STABILITY ANALYSIS AND APPLICATION OF THE CENTER MANIFOLD THEORY FOR A NONLINEAR SPRAG-SLIP MODEL

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ABSTRACT

This paper presents a research devoted to the study of instability phenomena in non-linear model with a constant brake friction coefficient. We consider brake vibrations and more specifically heavy truck judder. The condition of stability is based on the resolution of a generalised eigenvalue problem and the limit cycle amplitudes are determined by the center manifold reduction. A model is presented for the analysis of a non-linear sprag-slip phenomena: the goal is to study the stability analysis and to validate the center manifold theory for a non-linear model by comparing results obtained by solving the full system and by using the center manifold approach.

In this study, a non-linear stiffness is considered. This non-linearity is expressed as a polynomial with quadratic and cubic terms. The model does not require the use of brake negative coefficient, to predict the instability phenomena. The center manifold approach is used to obtain equations for the limit cycle amplitudes. The brake friction coefficient is used as unfolding parameter of the fundamental Hopf bifurcation point. The analysis shows that stable and unstable limit cycles can exist for a given constant brake friction coefficient.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>( C )</td>
<td>damping matrix</td>
</tr>
<tr>
<td>( K )</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>( M )</td>
<td>mass matrix</td>
</tr>
<tr>
<td>( a_{k,ij} )</td>
<td>coefficients of the center manifold</td>
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<td>( f_{ji}^{(2)} )</td>
<td>coefficients of quadratic non-linear terms</td>
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<td>( f_{ijk}^{(3)} )</td>
<td>coefficients of cubic non-linear terms</td>
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<tr>
<td>( \otimes )</td>
<td>Kronecker product</td>
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<tr>
<td>( x )</td>
<td>vector of displacement</td>
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<tr>
<td>( \dot{x} )</td>
<td>vector of velocity</td>
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<tr>
<td>( \ddot{x} )</td>
<td>vector of acceleration</td>
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<tr>
<td>( x_0 )</td>
<td>equilibrium point</td>
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<tr>
<td>( \varepsilon )</td>
<td>small perturbation</td>
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<td>( \mu )</td>
<td>brake friction coefficient</td>
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<tr>
<td>( \eta_{ji}^{(2)} )</td>
<td>quadratic non-linear terms in state variables</td>
</tr>
<tr>
<td>( \eta_{ijk}^{(3)} )</td>
<td>cubic non-linear terms in state variables</td>
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1 INTRODUCTION

The customers’ requests induce to consider the optimisation of all elements of structure and the dynamic design of products becomes one of the most important factors for manufacturers. Of course they developed robust softwares to solve differential-algebraic equations corresponding to systems including several nonlinearities. The time history response solutions of the full set of non-linear equations can determine the vibration amplitude but are both time consuming and costly to perform when extensive parametric design studies are needed. For this reason, an understanding of the behaviour of systems with many degrees of freedom requires simplification methods to reduce the order of the system of equations and/or eliminate as many nonlinearities as possible in the system of equations. Considerable work has been devoted to carry out explicit reductions: Perturbation methods (Nayfeh [1]), normal form approach (Guckenheimer [2], Jezequel and Lamarque [3] and looss [4]) and center manifold approach (Nayfeh [1], Guckenheimer [2] and Knoblock [5]). The center manifold approach can be compare as a simplification method that reduces the number of equations of the original system to obtain a simplified system without losing the dynamics of the original system and without losing contributions of non-linear terms.

In this present paper, we applied the center-manifold reduction to a self-excited system with many-degrees-of-freedom containing quadratic and cubic non-linear terms that characterise the modelling of heavy trucks judder.

2 FRICTION INDUCED VIBRATION

A serious difficulty to study the stability analysis is that the dynamic stability of a brake system depends on a number of
factors such as friction coefficient, mechanical interaction and stiffness for example. As a result, much effort has gone into determining models and mechanisms to predict friction induced vibrations. Recent years have seen a greater concentration of work on the brake noise and vibration. Although there have been no uniformly accepted theory to characterise the problem and various types of vibrations have been investigated, such as disk brake squeal (Chambrette [6], Moiriot [7], North [8], Earles [9], Millner [10]), aircraft brake squeal (Liu and Ozbek [11]) and railway wheel squeal (Rudd [12]).

One of the most important phases in study of brake systems is the determination of the mechanism of the unstable friction induced vibration in brake systems. There is no unique mathematical model and theory to explain the mechanisms and dynamic phenomena associated with friction. According to Ibrahim [13-14] and Crolla [15], there are four general mechanisms for friction-induced system instability and more specifically friction-induced vibration in disc-brake systems: stick-slip, variable dynamic friction coefficient, sprag-slip and coupling mechanism. The first two approaches rely on changes in the friction coefficient with relative sliding speed to affect system stability. The latter two approaches use kinematic constraints and modal coupling to develop the instability. In these cases, instability can occur with a constant brake friction coefficient.

Effectively, there are many types of brake vibration problem with various phenomena. Specialists as Crolla and Lang [15] split them into three headings: disc brake noise, brake judder and brake drum noise.

Generally, brake noises are divided into categories according to the sound frequency. On the basis of previous brake experiments, there are many types of brake noises with varying phenomena as squeal noise, groan noise, judder noise, squech noise, pinch-out noise. Squeal noise and groan noise are the two important phenomena of brake noise. Technically speaking, noise is the result of a self-excited oscillation or dynamic instability of the brake. Squeal is accepted as being the result of such instabilities. For example, squeal can be due to a resonance of drums, rotors or back plates. The frequency spectrum of squeal is in the 1 – 10 kHz ranges. In contrast to squeal, groan occurs at very slow vehicle speed. It is caused by stick-slip at the rubbing surface. The frequency spectrum of groan is in the 10 – 300 Hz ranges.

Unlike brake noise, judder is a lower frequency vibration that is generally felt rather than heard. Judder is defined as a forced vibration. To find a solution to this friction-induced vibration and minimize vibration, the effect of suspension and vehicle body dynamics on the transmission of judder to the driver are being investigated. The frequency spectrum of judder vibration is in the 10 – 100 Hz ranges.

3 JUDDER MODELING

Friction induced vibration has been described and analysed in a number of published studies of various degrees of sophistication. In a previous work [16], Boudot presents heavy trucks judder where the dynamic characteristics of the whole front axle assembly is concerned, even if the source of judder is located in the brake system. The modelling uses a structural phenomenon by introducing sprag-slip mechanism based on dynamic coupling due to buttressing.

The dynamic behaviour of the braking system is expressed by two free-free modes of the structure: the first mode \( (k_2, m_2) \) is tangential to the friction contact and the second mode \( (k_1, m_1) \) is normal to the friction contact.

In the case of the grabbing of brake system, \( k_2 \) and \( m_2 \) defined the torsional mode of the front axle excited by the tangential forces of the disc. The normal forces provide by the brake control whose dynamic behaviour is described by the second mode \( (k_1, m_1) \). Consequently, tangential and normal degree-of-freedom are coupled only induced by friction forces. This expresses the braking system contribution.

In order to simulate braking system placed crosswise because of overhanging due to static force effect, we may consider the moving belt slopes with an angle \( \theta \). This slope couples the normal and tangential degree-of-freedom only induced by the brake friction coefficient. This consideration called sprag-slip is based on dynamic coupling due to buttressing motion. Moreover, we consider the effect of braking force that is an important parameter in friction-induced vibration. The force \( F_{brake} \) transits through the braking command that have non-linear behaviour. The dynamic system is modelled here as a three-degrees-of-freedom system: translational and normal displacement in the frictional \( x \)-direction of the mass \( m_2 \) defined by \( X(t) \) and \( Y(t) \), respectively and the translational displacement in the \( y \)-direction of the mass \( m_1 \) defined by \( y(t) \).
Therefore, we consider the possibility of having a non-linear contribution. Then, we express this non-linear stiffness as a quadratic and cubic polynomial in the relative displacement:

\[ k = k_{11} + k_{12} \Delta + k_{13} \Delta^2 \]  \hspace{1cm} (1)

Where \( \Delta \) is the relative displacement between the translational displacement in the \( y \)-direction of the mass \( m_1 \) defined by \( y(t) \) and the normal displacement in the frictional \( x \)-direction of the mass \( m_2 \) defined by \( Y(t) \). This nonlinearity is applied to show the influence and importance of non-linear terms to understand the dynamic behaviour of systems with non-linear phenomena and to predict dangerous or favourable conditions and exploit the full capability of structures by using system in the non-linear range.

We assume that the tangential force \( T \) is generated by the brake friction coefficient \( \mu \), considering the Coulomb’s friction law:

\[ T = \mu N \]  \hspace{1cm} (2)

Finally, the three equations of motion can be expressed:

\[
\begin{align*}
    m_1 \ddot{y} + c_1 (\dot{y} - \dot{Y}) + k_{11} (y - Y) + k_{12} (y - Y)^2 + k_{13} (y - Y)^3 &= F_{\text{brake}} \\
    m_2 \ddot{X} + c_2 \dot{X} + k_2 X &= -N \sin \theta + T \cos \theta \\
    m_2 \ddot{Y} + c_1 (\dot{Y} - \dot{y}) + k_{11} (Y - y) + k_{12} (Y - y)^2 + k_{13} (Y - y)^3 &= N \cos \theta + T \sin \theta
\end{align*}
\]  \hspace{1cm} (3)

By using the transformations \( y = X \tan \theta \) and \( x = \{X \ Y\}^T \), and considering the Coulomb’s friction law \( T = \mu N \), the non-linear 2-degrees-of-freedom system has the form

\[
\begin{bmatrix} M \end{bmatrix} \{\ddot{x}\} + \begin{bmatrix} C \end{bmatrix} \{\dot{x}\} + \begin{bmatrix} K \end{bmatrix} \{x\} = \{F\} + \{F_{\text{nonlinear}}\} \hspace{1cm} (4)
\]

Where \( \{\ddot{x}\} \), \( \{\dot{x}\} \) and \( \{x\} \) are the acceleration, velocity, and displacement response 2-dimensional vector of the degrees-of-freedom, respectively. \( [M] \) is the mass matrix, \( [C] \) is the damping matrix and \( [K] \) is the stiffness matrix. \( \{F\} \) is the vector force due to brake command and \( \{F_{\text{nonlinear}}\} \) contains moreover the non-linear stiffness terms. We have

\[
[M] = \begin{bmatrix} m_2 \tan^2 \theta + l & 0 \\ 0 & m_1 \end{bmatrix} \hspace{1cm} (5)
\]

\[
[C] = \begin{bmatrix} c_1 (\tan^2 \theta - \mu \tan \theta) + c_2 (1 + \mu \tan \theta) & c_1 (-\tan \theta + \mu) \\ -c_1 \tan \theta & c_1 \end{bmatrix} \hspace{1cm} (6)
\]

\[
[K] = \begin{bmatrix} k_{11} (1 + \mu \tan \theta) + k_{12} (\tan^2 \theta - \mu \tan \theta) & k_{11} (-\tan \theta + \mu) \\ -k_{11} \tan \theta & k_{11} \end{bmatrix} \hspace{1cm} (7)
\]

The general form of the equation of motion for the nonlinear judder model can be expressed in the following way:

\[
\begin{aligned}
&\left[ M \right] \{\ddot{x}\} + \left[ C \right] \{\dot{x}\} + \left[ K \right] \{x\} = \{F\} + \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}^T x_i x_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} f_{ijk}^T x_i x_j x_k
\end{aligned}
\]  \hspace{1cm} (8)

where \( f_{ij}^T \) and \( f_{ijk}^T \) are coefficients of quadratic and cubic non-linear terms, respectively. \([M] \), \([C] \) and \([K] \) are \( 2 \times 2 \) matrices.

### 4 METHODOLOGY

The study can be divided into two parts:

- The first step is the static problem: the steady-state operating point for the full set of non-linear equations is obtained by solving them for the equilibrium point. We obtain the linearized judder equations of motion by introducing small perturbations about the equilibrium point into the non-linear equations. Stability is investigated by determining eigenvalues of this linearized equations for each steady-state operating point of the non-linear system.

- The second step is the estimation of the limit cycle. The non-linear dynamic equations can be integrated numerically to obtain a time-history response and this way the limit cycle. However, this procedure is too much time consuming. So the center manifold reduction is used to obtain equations for the limit cycle. This approach simplifies the dynamics on the center manifold by reducing the order of the dynamical system while retaining the essential features of the dynamic behavior near the Hopf bifurcation point.

#### 4.1 Stability Analysis

Stability of the system is investigated on the linearized judder equations by assuming small perturbations \( \{\bar{x}\} \) about the equilibrium point \( \{x_0\} \) of the nonlinear system:

\[
\{x\} = \{x_0\} + \{\bar{x}\}
\]  \hspace{1cm} (9)

where the equilibrium point \( \{x_0\} \) is given by solving the nonlinear static equations for a given net brake hydraulic pressure:
Substituting Eq. (11) in the nonlinear judder system (10) and neglecting higher order terms, we obtain the linearized judder system:

\[
[M]\ddot{\mathbf{x}} + [C]\dot{\mathbf{x}} + [K]\mathbf{x} = \{F_{\text{nonlinear}}(\mathbf{x})\}
\]  

(13)

where

\[
F_{\text{nonlinear}}(\mathbf{x}) = \left. \frac{\partial F_{\text{nonlinear}}}{\partial \mathbf{x}} \right|_{\mathbf{x}_0}
\]

(14)

Considering \( \lambda \) the system eigenvalue of the judder equations Eq.(13). It can be expressed as

\[
\lambda = a + i b
\]

(15)

where \( a \) is the real part, and \( b \) is the imaginary part of the eigenvalue, respectively.

If \( a \) is negative or zero, the system is stable and we don’t have vibration. If \( a \) is positive, we have an unstable root and whirl vibration. Therefore, \( b \) represents frequency of the unstable mode.

Computations are conducted with various brake friction coefficients. The Hopf bifurcation point is detected for \( \mu_0 = 0.36 \).

A representation of the evolution of frequencies against brake friction coefficient is given in Figure 2. We show the coalescence for \( \mu = \mu_0 \) of two imaginary parts of the eigenvalues (frequency about 50 Hz). In Figure 3, the associated real parts are plotted.

For \( \mu = \mu_0 \), there is one pair of purely imaginary eigenvalues. All other eigenvalues have negative real parts. As illustrated in Figure 3, the system is unstable for \( \mu > \mu_0 \).

For \( \mu < \mu_0 \), the system is stable.

This stability analysis indicates that system instability can occur with a constant friction coefficient.

Moreover, frequency \( \omega_0 \) of the unstable mode obtained for \( \mu = \mu_0 \) is near 55 Hz. There is a perfect correlation with experiments tests where judder vibration is observed in the 40-70 Hz range.

\[ K.\{x_0\} = \{F\} + \{F_{\text{nonlinear}}(x_0)\} \]  

(12)

4.2 Complex nonlinear problem

Stability analyses were investigated by determining eigenvalues of the linearized perturbation equations about each steady-state operating point.

In order to obtain time-history responses, the complete set of nonlinear dynamic equations may be integrated numerically. But this procedure is time consuming when parametric design studies are needed. So it is necessary to use nonlinear analysis: we will present the center manifold approach to obtain equations for the limit cycle amplitude.

In order to use the center manifold approach, we write the nonlinear judder equation in state variables.

\[
\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \sum_{i=1}^{2n} \sum_{j=1}^{2n} \eta_{ij}^H y_i y_j + \sum_{i=1}^{2n} \sum_{j=1}^{2n} \sum_{k=1}^{2n} \eta_{ijk}^H y_i y_j y_k
\]

(16)

where
\[ y = \begin{bmatrix} \bar{x} \\ \bar{x} \end{bmatrix} \]

\[ \dot{y} = \begin{bmatrix} \bar{x} \\ \bar{x} \end{bmatrix} \]

\[ A = \begin{bmatrix} C & M \\ I & 0 \end{bmatrix}^{-1} \begin{bmatrix} K & 0 \\ 0 & I \end{bmatrix} \]

\[ \eta_{(2)} = \begin{bmatrix} C & M \\ I & 0 \end{bmatrix}^{-1} \begin{bmatrix} f_{(2)}(\mu) \\ 0 \end{bmatrix} \]

\[ \eta_{(3)} = \begin{bmatrix} C & M \\ I & 0 \end{bmatrix}^{-1} \begin{bmatrix} f_{(3)}(\mu) \\ 0 \end{bmatrix} \]

\[ \eta^{ij}_{(2)} \text{ and } \eta^{ij}_{(3)} \text{ are quadratic and cubic non-linear terms of the state variables, respectively.} \]

This system can be written by using the Kronecker product \( \otimes \) :

\[ \dot{y} = A \cdot y + \eta_{(2)} \cdot y \otimes y + \eta_{(3)} \cdot y \otimes y \otimes y \]  

(22)

where \( y \otimes y \) is defined as the basis of quadratic terms and \( y \otimes y \otimes y \) is defined as the basis of cubic terms. \( A \) is an \( 2n \times 2n \) matrix.

### 4.3 The center manifold Approach

In this section, we describe the method to obtain the lower dimensional system, defined on the center manifold. Locally, the stability of the center manifold is equivalent to the stability of the original system.

Consider the nonlinear ordinary differential \( 2n \)-degree-of-freedom equations (with \( n = 2 \) for the judder non-linear system)

\[ \dot{y} = f(y, \mu) = M(\mu) \cdot y + \eta^{ij}_{(2)}(\mu) \cdot y \otimes y + \eta^{ij}_{(3)}(\mu) \cdot y \otimes y \otimes y \]  

(23)

where \( \mu \) is a parameter. This system has an equilibrium point \( X_0(\mu) \) if \( f(X_0 \cdot \mu) = 0 \). We may suppose, without loss of generality, that \( X_0 = 0 \). The stability of this point is obtained by the analysis of eigenvalues of the linearized system. The bifurcation appears when one or several eigenvalues cross the imaginary axis in the complex plane (as \( \mu \) is varied).

Therefore, the problem can be put into Jordan normal form by means of the eigenbasis. At the Hopf bifurcation point, we may write the previous system in the form

\[ \dot{v}_c = J_c(v_c) + G_c(v_c, v_s) + G_s(v_c, v_s) \]

\[ \dot{v}_s = J_s(v_s) + H_s(v_c, v_s) + H_s(v_c, v_s) \]  

(24)

where \( J_c \) and \( J_s \) have eigenvalues \( \lambda \) such that \( \text{Re}[\lambda] > 0 \) and \( \text{Re}[\lambda] < 0 \). \( G_c, G_s, H_c, H_s \) are polynomials of degree 2 and 3 in the components of \( v_c \) and \( v_s \). Here, we consider the physically interesting case of the stable equilibrium losing stability so we may assume that all eigenvalues of \( J_s \) have negative real part. Moreover, we considered the first coupling modes. For a Hopf bifurcation, the center manifold is two dimensional. Consequently, \( v_c \) is composed by two terms

\[ v_c = \begin{bmatrix} v_{c1} \\ v_{c2} \end{bmatrix} \]

So, there exists a local center manifold and the center manifold theory enables us to express the variables \( v_s \) as a function of \( v_c \) (Carr [17]):

\[ v_s = h(v_c) \]  

(25)

Using the tangency conditions at the bifurcation point to the center eigenspace, the function \( h \) satisfies:

\[ h(0) = 0 \]

\[ Dh(0) = 0 \]  

(26)

It is very important to note that \( v_s \) is a local invariant manifold because the expression of \( v_s \) as a function of \( v_c \) satisfies equations (26) for only small \( \|v_c\| \). Moreover, the center manifold \( v_c \) is a local center manifold due to equations (24).

We can not solve explicitly the expression of \( h \). But, it is possible to define an approximate solution of \( h \) by a power expansion and by equating the coefficients. Now, we show how \( h \) can be approximated. In order to satisfy the conditions (26), the polynomial approximations do not contain constant and linear terms (\( m \geq 2 \)):

\[ v_s = h(v_c) = \sum_{p+i+j=2}^{m} \sum_{j=0}^{n} a_{ij} v_{c1} v_{c2} \]  

(27)

where \( a_{ij} \) is a matrix with constant coefficients.

So \( v_s \) is composed by two terms:

\[ v_s = \begin{bmatrix} v_{s1} \\ v_{s2} \end{bmatrix} = \begin{bmatrix} h_1(v_c) \\ h_2(v_c) \end{bmatrix} = \begin{bmatrix} \sum_{p+i+j=2}^{m} \sum_{j=0}^{n} a_{1ij} v_{c1} v_{c2} \\ \sum_{p+i+j=2}^{m} \sum_{j=0}^{n} a_{2ij} v_{c1} v_{c2} \end{bmatrix} \]  

(28)

where \( a_{kij} \) are constant coefficients (\( k = 1, 2 \)).
Upon differentiating Eq. (24), the relation between $\dot{v}_c$ and $\dot{v}_s$ is obtained:

$$
\dot{v}_s = D_{v_s} h(v_c) \dot{v}_c
$$

(29)

Substituting Eq.(29) and Eq.(28) into the second equation of Eq.(24), we obtained

$$
D_{v_s} \left\{ \sum_{p=1}^{m} \sum_{j=0}^{p} a_{ij} v_c^j v_s^p \right\} J_{ij} v_c^j + G_2 [v_s, v_c, v_c] + \sum_{p=1}^{m} \sum_{j=0}^{p} a_{ij} v_c^j v_s^p \right\}
+ G_3 [v_s, v_c, v_c, v_c] + \sum_{p=1}^{m} \sum_{j=0}^{p} a_{ij} v_c^j v_s^p \right\}
= J_s \sum_{j=0}^{m} a_{ij} v_c^j v_s^j + H_2 [v_s, v_c, v_c] + \sum_{p=1}^{m} \sum_{j=0}^{p} a_{ij} v_c^j v_s^p \right\}
+ H_3 [v_s, v_c, v_c, v_c] + \sum_{p=1}^{m} \sum_{j=0}^{p} a_{ij} v_c^j v_s^p \right\}
$$

(30)

By equating the polynomial coefficients of same order, one obtains a system of algebraic equations for the polynomial coefficients. Solving these equations we obtain the approximation to the center manifold $v_s = h(v_c)$.

Provided that an polynomial approximation of $h$ up to sufficient order is obtained, the dynamics of Eq.(24) restricted to the center manifold is defined by the system:

$$
\dot{v}_c = J_{v_c} v_c + G_2 [v_s, h(v_c)] + G_3 [v_c, h(v_c)]
$$

(26)

where $G_2 [v_c, h(v_c)]$ and $G_3 [v_c, h(v_c)]$ are given as a power series in $v_c$ for the parameter $\mu = \mu_0$ and $h(v_c)$ defined in Eq.(23). The stability of this reduced system is equivalent to the original system.

4.4 Limit cycles

Now, we describe the procedure to obtain limit cycles for parameter values near the bifurcation point

$$
\mu = \mu_0 + \overline{\mu}
$$

(32)

where $\mu_0$ is the bifurcation point and $\overline{\mu} = \varepsilon \mu_0$ (with $\varepsilon \ll 1$).

We may use an application of the center manifold approach by adding to Eq. (26) the equation $\dot{\mu} = 0$. Moreover, we approximate the non-linear terms: they are evaluated at the bifurcation point $\mu = \mu_0$. Then the dynamics is given by

$$
\dot{v}_c = J_{v_c} v_c + G_2 [v_s, h(v_c)] + G_3 [v_c, h(v_c)] + \dot{\mu} = 0
$$

(33)

with $h(v_c)$ defined in Eq.(27).

This reduced system is easier to study than the original system. By using an approximation of $h$ up the 4th order, evolutions of limit cycle amplitudes diverge. This problem lies in the fact that a polynomial approximation of $h$ up to the 4th order is insufficient. Then, we may determine the limit cycles of the system by using an approximation of $h$ up to the 5th order.

In Figure 4-5, limit cycle are plotted for the two degree-of-freedom of the physical system (4). Thin lines and star lines show limit cycle by integrating the original system and by using center manifold approach, respectively. We have a
good correlation between the integrated system and the center manifold approach by using an approximation of $h$ up to the 5th order. Consequently, the center manifold approach is validated and reduces the number of equations of the original system to obtain a simplified system without losing the dynamics of the original system and without losing the non-linear terms.

5 SUMMARY AND CONCLUSION

A nonlinear model for the analysis of mode heavy truck judder has been developed. Results from stability are investigated by determining eigenvalues of this linearized equations for each steady-state operating point of the nonlinear system. This stability analysis indicates that system instability can occur with a constant friction coefficient. Moreover, this paper present the center manifold approach to obtain equations for the limit cycle amplitude. This approach simplifies the dynamics on the center manifold by reducing the order of the dynamical system while retaining the essential features of the dynamic behavior near the Hopf bifurcation point. One of the most important point is the determination of polynomial approximations and the determination of power that defines expressions of stable variables versus center manifold.

Finally, we valid the center manifold theory for this nonlinear model by comparing results obtained by solving the full system and by using the center manifold approach.

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REFERENCES