SYSTEM IDENTIFICATION OF AN INFLATABLE HEXAPOD REFLECTOR

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ABSTRACT

This paper presents an experimental investigation of the dynamics of an inflatable/rigidizable hexapod structure with tensioned membrane. Compared to conventional structures, this structure offers several advantages, such as mass reduction, launch volume reduction, and cost reduction, for further space exploration and discovery. This 3m-diameter hexapod membrane structure was designed and built for research on modeling and vibration control of inflatable gossamer observatories. Modeling this structure is a challenging task because of the flexibility of the structure and the substantial number of modes found in the low frequency range. A series of experiments with different kinds of excitation, such as random input and sine-sweep input, are conducted to investigate the structural dynamics of this hexapod. To obtain the structural dynamic model, we apply time-domain and frequency-domain system identification techniques to experimental data collected from the hexapod.

INTRODUCTION

There is an increasing interest in large ultra-lightweight space structures for further space exploration, the applications include solar sails, large solar arrays, large aperture telescopes, and communication antennas. New technologies for reducing construction cost, weight and pre-deployment size of space structures are needed in the development of ultra-lightweight space structures. Recently, research related to these applications has been conducted in NASA's Gossamer Spacecraft initiative [1-3]. Modeling such a structure is a challenging task because of the structure's flexibility and the substantial number of modes. To date, only a few experimental studies have been conducted for the vibration of membrane structures [4,5].

The hexapod used for this study is a generic structure that incorporates design features and materials that could be used on a variety of future missions [4,6]. For vibration testing, conventional instrumentation, such as accelerometers, could greatly increase the mass of the structure, altering its structural vibration. Instead, laser vibrometry, a non-contact measurement technique, is used for the vibration testing.

Nearly all the missions envisioned for inflatable and rigidizable structural systems will benefit from vibration suppression, line of sight control and shape control systems. The design of these control systems requires accurate analytical models. The research reported in this paper was conducted to investigate how to obtain an accurate dynamic model of this structure from both time-domain and frequency-domain experimental data. A series of experiments with different kinds of excitation, such as random input and sine-sweep input, are conducted to investigate the structural dynamics of this hexapod. To obtain the identified parameters of each mode accurately from Frequency Response Function (FRF) data, sine-sweep signal in a specified narrow frequency range is used to excite the structure. A least-squares technique is developed for curve fitting FRF data to obtain the identified parameters. In the time-domain
system identification, random white noise at various magnitude levels is used to excite the structure. Models for the system are synthesized with the Eigensystem Realization Algorithm (ERA) [7] curve fitting the impulse response obtained from time-domain input/output data.

Figure 1: Hexapod structure for testing.

Figure 2: Hexapod with dimensions in Meters.
DESCRIPTION OF THE TESTBED

The assembled test-bed is shown in Figure 1. The torus and struts were fabricated from a lightly cross-linked epoxy developed by ILC-Dover, Inc. When heated above its glass transition temperature, its modulus decreases significantly allowing the entire structure to be flattened, rolled, or Z-folded into a smaller volume for packing. Hexapod components made from this material were rigidized prior to assembly.

The dimensions of the hexapod are shown in Figure 2. The torus was assembled from twelve 0.181m diameter graphite epoxy composite tubes. Rigid plastic joints, cast from glass filled urethane, connect the tubes. The struts are made from the same material as the torus and tapered from 0.0795m to 0.130m in diameter. Six struts connect the torus to a 0.0064m thick triangular primary machined from 6061-T6 Aluminum. In the center of the torus is a 2.54 x 10^-5 m thick tensioned reflective membrane made from kapton with a vapor deposited aluminized surface. It is attached to the torus with twelve cables of length 0.101m and diameter 9.65 x 10^-4m.

To minimize gravity effects on the tensioned membrane, the torus was mounted vertically on a pair of steel rods rigidly attached to an opposing pair of torus joints as shown in Figure 1. For dynamic testing, an input force was applied at a hard point (Plastic Collar) on the torus, and displacement measurements at this position were taken by using an OMETRON VH300+ single point laser vibrometer.

LEAST-SQUARES APPROACH

In this paper, a least-squares technique is developed for curve-fitting frequency domain response data. The transfer function of a single input and single output system can be written as

\[ f(s) = c + \sum_{i=1}^{n} f_i(s) \]  

where \( f_i \) is the \( i \)th component of the system transfer function. For example, \( f_i \) can be chosen as the transfer function corresponding to the \( i \)th mode. The \( i \)th component \( f_i \) is expressed as

\[
\begin{align*}
  f_i(s) &= \frac{f_{i(m+1)}s^{m_i} + \cdots + f_{i(2m+1)}}{s^m + f_{i1}s^{m_i-1} + f_{i2}s^{m_i-2} + \cdots + f_{i(m-1)}s + f_{i(m+1)}} - f_{i(m+1)} \\
  &= \frac{n_i(s)}{d_i(s)} - f_{i(m+1)} \\
  &= \frac{f_{i0}^0}{s^m + f_{i1}s^{m_i-1} + f_{i2}s^{m_i-2} + \cdots + f_{i(m-1)}s + f_{i(m+1)}} \\
  &= \frac{n_{i0}(s)}{d_i(s)}
\end{align*}
\] 

(2)
To obtain the initial value of $f_i$, the experimental data $y(j \omega_l)$ in the $i$th frequency range are used to curvefit $f_i$ and form the cost function as

$$V_i = \sum_{l=k_i+1}^{k_{i+1}} |y(j \omega_l) d_i(j \omega_l) - n_i(j \omega_l) |^2$$  

(3)

This is a linear least-squares problem of the variables $f_i$, and a unique solution can be obtained by minimizing this cost function. To get the solution of the identified parameters of transfer function, the following procedures are used. First, the initial value of constant $c$ is computed as

$$c = \frac{1}{n} \sum_{i=1}^{n} f_{i(m+1)}$$  

(4)

To compute the variables of $f_i$, the experimental data $y(j \omega_l)$ in the $i$th frequency range are used to form the cost function as

$$V_i = \sum_{l=k_i+1}^{k_{i+1}} |y(j \omega_l) - c - y_0(j \omega_l) d_i(j \omega_l) - n_{i0}(j \omega_l) |^2$$  

(5)

where

$$y_0(j \omega_l) = \sum_{k=1,k \neq i}^{n} f_k(j \omega_l)$$  

(6)

is the effect of the transfer function from other modes. The updated variables of $f_i$ are computed. Then the system cost function is computed as

$$V = \sum_{i=1}^{n} V_i(c)$$  

(7)

where $V_i(c)$ is a function of $c$. This is a nonlinear optimization problem with one variable $c$. An optimization technique can be used to find an optimal value of $c$ to minimize $V$. The Matlab program *fmins* is used to get the solution of $c$ and the identified parameters of $f_i$.

**EXPERIMENTAL RESULTS**

First, experiments are conducted to obtain FRF data, where sine-sweep excitation is used. The frequency range of interest is between 1 and 40 Hz [4]. Figure 3 shows the FRF data in the frequency range between 1 to 40 Hz for sine-sweep excitation. To get accurate identified parameters for each mode, sine-sweep excitation in a specified narrow frequency range is used. Figure 4 shows the FRF data in six specified frequency ranges, which include the dominant structural modes within 40 Hz. The least-squares technique in the preceding section is applied to curve fit FRF data. Figure 5 shows the results of the initial estimation, and the optimal results are shown in Figure 6. The initial value of $c$ is -0.0693, and the optimal value of $c$ is -0.0243. From
Figure 6 and other results, the model error is one order smaller than experimental data. Table 1 shows the results of identified parameters.

To obtain time-domain response data, Dspace system is used to generate random input force with sampling rate of 200 Hz. Models for the system are synthesized with the Eigensystem Realization Algorithm (ERA) [7] curve fitting the impulse response obtained from time-domain input/output data. The frequency range of interest is between 1 and 40 Hz. Four levels of random input are used to excite the hexapod structure. Table 2 shows the identification results when the model order in ERA is chosen as 24 (12 modes). Figure 7 shows the results of impulse response for the input excitation with gain 1.5. The mean-square root of error is about 10% of that of experimental impulse response data.

CONCLUDING REMARKS

Testing of inflatable/rigidizable structures presents many challenges such as high modal densities, tension stiffened membranes, need for non-contacting measurement techniques, and many others. Our main goal was to test, characterize, and develop analytical models for use in efforts to control the structure. Both time-domain and frequency-domain response data were used to investigate the structural dynamics of the hexapod. To identify the parameters of each mode precisely from FRF data, sine-sweep excitation is a specified narrow frequency range is used. A least-squares technique is developed to curve fit the FRF data. For both frequency-domain and time-domain experiments, the identified parameters fit the experimental data quite well with model error one order smaller than testing data. The identified parameters based on frequency-domain response data are coincident with those identified from time-domain response data.

REFERENCES

Figure 3: FRF data for sine-sweep excitation.

Figure 4: FRF data for each specified frequency range from sine-sweep excitation.
Figure 5: Identification results of initial estimation from FRF data: — experimental data, ⋯ model, - - model error.

Figure 6: Identification results of optimal estimation from FRF data: — experimental data, ⋯ model, - - model error.
Table 1: Identification results from FRF data.

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<th>Optimal</th>
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Figure 7: Impulse response for random excitation with gain of 1.5: — experimental data, ⋅⋅⋅ model, - - model error.

Table 2: Identification results from time-domain response data with various excitation gains.

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