Blind Source Separation Technique
Applied on Fault Diagnosis of Rotors’ Vibration

S M Li
College of energy and power engineering,
Nanjing University of Aeronautics and Astronautics, 210016, Nanjing
China

ABSTRACT: Some research on sound source estimation by blind source separation (BSS) have been achieved and been satisfactory. On the aspect of sound source identification of rotating machine, some research is been achieved too. How to apply blind source separation to fault character identification and improve veracity of fault diagnosis is the research content of this paper. Based on the principle of minimum mutual information, the speedup grads method that use independency component analysis (ICA) to estimate separation matrix is advanced. The realization step of the method is presented. By the method, the numerical simulation of blind source separation on vibration signal is carried out. The effect is better. Then the real rotors’ fault signals are been collected and are been analyzed by above BSS method. On the power spectrum picture of separation signals, the fault characters are been better separated. This illuminated the validity of blind source separation to rotors’ fault diagnosis.

1 INTRODUCTION

The blind source separation (BSS) of signals is a new signal identification method developed in last decade and is a new research hotspot. As an unknown and mixed state, the signals of multi-independent source first be accepted and transmit by transducer. Then the signals are been identified at the analysis terminal [1,2]. After a sort of nerve blind source separation method being advanced by Herault and Jutten [3], a lot of effective research on BSS is obtained. It is developed rapidly and popularized in the fields of communications, voice process, biomedicine and other more widely fields. A great deal of new methods and more perfect theory is advanced, and is perfected continually [4,5].

The vibration signals of mechanical facility are important information source for faults character identification and diagnosis. Usually the signal collected is a mixed signal from several vibration signals. This results in some difficulty for character identification and diagnosis. In the collection of vibration signal, the measurement results of sensor are the projection values on measurement direction of actual vibration signal. For ensure the picking-up of fault character, the setting direction of sensor should be most large projection direction of fault character signal, and the disturbing should be smaller. On the other aspect, vibration faults induce new vibration source to produce. The complexity of machine faults result in the difficulty on improving the capability of facility health inspect and on increase the accuracy rate of fault diagnosis. The research of BSS method provides a new approach for separation of source vibration signal and fault character identification. At present, the BSS research on the character identification of vibration signal is regarded but only in exploring stage. The independence component analysis (ICA) method of BSS is applied in source separation of rotors’ vibration signal for increasing the accuracy rate of diagnosis of rotors’ faults.
2 BASIC PRINCIPLE OF BLIND SOURCE SEPARATION

2.1 Universal

The blind source separation of signal is a class of methods of signal processing, it can recover mixture of several observation signals into each separate signals. Usually the observation signal is obtained from the output of sensor, and the receiving signal of sensor is unknown signal mixed from several source signals. Here the ‘blind’ indicates that the source signals cannot be observed directly and also that no information is available about the mixture. In the BSS, assuming the mutual independence of the sources compensates the lack of knowledge about the mixture and the sources. This is that assumes the existence of \( m \) statistically independent source signals \( S(n) = [s_1(n), \ldots, s_m(n)] \) and the observations at least \( m \) mixture \( X(n) = [x_1(n), \ldots, x_m(n)] \), they satisfy the equation

\[
X(n) = f [S(n), S(n-1), \ldots, S(0)] + B(n)
\]

where, \( B(n) = [b_1(n), \ldots, b_m(n)] \) denotes an additive noise which can be Gaussian or not.

The solution of equation (1) is usually to find an estimation \( Y(n) \) of source signal \( S(n) \). This is actually to find an unknown separation function for satisfying independence condition of estimation \( Y(n) \). In many fields, observation \( X(n) \) is commonly assumed to be a linear, time-independent, combination of the sources. In this case, the mixing system is called instantaneous mixture.

Since vibration of mechanical structure is delayed when it is transfer through the structure in which an instantaneous assumption does not always hold true. Usually, the model used for mechanical system is a linear time-dependent mixture, called a convolute mixture. However, the transmission delays of vibration can be neglected and an instantaneous model can be hold when the structure under investigation has a high rigidity and a small size\(^8\).

2.2 The instantaneous mixing model

If the noise in equation (1) is not considering, the instantaneous mixing model can be expressed as

\[
X(n) = AS(n)
\]

Where, \( A \) is mix matrix, both \( A \) and \( X \) is unknown. This instantaneous mixing model is also called an independent component analysis (ICA) model. The aim of ICA is to estimate a separation matrix \( W \) for satisfying equation

\[
Y(n) = WX(n) = WAS(n)
\]

Where, \( Y(n) \) is the estimation of vector \( S(n) \). The estimation of vector \( S(n) \) can be denoted by symbol \( \hat{S}(n) \), that is \( Y(n) = \hat{S}(n) \).

The ideal result is \( W = A^{-1} \). This cannot be realized if there is no any assumption. As an estimation of \( W \), ICA assumes production \( WA \) equal to diagonal matrix \( D \) which is also called permutation matrix. The model and separation block diagram of mixing system is showed as Fig. 1.
It is obvious that as long as the separation matrix \( W \) is estimated, the source signals \( S(n) \) can be estimated from equation (3). So the problem of blind signal separation is translated into a problem of searching separation matrix \( W \).

3 ACCELERATE GRADS METHOD FOR ESTIMATING SEPARATION MATRIX

3.1 Indeterminacies and uniqueness

Intrinsically, the BSS problem is confronted with two inherent ambiguities.

First it is impossible to know the original order of the source due to the insensitivity of mathematical independence to the permutation of the source. For example, assuming that \( P \) is a permutation matrix, the noise free case \((B(n) = 0)\) can be written as

\[
X = [AP^{-1}]PS
\]

(4)

where the elements of \( PS \) are the permuted original source and the mixing matrix \( AP^{-1} \) is a new mixing matrix estimated by BSS algorithm.

The second indeterminacy is that it is impossible to identify uniquely the Amplitude of the sources due to the insensitivity of mathematical independence to the scale factors applied on the source. Hence,

\[
X(n) = [AD^{-1}]DS(n)
\]

(5)

So, for an instantaneous mixture, any scalar multiplier of amplitude on one of the sources \( x_i \) can be cancelled by corresponding column of matrix \( A \). Therefore the exact values of source cannot be obtained but a class of solutions of every independency source can be obtained. For a convolution mixture, this indeterminacy of scale becomes a filtering indeterminacy and equation (2) cannot confirm \( A(z) \) uniquely (i.e. they can not be identified) but to a linear filtering \( D(z) \). The indeterminacy of BSS brings many problems to user, especially in detecting and diagnosis.

3.2 The principle minimum mutual information

For estimating separation matrix \( W \), a lot of methods are advanced. In it, minimum mutual information\(^{(9)}\) (MMI) is one of method, which is sententious and efficacious. The basic idea of MMI is to choose an appropriate power matrix \( W \) of nerve network (That is separation matrix \( W \) which is concerned) and makes the dependency each other least between each component of output \( Y \). On the ideal instance, the dependency each other are going to zero. Hence, mutual information is the scale to measure independency between random variables, and can be expressed by information entropy. The expression by mutual information entropy can express as

\[
I(W) = -H(y, W) + \sum_{i=1}^{n}H(y_i, W)
\]

(6)

Where \( H(y, W) \) is expressed as unite information entropy of \( y \) with \( W \). Under the condition of double dimension,

\[
H(y, W) = -\sum_{i=1}^{n} \sum_{j=1}^{m} p(y_i, W_j) \log p(y_i, W_j)
\]

(7)

Under the condition of more than double dimension, the entropy of vector \( y \) and its’ edge entropy can be defined as
mutual information is a nonnegative, that is \( I(W) \geq 0 \). Mutual information is the contrast function of ICA, which is proved by Comon in 1994\cite{10}. That is to express matrix \( W \) as the product of three matrixes by singular value decomposition (SVD):

\[
W = VPA^{-1}
\]

Then it can be obtained that \( I(W) = 0 \). Where \( P \) is any metathesis matrix, and \( V \) is a reversible diagonal matrix. Expressing mutual information by nonnegative entropy, which can write out as

\[
I(W) = H(y, W) - \sum_{i=1}^{n} H(y_i, W)
\]

It can be approximated by equation

\[
J_G(w) \approx c[E\{G(w)\} - E\{G(v)\}]^2
\]

In it, \( c \) is a constant, \( v \) is a gauss variable which have zero mean value and unit variance, \( w \) is a variable which have zero mean value and unit variance, \( E \) express the operation of mathematical expectation, \( G(\cdot) \) is a contrast function which is non linear and non quadratic. Usually name \( G(w) \) as \( G(w) = a w^k \). Where \( a \) is a constant factor, \( k \) is positive integer and \( k \geq 3 \).

Then the problem is translated into finding separate matrix \( W \) so that the estimate signal \( y = W^T x \) cause function \( J_G(w) \) taking maximum.

### 3.3 Realization of accelerated grads method

Define object function as\cite{11}

\[
\min \{-J_G(w)\} = -E\{G(w^T x)\}
\]

and makes it satisfy \( ||w||^2 = 1 \). Where, \( w \) is line vector of matrix \( W \).

For observation signal \( x \) that has been mixed with source and noise, the \( w \) in equation (12) is a variable, which have zero average and unit variance. That is equivalent to restrict the Norms of \( w \) to 1. It can be translated to nonrestraint optimization problem, and can be expressed as castigation function:

\[
F(w) = -E\{G(w^T x)\} + c(||w||^2 - 1)
\]

It grads is

\[
\frac{\partial F(w)}{\partial w} = -E\{x G'(w^T x)\} - \alpha w
\]

where, \( \alpha \) is a constant, and \( G' \) is the derivative differential coefficient.

The calculation approach of accelerated grads method can be summarized as below:

Step one, suppose original point \( w^1 \), allowable error \( \epsilon > 0 \), and set \( d^1 = -\partial F(w^1) / \partial w \).
Step two, when \( |\partial F(w^k)/\partial w| \leq \varepsilon \) satisfying, the calculation is stopped. Otherwise \( w^{k+1} = w^k + \lambda_k d^k \), and do Normalization

\[
w^{k+1} : = w^{k+1} / \|d^{k+1}\|
\]

For accelerating convergence, the step length \( \lambda_k \) should choose in \( \{1, \alpha^1, \alpha^2, \alpha^3, \ldots\} \) \((0<\alpha<1)\) for satisfying the equation below

\[
F(w^k + \lambda_k d^k) = \min_{\lambda} F(w^k + \lambda d^k)
\]

Step three, set

\[
d^{k+1} = -\partial F(w^{k+1}) / \partial w + \beta^k d^k
\]

In it, \( \beta^k \) is modifying coefficient, \( \beta^k = \max \left[ -\|\partial F(w^{k+1})/\partial w\|^2 / (d^k)^T \partial F(w^{k+1})/\partial w, 0 \right] \).

Step four, set \( k = k + 1 \), and turn to step two.

4 EXPERIMENTATION

4.1 Digital imitating

As a example, using a mixing matrix of two source for imitate

\[
A(z) = \begin{bmatrix} 1 & A_{12}(z) \\ A_{12}(z) & 1 \end{bmatrix}
\]

where, suppose the mixing coefficients \( A_{12} \) and \( A_{12} \) as

\[
A_{12} = [0.01, 0.08, 0.04, 0.19, 0.55, 0.48]^T,
\]

\[
A_{12} = [0.11, 0.20, 0.57, 0.48, 0.22, 0.05]^T.
\]

It may be seen that the choosing of two filter coefficients is optional obviously. Suppose two source signal is \([12]\]

\[
X_1 = \sin(2\pi 0.010n) + 0.3 \sin(2\pi 0.0010n) + 0.3 \sin(2\pi 0.075n)
\]

\[
X_2 = \sin(2\pi 0.06n) + 1.2 \sin(2\pi 0.012n) + 0.8 \sin(2\pi 0.018n)
\]

Choose \( G(w) = w^4/4, \ \alpha = 0.2, \ \varepsilon = 0.0001 \), and do iterative calculation. When the precision is satisfied, the estimation \( W \) is obtained. So the estimation of sources is obtained. The source signals, mixed signals, and separation signals in time domain are shown in Fig. 2. It can be seen from the Fig.2 that the source signals is better separated.

4.2 BSS of real vibration signals on rotor

The interested source signals can be separated when diagnosing vibration signals by BSS. This will realize health
examination on line for vibration signals, and need not stop machine running for avoiding production. For this purpose, BSS can be seen as the pretreatment step for improving diagnosis. Then the traditional method is used to inspecting and testing of fault character. For real record of measure signals, separation is quite difficulty. In the experimentation here, fore knowledge of source is used. That is artificial fault character. The recording signal is the harmonic wave of mixture by artificial source signals. These source signals are separate virtually.

On the rotor test-bed, the diameter of rotor axis is 25 mm, a disk is installed right on the symmetry central of length direction and its' mass is 1kg. On across direction, make a tiny crack near the disk. The rotor system is driven by timing electromotor. In the experiment, do some adjusting to experiment state as below:

(1) Increase the prejudicial mess on the disk at the right about of the creak.

(2) Decrease the supporting stiffness of supporting bearing on left, and makes the bearing having some sway.

(3) Change the bearing on right of axis with a fault bearing which the ball touch and rub the outside ring of the bearing.

To designed experiment system, three sensors are installed. On the vertical direction of bearing sets at both end of axis, the accelerate sensors are installed. On the vertical direction of axis at creak position, a displacement sensor is installed. The signal is collected and recorded by dynamic collection and analysis system based on computer. That is shown as Fig. 3.

The crack rotor is excited first by vibration exciter on the vertical direction of axes. And the natural frequencies are found out by the method of slowly scanning frequency. Then the rotor is derived by timing electromotor. The research before shown that when the rotate speed exceed critical speed $2580 \text{ r.p.m (43Hz)}$ and reach certain speed, the crack character of signal can be received after the collected signal come from whirlpool displacement sensor is decomposed with wavelet method. This character is the appearance of fractional spectrum peak at half and $3/2$ times the natural frequency. If do any processing to collected signal at this rotating speed, the power spectrum of the signal is shown Fig. 4. Make using of above accelerate grads method, the separation matrix $W$ can be estimated, and the source signals can be separated. The separation results are as shown in Fig. 5. Where, the same value is adopted with the example of numerical value imitation on contrast function $G(w)$, coefficient $\alpha$ and precision $\varepsilon$.

It can be seen from Fig.4 and Fig.5 that the separation matrix can be estimated better by accelerated grads method, thereby the vibration sources can be separated better. The analysis results is as below:

(1) From Fig.4 of collected signals, excitation frequency (68Hz) from rotation of motor can be seen evidently.

(2) Due to the transmitting of structure vibration, the frequency of fault character of collected signals are mixed on one power spectrum. It cannot confirm the existent faults. Under the instance of unknown faults, the every fault can difficult to diagnoses with accuracy.

(3) After BSS, the power spectrum in Fig.5 shown the separation of faults character better. Each power spectrum of
sensor signal only show one fault character ultimately. Fig.5(a) show the character of support looseness only. The rotor’s fault character of crack in Fig.5(b) and of touch and rub in Fig.5(c) are presented definitely. Compare Fig.5(c) with Fig.5(a), although the sub-spectrum characters of faults are similitude, but a peak appears at the $3/2$ times of base frequency only in Fig.5(c). Now the faults of touch and rub and support looseness are separated.

(4) The effect of rotational frequency is eliminated by BSS in Fig.5(a). But the effect of rotational frequency can not be eliminated in Fig.5(b) and Fig.5(c). The results indicate that the existence of rotational frequency have no relation to iterative times. The peaks of rotational frequency in Fig.5(b) and Fig.5(c) can not be eliminated when increasing iterative times. In fact the rotational frequency can be tested simply from the electromotor rotate speed.

5 CONCLUSION

The research above indicates that the accelerated grads method based on minimum mutual information can estimate the separation matrix better. The realization step of estimating separation matrix by independence component analysis is proved to be appropriate. The source signals can be estimated better by digital simulation.
results. The observations signals come from sensors on the real rotor that has faults are separated by BSS. Different fault characters in mixed signals are distinguished and extracted. On each power spectrum fig of sensor signal, only one fault character can be shown probably after separation. Analysis results shown that the peak of rotational excitation cannot be eliminated completely from separate results. BSS provide a new method for improving veracity of vibration diagnosis on rotational machine.

REFERENCE