Uncertainty Quantification for Structural Dynamics and Model Validation Problems

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ABSTRACT
The application of uncertainty quantification methodologies for structural dynamics requires the use of efficient and accurate analysis tools that can predict the uncertainty in a response due to uncertainties in the model formulation and input parameters. Uncertainty quantification techniques such as probabilistic methods are also being used to help develop quantitative model validation strategies. Using four different application problems—automotive crashworthiness, blast containment, spinal injury and space shuttle flowliner—this paper provides a broad overview of some techniques of applying probabilistic methods to computationally expensive structural dynamics simulations. For each application problem, a discussion on the model validation strategy used is also given.

KEYWORDS
Probabilistic, Structural Dynamics, Uncertainty Quantification, Stochastic, Model Validation

1 INTRODUCTION AND BACKGROUND
As performance requirements and testing costs for engineered systems continue to increase, computational simulation is being increasingly relied upon to serve as a predictive tool. To meet these requirements, analysts are developing higher fidelity models in an attempt to accurately represent the behavior of the physical system. It is not uncommon nowadays for these models to involve multiple physics, complex interfaces, and several million finite elements. Despite the recent extraordinary increase in computer power, analyses performed with these high fidelity models continue to take hours or even days to complete for a single deterministic analysis.

Structural performance is directly affected by uncertainties associated with models or in physical parameters and loadings. The traditional design approach has been to adopt safety factors to ensure that the risk of failure is sufficiently small, albeit not quantified. However, probabilistic analysis permits a more rigorous quantification of the various uncertainties, and ultimately will facilitate a more efficient design process. Areas in which probabilistic methods are being successfully applied include engineered or naturally occurring systems with high consequences of failure. Some of these areas include aircraft propulsion systems, airframes, biomedical...
prosthetics, weapon systems, space vehicles, pipelines, nuclear waste disposal, offshore structures and automobiles.

1.1 Uncertainty Quantification for Structural Dynamics

Probabilistic analysis requires multiple solutions of the deterministic dynamic model. Consequently, efficient and accurate probabilistic analysis methods and software tools are required. Southwest Research Institute (SwRI) has been addressing the need for efficient probabilistic analysis methods for over twenty years. Much of the reliability technology developed and implemented by SwRI researchers is available in the NESSUS probabilistic analysis software. \[1\] NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) is a general-purpose tool for computing the probabilistic response or reliability of engineered systems. The software was originally developed by a team led by SwRI as part of a NASA project entitled “Probabilistic Structural Analysis Methods (PSAM) for Select Space Propulsion Components.” \[2\] NESSUS can be used to simulate uncertainties in loads, geometry, material behavior, and other user-defined random variables to predict the probabilistic response, reliability and probabilistic sensitivity measures of engineered and naturally occurring systems.

NESSUS allows the user to perform probabilistic analysis with analytical models, external computer programs such as commercial finite element codes, and general combinations of the two. As an example, consider the problem of estimating the damage to the high-strength steel used in a containment vessel that confines high explosive experiments. In NESSUS the user can define a simulation to include 1) an explosive burn calculation to compute the pressure history at the containment wall boundary, 2) a finite element stress analysis using the computed pressure history as a load input, and 3) a cumulative damage life calculation based on the computed stresses. Each dynamic model in the simulation can include random variables. This sequentially linked hierarchy of models allows the user to quickly and easily create complex multi-physics based probabilistic simulations.

Recent work in NESSUS has been focused on reducing the time required to define complex dynamic probabilistic problems, improving support for large-scale numerical models (e.g., in excess of one million elements), and improving the robustness of the low-level probability integration routines. \[3-7\] A practical aspect of dynamic simulations is efficiently dealing with response histories, i.e., stress or strain as a function of time. Time-variant reliability methods are computationally expensive; therefore, NESSUS offers several techniques to transform the generally time-variant reliability problem to a time-invariant one. These include built-in functions for capturing maximum, minimum or RMS response values; transformations to compute whole-model features such as principal components or eigenvalues; and smoothing filters to remove unwanted harmonics. Facilities are also available for the user to define tailored functions, transformations, and filters.

1.2 Model Validation

Model verification and validation (V&V) is an enabling methodology for the development of computational models that can be used to make predictions with quantified confidence. Model V&V procedures are needed to reduce the time, cost and risk associated with component and full-scale testing of products, materials, and engineered systems. Quantifying the confidence and predictive accuracy of model calculations provides the decision-maker with the information necessary for making a risk-informed decision.

Model V&V is the primary process for quantifying and building credibility in computational models. Verification is the process of determining that a model implementation accurately represents the developer’s conceptual description of the model and its solution. Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model. \[8\] In short, verification is a math issue, whereas validation is a physics issue.

Uncertainty quantification plays a key role in model V&V. Nondeterminism refers to the existence of errors and uncertainties in the outputs of computational simulations due to inherent and subjective uncertainties in the model. Likewise, the measurements that are made to validate these simulation outputs also contain errors and uncertainties. While the experimental outcome is used as the reference for comparison, the V&V process does not presume the experiment to be more accurate than the simulation. Instead, the goal is to quantify the uncertainties in both experimental and simulation results such that the model fidelity requirements can be assessed (validation) and the predictive accuracy of the model quantified. The role of non-determinism in model V&V is more fully discussed in Ref. \[9\].
2 SELECT STRUCTURAL DYNAMICS APPLICATIONS

In the following sections, four structural dynamic problems are summarized to demonstrate the application of probabilistic analysis methods and to highlight, where possible, issues relative to model validation. All applications involve high-fidelity computationally expensive numerical models.

2.1 Stochastic Crashworthiness

In this application, the reliability is computed of a sport utility vehicle impacting a small vehicle, as shown in Figure 1. The crash simulates a frontal offset impact. The problem was conceived to identify important variables contributing to the crashworthiness reliability and to use this information to improve the design and manufacturing processes. The ultimate goal of the analysis is to improve vehicle reliability using a computational approach to reduce expensive crash testing. Additional details about this analysis can be found in Ref. [10].

Figure 1 Vehicle-to-vehicle frontal offset crash simulation model.

2.1.1 Problem Description

An LS-DYNA finite element model of a vehicle frontal offset impact and a MADYMO model of a 50th percentile male Hybrid III dummy were integrated with the NESSUS probabilistic software. A number of different response quantities from the models were used to define four occupant injury acceptance criteria and six compartment intrusion criteria. The NESSUS problem statement for the Head Injury Criteria (HIC) is shown in Figure 2. An acceleration history from the LS-DYNA vehicle model is used as the crash pulse input to the occupant injury model in MADYMO. The other three occupant injury criteria are modeled in the same fashion. The compartment intrusion criteria are determined from relative displacements of the points in the small vehicle model. These ten acceptance criteria were used as events in a probabilistic fault tree to compute the overall system reliability.

Figure 2 NESSUS problem statement for the head injury criterion (HIC).
Uncertainty inputs to the model consist of 16 random variables. These random variables include parameters that define key energy absorbing components of the vehicles such as material properties for bumpers and rails, test environment uncertainties such as impact velocity and angle, manufacturing variations in the form of rail and bumper installation parameters, and inherent uncertainty of material characteristics. Each of these random variables is characterized by a statistical distribution defined from manufacturing data, literature and/or expert opinion. The distributions for parameters that affect the geometry are based on design/manufacturing tolerances.

2.1.2 Results
A probabilistic analysis using NESSUS/LS-DYNA/MADYMO was performed for each criterion and the results are shown in Table 1. The femur axial load acceptance criteria event has the lowest reliability followed by the HIC event and the door aperture closure event. All other acceptance criteria have relatively high reliability.

<table>
<thead>
<tr>
<th>Acceptance Criteria</th>
<th>NESSUS Variable</th>
<th>Original Design</th>
<th>Final Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIC</td>
<td>g_hic</td>
<td>57.7910%</td>
<td>94.0120%</td>
</tr>
<tr>
<td>Chest acceleration</td>
<td>g_cg</td>
<td>92.2970%</td>
<td>98.8240%</td>
</tr>
<tr>
<td>Chest deflection</td>
<td>g_chestd</td>
<td>99.9752%</td>
<td>99.9999%</td>
</tr>
<tr>
<td>Femur axial load</td>
<td>g_femurl</td>
<td>46.4020%</td>
<td>92.9330%</td>
</tr>
<tr>
<td>Footrest intrusion</td>
<td>g_fri</td>
<td>99.9623%</td>
<td>100.0000%</td>
</tr>
<tr>
<td>Toepan deflection</td>
<td>g_tpd</td>
<td>100.0000%</td>
<td>100.0000%</td>
</tr>
<tr>
<td>Brake pedal location</td>
<td>g_bpd</td>
<td>100.0000%</td>
<td>100.0000%</td>
</tr>
<tr>
<td>Instrument panel def.</td>
<td>g_ipd</td>
<td>99.6870%</td>
<td>99.9719%</td>
</tr>
<tr>
<td>Door aperture closure</td>
<td>g_dac</td>
<td>72.6750%</td>
<td>98.7460%</td>
</tr>
<tr>
<td>Engine location</td>
<td>g_engd</td>
<td>99.6000%</td>
<td>99.9997%</td>
</tr>
</tbody>
</table>

The objective of the redesign analysis was to provide a recommendation to improve the reliability of the small vehicle in a vehicle-to-vehicle frontal offset impact. The approach was to rely on the probabilistic sensitivity factors to identify the dominant parameters (random variable mean and standard deviation) that will improve system reliability. The computed probabilistic sensitivity factors are shown in Figure 3. From the figure, the nominal value of the yield strength of the small vehicle rail material can be most influential in increasing the reliability.

![Figure 3 Probabilistic sensitivity factors for the stochastic crashworthiness example.](image-url)
The reliability for each acceptance criteria in the new design is listed in Table 1. The dominant event for the original design was the femur axial load acceptance criteria. The femur axial load also shows the lowest reliability for the final design but increased from a reliability of 46% to 93%. The reliability improvements are shown in Figure 4 along with a description of the parameter changes to achieve the improvement. The system reliability for the final design is 86%.

![Figure 4 Vehicle system reliability improvement study performed with NESSUS.](image)

A system reliability analysis is critical to the correct evaluation of the vehicle performance especially for evaluating the probabilistic sensitivity factors at the system level for redesign analysis. Certain parameters such as stiffness/strength parameters can improve reliability for compartment intrusion performance measures but may be detrimental to the crash pulse attenuated to the vehicle occupant. The system model correctly accounts for events with common variables (correlated events) and thus correctly identifies the important variables on the system level.

This example demonstrates how the results of a probabilistic analysis considering complex dynamics, nonlinear material behavior and large-scale contact and material deformations can be used to improve the crashworthiness of a vehicle, and the safety of the vehicle for its occupants. Specifically, by making a series of 11 specific design changes, the occupant safety is increased from under 30% to nearly 90%.

### 2.2 Blast Containment Vessel

Over the past 30 years, Los Alamos National Laboratory (LANL) has been conducting confined high explosion experiments utilizing large, spherical, steel pressure vessels. These experiments are performed in a containment vessel to prevent the release of explosion products to the environment. Design of these spherical vessels was originally accomplished by maintaining that the vessel's kinetic energy, developed from the detonation impulse loading, be equilibrated by the elastic strain energy inherent in the vessel. Within the last decade, designs have been accomplished utilizing sophisticated and advanced 3D computer codes that address both the detonation hydrodynamics and the vessel's highly nonlinear structural response. Additional details about this analysis can be found in Refs. [11,12].
2.2.1 Problem Description

The containment vessel, shown on the left side in Figure 5, is a spherical vessel with three access ports: two 16-inch ports aligned in one axis on the sides of the vessel and a single 22-inch port at the top of the vessel. The vessel has an inside diameter of 72 inches and a 2 inch nominal wall thickness. The vessel is fabricated from HSLA-100 steel, chosen for its high strength, high fracture toughness, and no requirement for post weld heat treatment. The vessel’s three ports must maintain a seal during use to prevent any release of reaction product gases or material to the external environment. Each door is connected to the vessel with 64 high strength bolts, and four separate seals at each door ensure a positive pressure seal.

A series of hydrodynamic and structural analyses of the spherical containment vessel were performed using a combination of two numerical techniques. Using an uncoupled approach, the transient pressures acting on the inner surface of the vessel were computed using the Eulerian hydrodynamics code, CTH (Sandia National Laboratories), which simulated the high explosive (HE) burn, the internal gas dynamics, and shock wave propagation. The HE was modeled as spherically symmetric with the initiating burn taking place at the center of the sphere. The vessel's structural response to these pressures was then analyzed using the DYNA3D explicit finite element structural dynamics code.

![Figure 5 Containment vessel (left) and one-quarter symmetry mesh used for the dynamic analysis (right).](image)

The simulation required the use of a large, detailed mesh to accurately represent the dynamic response of the vessel and to adequately resolve the stresses and discontinuities caused by various engineering features such as the bolts connecting the doors to their nozzles. Taking advantage of two planes of symmetry, one quarter of the structure was meshed using approximately one million hex elements. The one-quarter symmetry model is shown on the right hand side of Figure 5. The structural response simulation used an explicit finite element code called PARADYN (Lawrence Livermore National Laboratory), which is a massively parallel version of DYNA3D, a nonlinear, explicit Lagrangian finite element analysis code for three-dimensional transient structural mechanics. PARADYN was run on 504 processors of LANL’s “Blue Mountain,” massively parallel computer, which is an interconnected array of independent SGI (Silicon Graphics, Inc.) computers. The containment vessel model can be solved on the Blue Mountain computer with approximately 2.5 hours of run time. The same analysis run on a single process would have required 35 days.

Four random variables considered: radius of the vessel wall (radius), thickness of the vessel wall (thickness), modulus of elasticity (E), and yield stress ($S_y$) of the HSLA steel. A summary of the probabilistic inputs is included in Table 2. The properties for radius and thickness are based on a series of quality control inspection tests that were performed by the vessel manufacturer. The coefficients of variation for the material properties are based on engineering judgment. In this case, the material of the entire vessel, excluding the bolts, is taken to be a single random variable.
Table 2 Probabilistic inputs for the containment vessel application.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Standard Deviation</th>
<th>Mean Value</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (in)</td>
<td>Normal</td>
<td>37.0</td>
<td>0.0521</td>
<td>0.00141</td>
</tr>
<tr>
<td>Thick (in)</td>
<td>Lognormal</td>
<td>2.0</td>
<td>0.08667</td>
<td>0.04333</td>
</tr>
<tr>
<td>E (lb/in$^2$)</td>
<td>Lognormal</td>
<td>29.0E+06</td>
<td>1.0E+06</td>
<td>0.03448</td>
</tr>
<tr>
<td>S_y (lb/in$^2$)</td>
<td>Normal</td>
<td>106.0E+03</td>
<td>4.0E+03</td>
<td>0.03774</td>
</tr>
</tbody>
</table>

When the thickness and radius random variables are perturbed, the nodal coordinates of the finite element model change with the exception of the three access ports in the vessel, which remain constant in size and move only to accommodate the changing wall dimensions. This was accomplished in NESSUS using the model mapping facility where a vector of direction cosines and magnitudes are defined to describe how much and in what direction each nodal coordinate moves for a given perturbation in both thickness and radius. The NESSUS mapping procedure allows the perturbations in radius and thickness to be cumulative so these variables can be perturbed simultaneously. Once the scale factors are defined and input to NESSUS, the probabilistic analysis, regardless of method, is performed without further user intervention.

The response metric for the probabilistic analysis is the maximum equivalent plastic strain occurring over all times at the bottom of the vessel finite element model. This maximum value occurs well after the initial pulse and is caused by bending modes created by the ports.

2.2.2 Results

The AMV+ method in NESSUS was used to calculate the CDF of equivalent plastic strain. Also, Latin Hypercube Simulation (LHS) was performed with 100 samples to verify the correctness of the AMV+ solution near the mean value. The CDF is plotted on the left in Figure 6 on a standard normal probability scale. As shown, the LHS and AMV+ results are in excellent agreement. However, in contrast to the LHS solution, the AMV+ solution predicts accurate probabilities in the extreme tail regions with far fewer PARADYN model evaluations.

Probabilistic sensitivities are shown in on the right in Figure 6. The sensitivities are multiplied by $\sigma_i$ to nondimensionalize the values and facilitate a relative comparison between parameters. The values are also normalized such that the maximum value is equal to one. It can be concluded that the reliability is most sensitive to the mean and standard deviation of the thickness of the containment vessel wall.

![Figure 6 Cumulative distribution function of equivalent plastic strain plotted on standard normal scale (left) and probabilistic sensitivity factors ($u = 3$) (right).](image-url)
Because of the statistical variations in thickness and radius, the probability of exceeding 0.5% plastic strain was also computed over the complete mesh domain. The analysis entailed post-processing the results from the AMV+ probabilistic analysis, i.e., no additional PARADYN runs were required. Color contours displaying equivalent plastic strain (deterministic PARADYN results) are shown on the left in Figure 7; contours displaying the probability of exceeding 0.5% equivalent plastic strain are shown on the right of the figure. The difference in the two contour plots clearly shows the additional (and different) information the probabilistic results produce.

2.3 Cervical Spine Impact Injury

Cervical spine injuries can occur as a result of impact or from large inertial forces such as those experienced by military pilots during ejections, carrier landings, and ditchings. Other examples include motor vehicle, diving, and athletic-related accidents. Reducing the likelihood of injury by identifying and understanding the primary injury mechanisms and the important factors leading to injury motivates research in this area. [13]

Injury of the cervical spine is important due to the severe consequences associated with injuries in this region of the spine. Southwest Research Institute (SwRI) is currently developing an anatomically and kinematically correct finite element model of the cervical spine that accounts for uncertainties in geometry, material properties, loading, and boundary conditions. This work is being performed for the Naval Air Warfare Center Aircraft Division (NAWCAD) for application to military aviator safety (ejection and long-term high-G exposure)

2.3.1 Problem Description

The smallest functional unit of the spine that can reproduce fundamental important kinematics is a motion segment composed of two vertebra and an intervertebral disc. For this application, a parametric model of the C5-C6 motion segment was developed. [14] The parametric nature of the model allows different statistical populations (e.g., male or female, young or old, etc.) to be easily modeled as well as inherent variations within a given population (e.g., variations within 26-year old females) to be represented. The statistical model parameters are characterized using quantitative computed tomography (QCT) image data. Measurements from the QCT data provide information needed to characterize probability density functions for all of the model parameters. Quasi-static and dynamic response is computed using the LS-DYNA finite element software (LSTC, Version 970). Probabilistic calculations are performed with NESSUS.

A hierarchical approach, shown in Table 3, is taken to develop the cervical spine model. Unit and component models of fundamental physics under static and dynamic conditions are constructed and validated before moving on to more complex behavior. Next, these component models are assembled and validated as a motion segment sub-system model. The motion segment model validation focuses on overall kinematic behavior, interaction between components, and contact interface behavior. Finally, the motion segment components are assembled together to yield the complete cervical column.
### Table 3 Cervical spine model validation hierarchy.

<table>
<thead>
<tr>
<th>Unit Tests</th>
<th>Unit Tests (cont’d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertebral Body</td>
<td>Ligaments (ALL, PLL, LF, JCL)</td>
</tr>
<tr>
<td>Stiffness (force deflection)</td>
<td>Stiffness</td>
</tr>
<tr>
<td>Strength (inelastic behavior)</td>
<td>Failure</td>
</tr>
<tr>
<td>Failure</td>
<td>Rate</td>
</tr>
<tr>
<td>Endplate Stiffness</td>
<td>Component Tests</td>
</tr>
<tr>
<td>Endplate Strength</td>
<td>C5/C6 vertebral response</td>
</tr>
<tr>
<td>Pedicle</td>
<td>C5/C6 facet response</td>
</tr>
<tr>
<td>Stiffness</td>
<td>Disc response</td>
</tr>
<tr>
<td>Strength</td>
<td>Facet joint</td>
</tr>
<tr>
<td>Failure</td>
<td></td>
</tr>
<tr>
<td>Disc Annulus</td>
<td>Sub-System Tests</td>
</tr>
<tr>
<td>Stiffness</td>
<td>Ligament preload</td>
</tr>
<tr>
<td>Strength</td>
<td>Axial and flexion-extension coupling</td>
</tr>
<tr>
<td>Failure</td>
<td>Global kinematics</td>
</tr>
<tr>
<td>Disc Nucleus</td>
<td>Stiffness</td>
</tr>
<tr>
<td>Stiffness</td>
<td>Failure</td>
</tr>
<tr>
<td>Strength</td>
<td></td>
</tr>
<tr>
<td>Failure</td>
<td></td>
</tr>
</tbody>
</table>

At each level in the hierarchy, model predictions are compared to experiment and agreement assessed. Initially, for example, a transversely isotropic hyperelastic material model in LS-DYNA is fit to experimental ligament data as shown in Figure 8 (left side). Once all ligament material models have been established, the motion segment is subject to various loadings and compared to experimental data. A typical comparison is shown in Figure 8 (right side) for axial compression. If the agreement is insufficient, the reason for the discrepancy is determined and the validation repeated. By using the hierarchical approach to the model development, credibility is incrementally established in predictions made with the complex dynamic nonlinear cervical spine model.

![Figure 8 Ligament Material Model Relaxation (left) and C5-C6 Compression vs. Experiment (right).](image)

### 2.3.2 Results

Ligament constitutive parameters were estimated with laboratory data using LS-DYNA3D and a nonlinear optimization procedure. A C5-C6 motion segment model was constructed and validated for quasi-static flexion, extension, and compression-flexion loadings. A C2-C7 column has also been constructed. Validation and probabilistic analysis of the motion segment and cervical column are underway.
2.4 Space Shuttle Flowliner

In May of 2002, three cracks were found in the downstream flowliner at the gimbal joint in the LH$_2$ feedline at the interface with the Low Pressure Fuel Turbopump (LPFP) of Space Shuttle Main Engine (SSME) #1 of orbiter OV-104. A solution involving weld repair of all cracks and polishing all slot edges to remove manufacturing discrepancies was carried out on all orbiters. Post-flight inspections after two subsequent orbiter missions did not find any cracks in the repaired flowliners. Although the implemented solution appeared to be successful, the NASA technical investigation continued because the root cause of the original cracks had not been conclusively established.

2.4.1 Problem Description

To investigate further, the NASA Engineering Safety Center (NESC) initiated an Independent Technical Assessment (ITA) of the flowliner feedlines. Details from the NESC ITA are given in Ref. [15]. To explore the effects of multiple compounding conservative assumptions inherent to the existing deterministic fracture mechanics analysis of the flowliner, Southwest Research Institute (SwRI), working as part of a multidisciplinary NESC team, was tasked to conduct a detailed probabilistic fracture mechanics analysis. The feedline under investigation supplies liquid hydrogen (LH$_2$) fuel to the high-pressure hydrogen turbo pumps (HPHTPs) of the SSME. A cross-sectional view of the flowliner and a three-dimensional representation of the feedline (flowliner in green) are shown in Figure 10.
A goal of the SwRI probabilistic fracture mechanics analysis was to replacing conservative modeling assumptions with more rigorous analysis techniques. Further details are provided in Ref. [16]. Areas where conservative assumptions had been made included: 1) initial defect sizes due to uncertainties in inspection capability, 2) load/stress history due to uncertainties in fluid-structure interactions, transient thermal stress, and welding residual stress, and 3) rate of fatigue crack growth due to the uncertain influence of the cryogenic environment (LH$_2$ at –423°F). In addition, uncertainties in modeling the crack driving forces (needed to predict remaining fatigue life) resulted in conservative assumptions regarding crack shape, as well as other factors.

To properly treat the complex loading, geometrical features, and interaction effects, a code named Flowliner Fatigue Life (FFL) was developed by SwRI. The FFL code was used to perform fatigue crack growth calculations for eight possible crack models consisting of four corner- and four through-crack geometries (corresponding to circumferentially oriented cracks at Locations B and C on a liner slot, and axially oriented cracks at Locations A and D, see Figure 10). The stress intensity factors for the four corner crack models are subject to bivariant stress fields and are calculated using a weight function method.

Assumed cracks in the flowliners are subjected to a number of different sources of stressing: Static stresses, which remain unchanged during a flight, and cyclic stresses. Static stresses can result, for example, from system loads (such as internal pressure), and residual stresses associated with machining or welding. The cyclic stresses are due to power variations during take-off and associated resonant vibration modes set up in the flowliners. An example of a representative loading profile is shown in Figure 11. RMS stress level as a function of time is shown in Figure 11(a); this same information in a normalized space (dimensionless cavitation parameter vs. flow rate/engine speed) is shown in Figure 11(b). The stress distributions at each slot in the flowliner are different for each stress source and flight stage. The FFL software allows each of the foregoing distributions to vary from slot ligament to slot ligament.

A probabilistic damage tolerance model was developed for the flowliner by integrating the deterministic FFL software with the NESSUS probabilistic analysis code. The probabilistic model treated the following uncertainties: magnitude of the cyclic stress amplitudes for each flight stage, fatigue crack growth material properties, and defect distribution resulting from periodic in-service inspection as characterized by several candidate probability of detection (POD) curves. A new model for treating uncertainty in fatigue crack growth rates was developed and implemented based on the three regimes of fatigue crack growth. (This physically based model resulted in an enhanced characterization of uncertainty in fatigue crack growth material properties—particularly in the near-threshold regime where difficulties had been encountered in implementing prior models in probabilistic analysis due to their asymptotic nature.)
The system risk assessment takes into account the variable loading on all 38 ligaments of the downstream flowliner. The most likely crack growth scenario is where the crack begins with a corner crack and transitions to a through-crack in response to the local stress gradient. Failure of the flowliner is defined as crack growth through 95% of the ligament length at any location. NESSUS keeps track of the critical life as well as which ligament is the most critical. This approach allows the computation of the relative contribution of each slot to the total failure probability.

2.4.2 Results

The probabilistic fracture analysis was carried out for a multiple load cases and various input assumptions such as flight spectrum, POD curve, etc. Figure 12 shows typical outputs from two load spectrum assumptions. The stair-stepped nature of the CDF shown in Figure 12 (left side) is the result of the ordering of the load stages required by the earlier deterministic analysis. A full probabilistic treatment of the random loadings would not require such an ordering. Such a treatment, however, was not possible within the constraints of the ITA schedule.

![Figure 12 Cumulative distribution of required number of SSME missions and probabilistic sensitivities.](image)

Probabilistic sensitivity factors were also computed and shown in Figure 12 (right side). For this scenario, the results indicate that the loading spectrum—including the local dynamic stress amplitudes and associated gradients—is the most significant uncertainty governing the flowliner reliability. The results also indicate that both the mean value and variance of the dynamic stresses significantly influence the number of missions that can be tolerated before cracking occurs. One of the key findings from analysis of all scenarios was that the greatest potential for reducing the probability of flowliner failure is to increase the POD from 50% at 75 mils (99% at 280 mils) to 50% at 20 mils (99% at 75 mils).

3 SUMMARY AND CONCLUSIONS

Although NESSUS was initially developed for aerospace applications, the methods are broadly applicable and their use warranted in situations where uncertainty is known or believed to have a significant impact on the structural response. Uncertainty quantification, in general, is also playing a key role in the development of model verification and validation procedures. The framework of NESSUS allows the user to link advanced probabilistic algorithms with analytical equations, commercial finite element analysis programs and “in-house” stand-alone deterministic analysis codes to compute the probabilistic response or reliability of a dynamic system.

Several examples were presented that demonstrated the application of uncertainty quantification to complex dynamic problems. Issues relative to model validation were also discussed. The advanced probabilistic analysis methods in NESSUS allow for use of high-fidelity dynamic models to define the system even when each model evaluation takes hours to run. In the application problems presented, the probabilistic results revealed useful insights that would not have been available from a deterministic approach.

Progress in probabilistic mechanics relies strongly on the development of validated analysis models, systematic data collection and synthesis to resolve probabilistic inputs, and identification and classification of failure modes.
Research and development in this area is needed to improve the robustness of the underlying probability integration methods, to develop alternative uncertainty modeling approaches and integrate these approaches with established probabilistic tools, and to apply probabilistic methods to model verification and validation, system certification and prognosis, component life assessment and integrity, and structural system health monitoring and management.

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5 REFERENCES