimensional zero-mean unmeasured disturbance that is in general independent of \( \mathbf{f}[t] \) but can have correlated components. \( \mathbf{A}(q), \mathbf{B}(q), \mathbf{C}(q) \) are the autoregressive (AR), exogenous (X), and moving average (MA) matrix polynomials in terms of \( q \), the backshift operator: \( x[t].q^j = x[t-j] \). \( \mathbf{I}_s \) is the \( s \times s \) identity matrix. \( \mathbf{A}(q) \) and consequently \( \mathbf{C}(q) \) are assumed to have a diagonal structure, which allows the multiple-input multiple-output (MIMO) ARMAX model to be broken up into a series of multiple-input single-output (MISO) models, and the diagonal elements of the AR and MA matrices can be manipulated as scalar polynomials. Note that in the deterministic case, where \( \mathbf{w}[t] \) is zero or insignificant, the ARMAX model reduces to an autoregressive with exogenous excitation (ARX) model, which is a matrix-fraction description of a structure’s transfer function. The zeros of \( \mathbf{A}(q) \) are the poles of the system transfer function and describe the global properties of the structure. The transfer function zeros are the zeros of each element of \( \mathbf{B}(q) \), which are affected by the position of the excitation and response measurements. In the case where \( \mathbf{w}[t] \) is significant, the zero-mean random sequence filtered by the rational function \( \mathbf{C}(q)/\mathbf{A}(q) \), which includes the global properties of the structure, is used to model random measurement noise as well as unmeasured excitations.

The estimation algorithm is a linear multistage method based on work by Fassois [13]. A number of modifications has been made to incorporate a simple method for distinguishing between spurious numerical modes and vibrational modes, and also to select the best model from a set of models of different order.

2.1 Stage 1

The first stage estimates a higher order ARX model. This model includes the structural dynamics and also the dynamics of noise and unmeasured excitations, and is of the form

\[
  \mathbf{H}_y(q) \cdot \mathbf{y}[t] = \mathbf{H}_f(q) \cdot \mathbf{f}[t] + \mathbf{w}[t],
\]

where the coefficient matrices are derived from the AR, X, and MA matrices [13]:
The ARX model in equation (6) can be solved efficiently using least squares techniques, in particular by a method described by Ljung [14], which uses QR factorisation. Using the diagonal structure of the AR and MA matrices, equation (6) can be rewritten as $s$ regression problems, which are solved for the parameters of the MISO ARX models:

$$y^{(k)}[t] = u^T_k [t] \cdot h_k + \varepsilon^{(k)}_i [t], \quad k = 1, \ldots, s,$$  

in terms of the regression vector $u_k [t]$ and parameter vector $h_k$:

$$u_k [t] = [-y^{(k)}[t - 1] \quad -y^{(k)}[t - 2] \quad \cdots \quad -y^{(k)}[t - p]]^T \cdot f^T [t] \quad \cdots \quad f^T [t - p] ]^T,$$

$$h_k = [h^{(k)}_1(1) \quad h^{(k)}_2(2) \quad \cdots \quad h^{(k)}_p(p) : h^{(k)}_i(0) \quad \cdots \quad h^{(k)}_i(p)]^T.$$

$y^{(k)}[t], \quad H^{(k)}_r(q), \quad \varepsilon^{(k)}_i[t]$ are the $k$th rows of $y[t], \quad H_r(q)$ and $\varepsilon_i[t]$, respectively; $h^{(k)}_r(q)$ is the $k$th diagonal element of $H_r(q)$. The order $p$ of the ARX models is chosen to be approximately $5 \cdot na$; setting $na, nb, nc$ will be discussed further below.

In practice, ‘backwards’ ARX models are formed. This can be achieved by replacing the backshift operator $q$, with a forward shift operator or simply reversing the order of the excitation and response data series. The backwards ARX models are adopted because they provide a useful method to distinguish between spurious numerical modes and vibrational poles. The effect of reversing the excitation and response data is that the poles that represent positively damped vibrational modes (decaying sinusoids) become negatively damped, and the least-squares solution for the model parameters results in a minimum-phase solution where the spurious numerical poles are stable and located inside the unit circle [15, 16]. Therefore, for moderate levels of noise, the vibrational modes can be distinguished from the spurious numerical modes on the basis of the sign of damping, or equivalently, on the position of the corresponding AR matrix poles on the complex $z$-plane.

2.2 Stage 2

The second stage of the algorithm involves separating the noise dynamics from the structural dynamics. Equation (7) is rewritten as [13]

$$\sum_{j=0}^{\min(i, nc)} C_j \cdot H_y(i-j) = A_i, \quad i = 0, 1, 2, \ldots$$

Approximating $A(q) = 0$, $q > na$, a set of equations can be written for $i = r - nc + 1, \ldots, r; \quad r \geq \max(na, nc) + nc$ [13], and solved for $C(q)$. An alternative method can be used to separate the noise and structural dynamics based on the position of the poles on the $z$-plane: poles that lie inside the unit circle can be used to define the MA matrix and poles that lie outside the unit circle define the AR matrix. Note that this method actually determines $C^{-1}(q)$.

The first method has the disadvantage that it approximates $A(q) = 0$, $q > na$, however, it has been found that the
results of the estimation algorithm are not very sensitive to the first estimate of the MA matrix, which is updated in subsequent stages of the algorithm. A further disadvantage of using the first method is that the estimated MA matrix can often include unstable poles, which leads to numerical problems in subsequent stages. The MA matrix can be stabilised by calculating the zeros of each scalar polynomial in the diagonal elements of $C(q)$ and reflecting the unstable zeros about the unit circle. This is a benefit of adopting the diagonal structure in the AR and MA matrices.

2.3 Stage 3

The MA matrix obtained in stage 2 can be used to filter the excitation and response vectors. It was shown in [13] that the ARMAX model in equation (2) could be rewritten as an ARX model if the MA matrix was known. Pre-multiplying equation (2) by $C^{-1}(q)$ leads to

$$
\hat{C}^{-1}(q) \cdot y[t] = \hat{C}^{-1}(q) \sum_{j=0}^{nb} B(j) \cdot f[t-j] - \hat{C}^{-1}(q) \sum_{j=1}^{na} A(j) \cdot y[t-j] + \epsilon_2[t]
$$

(13)

Fassois [13] used the identity \( \text{col}(ABC) = (C^T \otimes A) \cdot \text{col}(B) \) [17], where \( \text{col}(\cdot) \) stacks the columns of a matrix into a vector with the first column at the top and $\otimes$ is the Kronecker product, to rewrite equation (13) in terms of a filtered excitation $F_F$ and filtered response $Y_F$:

$$
y_F[t] = \sum_{j=0}^{nb} F_F[t-j] \cdot \text{col}(B(j)) - \sum_{j=1}^{na} Y_F[t-j] \cdot \text{col}(A(j)) + \epsilon_2[t],
$$

(14)

where

$$
y_F[t] = \left( y^T[t] \otimes \hat{C}^{-1}(q) \right) \cdot \text{col}(I_s) \quad [s \times 1] \tag{15}
y_F[t] = y^T[t] \otimes \hat{C}^{-1}(q) \quad [s \times s^2] \tag{16}
$$

The definition of the Kronecker product is used to separate the MIMO ARX model in equation (13) into $s$ MISO ARX models in terms of the filtered excitation and response vectors. For example, equation (16) can be expanded:

$$
Y_F[t] = \begin{bmatrix}
y^{(1)}[t] \cdot \hat{C}^{-1}(q) & \ldots & y^{(k)}[t] \cdot \hat{C}^{-1}(q) & \ldots & y^{(s)}[t] \cdot \hat{C}^{-1}(q)
\end{bmatrix},
$$

(17)

and each element of the RHS of equation (17) is an element of the response vector filtered by the inverse of the MA matrix applied as a finite impulse response (FIR) filter:

$$
Y_F^{(k)}[t] = y^{(k)}[t] \cdot \hat{C}^{-1}(q). \tag{18}
$$

If $C(q)$ is calculated instead of $C^{-1}(q)$ in stage 2, equation (18) can be modified to apply $C(q)$ as an infinite impulse response (IIR) filter. A similar procedure is applied to the filtered excitation vector, $F_F[t]$. The ARX model in equation (14) is reformulated as $s$ MISO ARX models, in terms of $F_F[t]$ and $Y_F^{(k)}[t]$

$$
y_F^{(k)}[t] = \sum_{j=0}^{nb} F_F[t-j] \cdot \text{col}(B(j)) - \sum_{j=1}^{na} Y_F^{(k)}[t-j] \cdot A_k(j) + \epsilon_2^{(k)}[t],
$$

(19)

and equation (19) is expressed as a linear regression problem

$$
y_F^{(k)}[t] = U_F^{(k)}[t] \cdot \theta^{(k)} + \epsilon_2^{(k)}[t]
$$

(20)
using the parameter vector
\[
\theta^{(k)} = \text{col} [ A_k (1) \cdots A_k (na) \cdots B(0) \cdots B(nb)],
\]
and the regression vector
\[
U^{(k)}_f [t] = [-Y^{(k)}_f [t - 1] \cdots - Y^{(k)}_f [t - na] \cdots F_f [t] \cdots F_f [t - nb]],
\]
where \( A_k (q) \) is the \( k \)th column of \( A(q) \). Equation (20) is solved as for stage 1, using the QR least-squares method described by Ljung [14]

2.4 Stage 4

Stage 3 yields an estimate of the backwards AR and X matrices of the ARMAX model introduced in equation (2). These can be used to define the transfer function relating the measured input and output, however, the estimate of \( A(q) \) can be used to obtain a more accurate estimate of the noise model. The definition of \( H_y(q) \) in equation (7) and the definition of the convolution of two polynomials (polynomial multiplication) [18]

\[
H_y (k) = \sum_j C^{-1}(j) \cdot A(k + 1 - j), \quad j = \max(1, k + 1 - (na + 1)), \ldots, \min(k, p - na),
\]
is used to set up a system of linear equations for \( k = 1, \ldots, p \), which is solved for \( C^{-1}(q) \). This procedure is different to that used in stage 2 as both \( A(q) \) and \( H_y(q) \) are known. The improved estimate of \( C^{-1}(q) \) can be used in further iterations of stages 3 and 4 to obtain more accurate estimates of the AR and X matrices, which are then used to obtain the modal parameters describing the dynamic behaviour of the structure.

2.5 Stage 5

Before calculating the modal parameters, the backwards estimates of the AR and X matrices are transformed into forwards AR and X matrices, which correspond to the original ARMAX model in equation (2). Denoting \( A_B(q) \) and \( B_B(q) \) as the backwards AR and X matrices, equations (24) – (25) are used to transform the backwards AR and X matrices into the forwards AR and X matrices.

\[
A(q) = A_B^{-1}_{\text{na}} \cdot (A_{B_{\text{na}}} + A_B(\text{na} - 1) \cdot q^1 + \cdots + A_B(1) \cdot q^{\text{na} - 1} + I_s \cdot q^\text{na})
\]
\[
B(q) = A_B^{-1}_{\text{na}} \cdot (B_{B_{\text{nb}}} + B_B(\text{nb} - 1) \cdot q^1 + \cdots + B_B(1) \cdot q^{\text{nb} - 1} + B_B(0) \cdot q^{\text{nb}})
\]
The poles of the transfer function are evaluated by either calculating the roots of the scalar polynomials in the diagonal elements of the AR matrix or by calculating the eigenvalues of the bottom companion matrix of the AR matrix. Equations (26) and (27) are then used to calculate the modal frequencies and damping [12].

\[
\omega_{n,r} = \frac{1}{T_s} \sqrt{\ln |z_r| \cdot \ln |z_r'|},
\]
\[
\zeta_r = -\frac{\ln (|z_r| \cdot |z_r'|)}{2 \cdot \omega_{n,r} \cdot T_s},
\]

where \( z_r \) is a transfer function pole, \( z_r' \) its conjugate (\( r = 1, \ldots, \text{na} \)), and \( T_s \) is the sampling period. Transmission zeros for the transfer functions relating each excitation-response pair can be calculated by finding the roots of
each element $b_i(q)$ of the X matrix. Alternatively, the MIMO transfer function matrix can be separated into $s \times m$
scalar transfer functions and factorised into partial fraction form [19]:

$$ \frac{b_i(q)}{a_i(q)} = \sum_{k=1}^{na} \left( \frac{R_{i,k}^{(k)}}{1 - z_k \cdot q} + \frac{R_{i,k}^{* (k)}}{1 - z_k^* \cdot q} \right). $$

(28)

The residues are used to define the $k$th mode shape:

$$ \phi_k = \begin{bmatrix} 1 & R_{1,k}^{(k)} & \cdots & R_{s,k}^{(k)} \\
R_{1,k}^{(k)} & \ddots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
R_{s,k}^{(k)} & \cdots & R_{s,k}^{(k)} \end{bmatrix}^T $$

(29)

Note that the $s$ MISO models yield $s$ estimates of the global parameters and these can be averaged and the sign
of the damping is used to assess whether the mode is a vibrational mode or spurious numerical mode. When the
response measurement signal-to-noise ratio around modal frequencies is very poor, for example if a response
measurement point was made close to a node for a particular mode, the estimated modal damping is often
estimated as negative. This is due to the negative bias on damping estimated from noisy data. In this case results
at other measurement points can be checked for positive damping estimates. Also note that $j = 1, \ldots, m$
estimates
of the mode shapes are obtained from equations (28) and (29) due to multiple excitation points.

2.6 Model Order Selection

The order of the AR, X, and MA matrices is related to the number of vibrational modes in the frequency range of
interest, the units of the response measurements, and the level of noise present in the measurements. The
diagonal structure imposed on the AR matrix requires that the theoretical order (for noise free measurements) is
$na = 2n$, where $n$ is the number of vibrational modes in the analysis frequency range. Fassois and Lee [20],
showed that the order of the X matrix is dependent on the units of the response measurements, for example $nb = na$
for acceleration measurements. The order of the MA matrix is dependent on the level of noise present in the
measurements. The number of modes to be identified and the signal-to-noise ratio is, in general, not known prior
to analysis and a typical approach is to estimate a number of different models of different order. The method used
to distinguish between spurious numerical modes and vibration modes using the sign of the estimated damping
can be applied as a simple model selection criterion. The model with the smallest order (above a particular
threshold, if required) and the greatest number of positively damped poles is likely to be the most accurately
estimated model. This model selection criterion compares favourably with other tests commonly used, for
example the BIC criterion [13] or statistical tests assessing the correlation of the innovations sequence with the
excitation [17], as the innovations sequence does not need to be calculated and the number of positively damped
poles (NPDP) criterion directly assesses one of the modal parameters of interest; i.e. the modal damping.

3 Experimental Testing

Numerical tests reported in [21] showed that accurate estimates of modal parameters could be obtained from data
corrupted with up to 10% random measurement noise and 100% unmeasured periodic excitations. Experimental
data were obtained from a cantilever aluminium beam (875×50×6 mm) to further verify the performance of the
ARMAX estimation algorithm and model selection criterion. Five experiments were carried out to obtain data from
the cantilever beam, and modal parameters estimated by the ARMAX algorithm were compared with FRF curve-
fitted results from a SIMO experiment using electromagnetic shaker excitation. A summary of each experiment is
included below.
1. SIMO modal analysis using random excitation applied by an electromagnetic shaker. Modal parameters estimated from FRFs (0 – 1600 Hz, ∆f = 0.5 Hz) using a rational fraction least squares (RFLS) curve-fitting method.

2. MIMO modal analysis using independent random excitation applied by two shakers. Modal parameters estimated from time records, length 2048 samples, $F_s = 4096$ Hz, using ARMAX estimation algorithm, $na = 60$, ..., 80, $nb = na$, $nc = 4$.

3. MIMO modal analysis using three sources of independent random excitation applied by pairs of piezoceramic actuators, bonded to the beam as shown in figure 1. Modal parameters estimated by ARMAX algorithm as for experiment 2.

4. MIMO modal analysis using three measured sources of independent random excitation and one source of unmeasured periodic excitation applied by pairs of piezoceramic actuators. The unmeasured periodic excitation was made up of summed sinusoids at 200, 500, 900, and 1200 Hz. The RMS level of the summed sinusoids was 219% of the RMS level of the summed measured excitations. Modal parameters estimated by ARMAX algorithm as for experiment 2.

5. As for experiment 4, with random noise added to the summed sinusoids resulting in unmeasured periodic and random unmeasured excitations with an RMS level 207% of the RMS level of the summed measured excitations. The amplitude of the random noise was approximately 12dB below that of the summed sinusoids.

6. As for experiment 3, with independent random noise used for unmeasured excitations with an RMS level 90% of the RMS level of the summed measured excitations.

Estimated model parameters from experiment 1 are listed in table 1 and results from experiments 2 – 6 are shown in figures 2 (a) – (e).

### Table 1: Modal frequency and damping values for experiment 1 data

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.76</td>
<td>-0.08136</td>
</tr>
<tr>
<td>2</td>
<td>37.6</td>
<td>0.00681</td>
</tr>
<tr>
<td>3</td>
<td>106.06</td>
<td>0.29889</td>
</tr>
<tr>
<td>4</td>
<td>207.19</td>
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<td>5</td>
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<td>6</td>
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<td>7</td>
<td>731.73</td>
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<td>8</td>
<td>971.19</td>
<td>0.17696</td>
</tr>
<tr>
<td>9</td>
<td>1240</td>
<td>0.16836</td>
</tr>
<tr>
<td>10</td>
<td>1550</td>
<td>0.26035</td>
</tr>
</tbody>
</table>

### 4 Discussion

The curve fitted results from experiment 1 data show that the cantilever beam has well-spaced and very lightly damped transverse modes. The first mode is estimated with negative damping and it was noted that the coherence for a number of measurements was poor around the first mode. The frequency resolution of the FRFs and the small response of the first mode at many measurement points contribute to the poorly estimated damping value. Experiment 1 results were used as a basis for assessing the MIMO results in experiments 2 – 6 estimated by the ARMAX estimation algorithm. The error of estimated natural frequencies is shown in figure 2 (a). Low order modes were poorly estimated by the ARMAX algorithm due to the relatively high sampling rate. It has been reported in [22] that estimation errors in discrete-time models are magnified when transformed into a continuous model (modal model), particularly when discrete poles are very close to 1 on the complex plane; i.e. very lightly damped and at low frequencies in the analysis.
frequency band. There is a negative bias on the experiment 2 natural frequencies and this is due to the additional shaker imposing a mass loading on the structure. Experiments 3 – 6 use piezoceramic actuators, which do not add significant mass but impose additional local stiffness and these effects can be seen in the positive bias in estimated natural frequencies. The addition of unmeasured excitations in experiments 4 – 6 does not have a significant effect on the accuracy of estimated natural frequencies.

Large variations in estimated damping can be seen in figure 2(b), particularly for low order modes. It was expected that additional structural damping would be imposed by the bonded piezoceramic plates but this was not clearly reflected in the results. Damping for modes 3 – 10 in experiments 2 and 3 was less than 0.5 % and the addition of unmeasured random excitations in experiment 6 resulted in modes 6, 8, 9, and 10 being estimated with negative damping.
Mode shapes are compared using the modal assurance criterion, which is a normalised dot product of the mode shape vectors. The MIMO ARMAX models yield mode shape estimates for each excitation point and MAC values comparing results for experiments 2 - 6 with experiment 1 results are shown in figures 2 (c) – (e). Note that two sources of excitation were used in experiment 2 and three in experiments 3 – 6. Mode shapes for experiment 2 are generally good for modes 3 – 10, with slightly lower MAC for modes 9 and 10 due to a small number of poorly estimated measurement points. Results for experiments 3 – 5 show the limitation of using piezoceramic plates as actuators, however, the addition of unmeasured periodic excitations does not lead to an obvious reduction in accuracy. The addition of significant unmeasured random excitation in experiment 6 does further reduce the accuracy of mode shapes. It was reported [23] that the effectiveness of a piezoceramic actuator in exciting a particular mode is related to the curvature of that mode over the contact area of the actuator. The effects of this are indicated by the poor MAC values for mode 6, as the middle of each actuator is located over a node. The distributed moment applied by the actuators does not excite that mode effectively, compared with locating the middle of the actuator over an anti-node. These arguments also apply to the mode shape estimates for mode 10, actuator pair 3.

5. Conclusion

A time domain modal parameter estimation algorithm has been introduced, which incorporates a simple model selection criterion based on the number of positively damped modes estimated from vibration excitation and response data. The sign of estimated modal damping also allows spurious numerical poles to be distinguished from vibrational modes for moderate levels of unmeasured excitations. The algorithm was shown to successfully estimate modal parameters from MIMO excitation and response data obtained from a cantilever beam; however, low order modes were poorly identified due to the relatively high sampling rate. The use piezoceramic actuators for structural excitation was investigated and the effect of positioning the actuator was observed; actuators placed close to nodes of a particular mode did not effectively excite those modes. The addition of unmeasured excitations did not affect the accuracy of estimated natural frequencies. However, significant levels of unmeasured random excitation (such as 90%) did affect the estimation of modal damping and mode shapes. Unmeasured periodic excitations (200%), and unmeasured periodic excitations (200%) with moderate levels (6%) of unmeasured random excitations did not significantly reduce the accuracy of estimated modal parameters.

References


