Identification of the Mechanical Joint Parameters with Model Uncertainty

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ABSTRACT Joint parameter identification is a key problem in the modeling of assembled complex structural. Dynamic behavior of mechanical joints may be with random feature due to the stochastic properties of preload, joint geometry, contact surface and its finishes, etc. A procedure for stochastic mechanical joint identification based on a novel model updating technique combined with probabilistic approach is proposed in this paper. Uncertainty of the mechanical joint of assembled structures is modeled via springs and dampers with normal distributed stochastic variables. Stochastic properties of the modal parameters of the assembled structure are obtained via EMA. Mean values of the joint parameters are then identified by a novel model updating technique, which is based on a meta-model via Response Surface Method (RSM). Variances of the parameters are finally estimated by using the probabilistic approach solving an inverse uncertainty propagation problem. A case study with a real assembled frame structure is conducted to demonstrate the feasibility and effectiveness of the proposed procedure.

Key words: Joint parameter identification, model updating, model uncertainty, meta-model, response surface

1. INTRODUCTION

Finite element analysis (FEA) has become a routine practice in dynamic analysis and design of engineering structures. However, it is well known that a FE model can be erroneous due to improper model structure, uncertainty of model parameters and boundary conditions of a structure, etc. For assembled complex structures, mechanical joints are often very complicated so that they can not always be accurately modeled (such as a contact FE model) by using purely analytical approach. Hence, identifying model parameters of a mechanical joint via dynamic testing combined with FEA is an attractive and promising approach [1-4].

The joint identification procedures based on combined experimental and analytical method can be generally divided into two main categories. One [5-7] is to derive a series of equations which denote the relationship between joint parameters and modal parameters or Frequency Response Functions (FRFs) by solving the motion equations of the system. This category of procedures may be very sophisticated and resulting in large errors in damping identification [7]. The other category is based on model updating or optimization, whose feasibility and accuracy have made it an effective procedure for mechanical joint identification [8-9]. However, one drawback of model updating should be overcome when dealing with joint identification. Model updating is effective normally under condition when the parameters have relatively small error. However, initial value ("guess") of joint parameters may have large errors (>50%), and need to be updated.
In structural dynamics, a mechanical joint can be modeled via springs, dampers and other element formulations. The model for mechanical joint can be nonlinear or has stochastic features. Modeling of joint in structural dynamics has been studied extensively. Ganggaharan \cite{10} presents a probabilistic system identification method by using static response. Howard Walther \cite{11} studied on model parameter uncertainty and experimental data variability of a bolted joint. Qiao \cite{12} developed a relaxation model of joints with the consideration of time-independent uncertainty. Aumann \cite{13} proposed a method to illustrate a bolted joint by a Smallwood model, etc.

The behavior of joints may be with uncertainty due to the stochastic properties of preload, joint geometries, contact surface and its finishes, etc. Normal and tangential springs with stiffness and damping coefficient as stochastic variables can then be applied to illustrate the uncertainty of a joint. In order to model those structures with joint model uncertainty, both mean values and variances are needed in FEM of the assembled structures for the purpose of response prediction, virtual prototyping simulation, etc.

Uncertainty is a very complicated problem in structural dynamics in general. So called “lack-of-knowledge” can be caused from complexity and/or variety of the structure & measurements. Uncertainties in the computational model typically arise from variability with respect to the input/design parameters. Uncertainties in the experiment are typically due to repeatability of the experiments, instrumentation and/or environment noise, etc., or variability of test articles. It is important to clarify uncertainty propagation, both direct propagation, i.e. from input parameter to response feature, and inverse propagation, i.e. from response feature to the input parameter. Since probabilistic methods, including Monte Carlo simulation, are widely utilized for uncertainty analysis & quantification, meta-model or fast-running-model is of great importance due to large number of samples.

In this paper, a procedure for mechanical joint identification based model updating technique combined with probabilistic approach \cite{15} is proposed. Uncertainty of the mechanical joint of assembled structures is modeled via springs and dampers with normal distributed stochastic variables. Stochastic properties of the modal parameters of the assembled structure are obtained via EMA. Mean values of the joint parameters are then identified by a novel model updating technique \cite{14}, which is based on a meta-model via Response Surface Method (RSM). Standard deviations of the stochastic parameters are finally estimated by using the probabilistic approach solving an inverse uncertainty propagation problem. A case study with a real assembled frame structure is conducted to demonstrate the feasibility and effectiveness of the proposed procedure.
2. BASIC THEORY

In this study we presume that both joint parameters and response features (such as modal frequencies and damping ratios) are random variables with normal or nearly normal distributions. A flowchart of joint identification is shown in Figure 1. In this paragraph, three major issues involved in the joint identification with uncertainty analysis & propagation will be discussed.

2.1 FE modeling of mechanical joint

A FEM with mass matrix , stiffness matrix and damping matrix, denote as $M, K, C,$ is applied to model a linear structure. For a lightly damped structure, damping matrix can be assumed as viscous and proportional, i.e. $C = \beta_1 M + \beta_2 K,$ where $\beta_1, \beta_2$ are the proportional coefficients. For those structures whose damping can not be expressed as the linear combination of $M$ and $K,$ a more complicated model should be introduced. [16]

The mechanical joint of an assembled structure can be modeled via joint stiffness matrix $K\Delta,$ joint damping matrix $C\Delta$ and joint mass matrix $M\Delta.$ By solving the eigenvalue problem of the assembled structure, modal frequencies and mode shapes $(\Lambda, \phi)$ can then be obtained. The modal model of the assembled structure can then be via following equations:

$$C_r = \phi^T [C + C\Delta] \phi, \quad M_r = \phi^T [M + M\Delta] \phi$$

$$K_r = \phi^T [K + K\Delta] \phi, \quad \xi_r = C_r / 2\sqrt{M_rK_r}$$

Where $\xi_r$ denotes the modal damping ratios of the assembly.

2.2 Identification of mean values of joint parameters via RSM-based model updating

Since the stochastic joint parameters can be represented by normal distributed stochastic variables, only mean values and standard deviations are required to describe joint parameters. The model updating based Identification of mean values of joint parameters includes the following three main steps: (1) Computing response features of every design point in the space spanned by the parameters based on Design of Experiment (DOE). (2) A high order polynomial model (or other meta-models such as a artificial neural network) is constructed via regression as: $y_j(p) = f_j(p) + \epsilon_j,$ where $f_j(p)$ describes response surface (RS) model and with $p$ as input parameter, $\epsilon_j$ is the bias error of regression. (3) Identification of mean values of joint parameters by RS-based model updating technique.

The updating problem can be expressed as an optimization procedure:

$$\min_p \| R(p) \|_2^2, \quad R(p) = \{y_E\} - \{y_A(p)\}$$

subject to $p_l \leq p \leq p_u$

where $\{y_E\} , \{y_A(p)\}$ are the mean values of experimental and analytical response features, respectively. $R(p)$ is the residue, $p_l, p_u$ is the lower and upper bound of input parameter. Obviously, both $R(p)$ and $\{y_A(p)\}$ are the functions of $p$. The optimization problem can be solved as general inverse problem by the following equation:

$$\Delta p = S^+ \Delta y$$

Where $\Delta y$ is the difference of the two sequential input parameters during updating / optimization iterations,
$S^+$ is the pseudo inverse of the sensitivity matrix \cite{14} of $S$ (known as Jacobian matrix). The sensitivity matrix can be derived directly from the RS model by differentiation.

### 2.3 Estimation of the variances of stochastic joint parameters

The equation for computing variances of response features is \cite{15}:

\[
\sigma_y^2 = \sum_{i=1}^{n} (\frac{\partial y}{\partial p_i})^2 \sigma_{p_i}^2 \tag{3}
\]

where $\sigma_y^2, \sigma_{p_i}^2$ is variances of response features and joint parameters, respectively. $\frac{\partial y}{\partial p_i}$ is the first order deviation of $y$ versus $p$. Suppose the number of response features is $m$, then Equation (3) can be rewritten as:

\[
\sigma_{y_j}^2 = \sum_{i=1}^{n} (\frac{\partial y_j}{\partial p_i})^2 \sigma_{p_i}^2 \quad (j=1 \cdots m) \tag{4}
\]

Or expressed in matrix formula:

\[
\left\{\sigma_{y_j}^2\right\} = Z \left\{\sigma_{p_i}^2\right\} \tag{5}
\]

where

\[
Z = \begin{bmatrix}
(\frac{\partial y_1}{\partial p_1})^2 & (\frac{\partial y_1}{\partial p_2})^2 & \cdots & (\frac{\partial y_1}{\partial p_n})^2 \\
(\frac{\partial y_2}{\partial p_1})^2 & (\frac{\partial y_2}{\partial p_2})^2 & \cdots & (\frac{\partial y_2}{\partial p_n})^2 \\
\vdots & \vdots & \ddots & \vdots \\
(\frac{\partial y_m}{\partial p_1})^2 & (\frac{\partial y_m}{\partial p_2})^2 & \cdots & (\frac{\partial y_m}{\partial p_n})^2
\end{bmatrix}
\]

Then standard deviations of joint parameters can then be obtained via general inverse:

\[
\left\{\sigma_{p_i}^2\right\} = Z^+ \left\{\sigma_{y_j}^2\right\} \tag{6}
\]

The pseudo inverse of the matrix $Z$ can be solved by singular value decomposition (SVD).

### 3. EXPERIMENTAL CASE STUDY

A case study is conducted to answer the following two questions. The first is whether the joint parameters and the response features of structures in practical have normal distributions or not. The second question is if the linear FE model with probabilistic joint parameters can be adopted to properly describe the dynamic properties of an assembled structure.

#### 3.1 Modeling of the flange joint structure

Since bolted joints and welded joints as well as rivets are very common in aerospace structures, an assembled frame structure with flange bolted joint, shown in Figure 2, is employed in the case study. A box section beam is connected with a square plate and forms the flange. The beam is fabricated with another fixed plate with four bolted joints with a piece of viscous-elastic damping material with 3mm thickness between them.

The FE model of the structure and the position of joints are shown in Figure 3. The FE model consist of 4000 shell elements and 48 (3DOF) spring elements along with 48 (3DOF) damper in the positions denoted by “Δ” and “∗”, standing for different stiffness and damper in z-direction. Hence, there are 6 joint parameters need to
be identified. Response features are the modal frequencies and damping ratios of the first five modes.

3.2 EMA and identified results

A series of EMA is designed in order to examine the stochastic modal parameters of the structure. During the test, excitations are generated using an instrumented hammer with force transducer, and all impacts are applied at the center of the structure and at approximately the same level. Accelerations are then measured by a piezoceramic accelerometer mounted with glue at the free end. The excitation force and acceleration response are captured using HP35670A Dynamic Signal Analyzer. The modal parameters are then extracted from measured FRFs via N-Modal software.

Eight experiments were designed to assess randomness via re-assemble the frame structure with respect to each experiment. The variance due to different testing themselves is examined by comparing several results of the same assembly (error of frequencies < 0.1%, damping ratios <1%). After statistic analysis of the data, the means and the standard derivations of the response features can be obtained. It can be seen from the normal probability plot that data distribution are nearly normal (shown in Figure 4 and Figure 5).

![Fig.2 An assembled frame structure with flange bolted joint](image.png)

![Fig.3 FE model of the structure and the position of joints](image.png)

In the case study, D-optimal design with 25 runs is employed in regression with a five order polynomials. Then the mean values and standard deviations of the joint parameters are obtained using the presented procedure.

In order to verify whether the identified parameters are consistent with test results, the mean values and variances of modal frequencies and damping ratios are computed by FEA combined with Monte Carlo simulations by using the identified parameters. By comparing the Normal Probability Distribution Function (PDF) plots of the identified result with respect to the test results, it is obviously true that the distributions of modal frequencies, as well as modal damping ratios, match each other quite well(Figure 6 and Figure 7).
4. CONCLUDING REMARKS

- A procedure for mechanical joint identification based model updating technique combined with probabilistic approach \[15\] is proposed.
  - A linear FE model combined with stochastic joint parameters is adopted. Uncertainty of the mechanical joint of assembled structures is modeled via springs and dampers with normal distributed stochastic variables.
  - Stochastic properties of the modal parameters of the assembled structure are obtained via EMA.
  - Mean values of the joint parameters are then identified by a novel model updating technique \[14\], which is based on a meta-model via Response Surface Method (RSM).
  - Standard deviations of the stochastic parameters are finally estimated by using the probabilistic approach solving an inverse uncertainty propagation problem.

- A case study with a real frame structure is conducted to demonstrate the feasibility and effectiveness of the proposed procedure.
  - Experimental studies have shown that the joint parameters, i.e. stiffness and damping parameters are very close to normal distribution. Therefore, proposed procedure, which is based on normal distributed joint parameter assumption, has sound theoretical background.
  - In the case study, response surface method is employed for meta-modeling of the assembled structure. D-optimal Design of Experiment is utilized in regression with a high order polynomials.
  - The comparison of Normal Probability Distribution Function (PDF) plots of the identified result with respect to the test results in the case study has shown the feasibility and effectiveness of the procedure.

- The presented procedure can be applied to assembled complex engineering structure, even structures with non-linearity with the novel model updating technique based on a meta-model via e.g. Response Surface Method (RSM).
The accuracy of standard deviation estimation can further be improved by taking account high order Taylor series in probabilistic approach.

REFERENCES