FATIGUE LIFE OF ALUMINUM BEAMS UNDER RANDOM VIBRATIONS

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ABSTRACT

Fatigue life of aluminum beams subjected to random vibrations while mounted on a shaker are evaluated. Finite element technique is used to obtain stress psd at critical location due to the prescribed acceleration psd. Various theories are used to estimate the fatigue life corresponding to the calculated stress psd. Theoretical values from various theories are compared with results obtained from experiment.

NOMENCLATURE

E [D]	Expected Damage
E [p]	Number of peaks
K	Constant
NB	Narrow band
S	Stress range
T	Time
a, b	Constants
m	slope of stress-life curve
p(s)	Peak Stress
γ	Irregularity Factor
ϵ	Bandwidth Parameter

THEORY

Fatigue under sinusoidal excitation is well understood for many years. However, there is no consensus on appropriate theory fatigue life under random vibration. As per Newland [1] actual lifetime can be 30% to 300% of the theoretical prediction. Notable work on random vibration and fatigue life has been many researchers including Wirsching et al [2], Ford [3] and Bishop [4].

First finite element analysis (ANSYS) was performed to compute stress psd as shown in Figure 3. The stress psd obtained from finite element analysis was used to estimate fatigue life using the following theories: Dirlik [5], Narrow Band [2],Tunna [6,7], Wirshing [2], Hancock, Kam and Dover [8] and Steinberg [9]. These theories are described next.

Tunna [6,7] proposed a formulation using a revised form of Rayleigh PDF for stress ranges as follows:

$$p(s) = (s/4\gamma m_0)\, e^{-\delta 2/8\, \gamma\, m}$$

For $\gamma = 1.0$, this formula becomes the narrow-band formula given earlier. Tunna's equation was developed with specific reference to the railway industry.

Wirsching's equation [2] is given by:

$$E[D] = E[D]_{NB}[a(b)+\{1-a(b)\}\{1-\epsilon\}^{c(b)}]$$

where $a(b) = 0.926-0.033b$, $c(b) = 1.587b - 2.323$, and $\epsilon = \sqrt{(1-\gamma^2)}$. ϵ is called the bandwidth parameter, which is an alternative version of the irregularity factor. This technique was developed with reference to the offshore industry, although it has been found to be applicable to a wider class of industrial problems.

Hancock's equation is given by:

$$S^b{}_h = \{2\sqrt{(2m_0)}\}^b[\gamma\tau\{(b/2)+1\}]$$

This solution is given in the form of an equivalent stress range parameter S_h, where $S^b{}_h = \int S^b p(S)dS$. Hancock's solution was developed for the offshore industry.

Kam and Dover's equation [8] was derived using a similar approach in which the fatigue damage can be easily obtained by substituting this into the general damage equation used when deriving the narrow-band solution:

$$E[D] = E[P](T/K)S^b_h$$

This was also developed for the offshore industry.

As a rough estimation for the fatigue damage, Steinberg [9] proposed a three-band technique. The basis for this method is the Gaussian distribution. The instantaneous stresses (or accelerations) between $+1\sigma$ (σ is the root mean square) and -1σ are assumed to act at the 1σ level 68.3% of the time. The instantaneous stresses (or accelerations) between $+2\sigma$ and -2σ are assumed to act at the 2σ level of 95.4 - 68.3, or 27.1% of the time. The instantaneous stresses (or accelerations) between $+3\sigma$ and -3σ are assumed to act at the 3σ level of 99 - 95.4, or 4.33% of the time. The fatigue damage is thus estimated based on these three stress levels. The Steinberg solution method is used by the electronics industry.

RESULTS

An aluminum beam as shown in Figure 1 is subjected to shaker excitation described in Table 1. Figure 2 shows the shaker controller reference created by Table 1.

Figure 2 shows the input created by the table. Figure 3 shows the stress psd obtained using the acceleration psd input. Fatigue life as computed by various theories using the software nSoft [10] is shown in Table 2.

Table 3 shows fatigue life observed in the experiment. One interesting observation may be made that while fatigue life predicted by various theories vary widely and fatigue life observed experimentally also vary widely. The mean life predicted by various theories is 34.09 min is very close to the mean value of 34.27 min observed experimentally.

CONCLUSION AND DISCUSSION

1. Vibration in the first mode seems to be dominant.

2. Uncertainties in experiment involving random vibration is reflected in the fact that standard deviation was about one third the mean value.

3. While average values predicted by various theories agreed very well with average value observed in experiment, that may be just a coincidence. Among various theories proposed, Dirlik, Hancock, and Kam and Dover seem to be in good agreement with experimental results. Wirshing seems to provide upper bound, but no method provided the lower bound.

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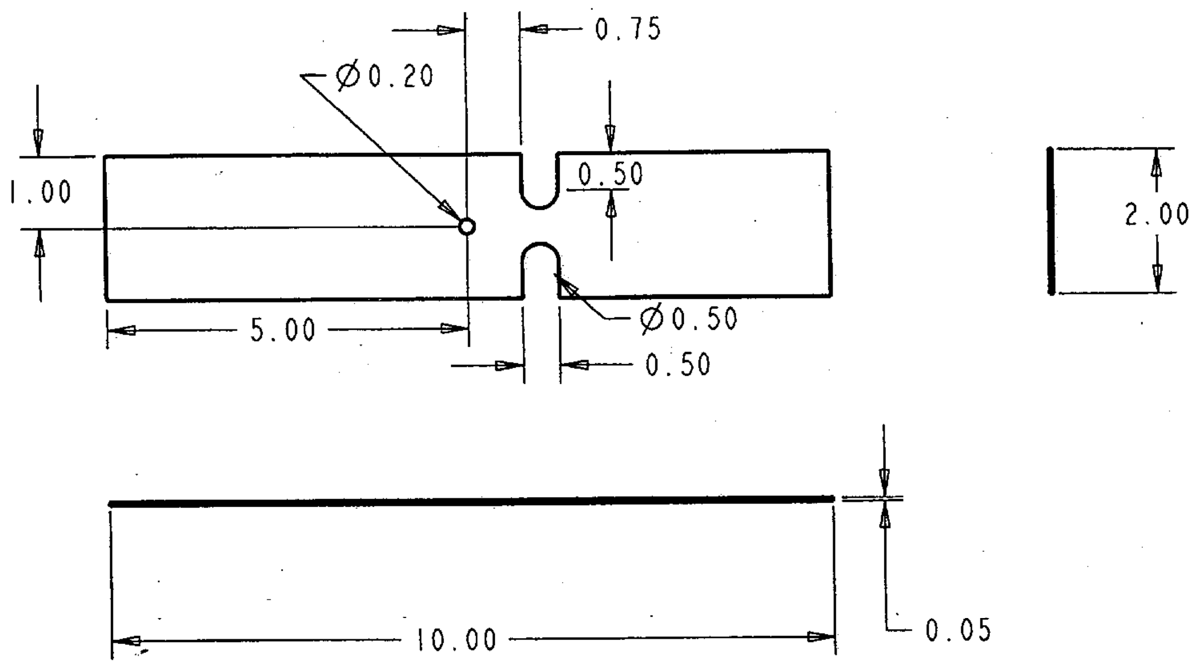

Figure 1. Aluminum Beam

Frequency (Hz)	20	30	90	200	500
PSD (g/Hz)	.05	.28	.28	.05	.05

TABLE 1. Base Excitation Random Input PSD

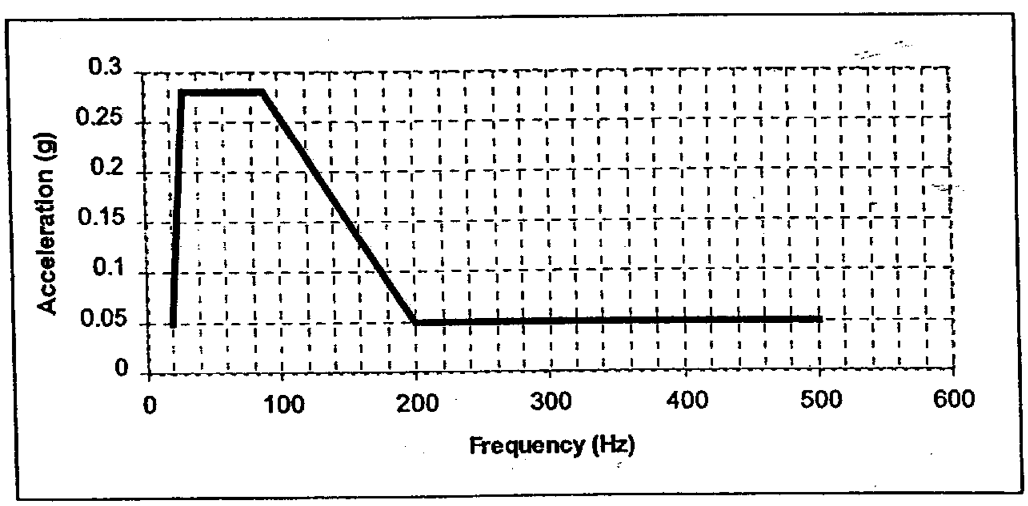

Figure 2. Base Excitation Input PSD in Random Vibration

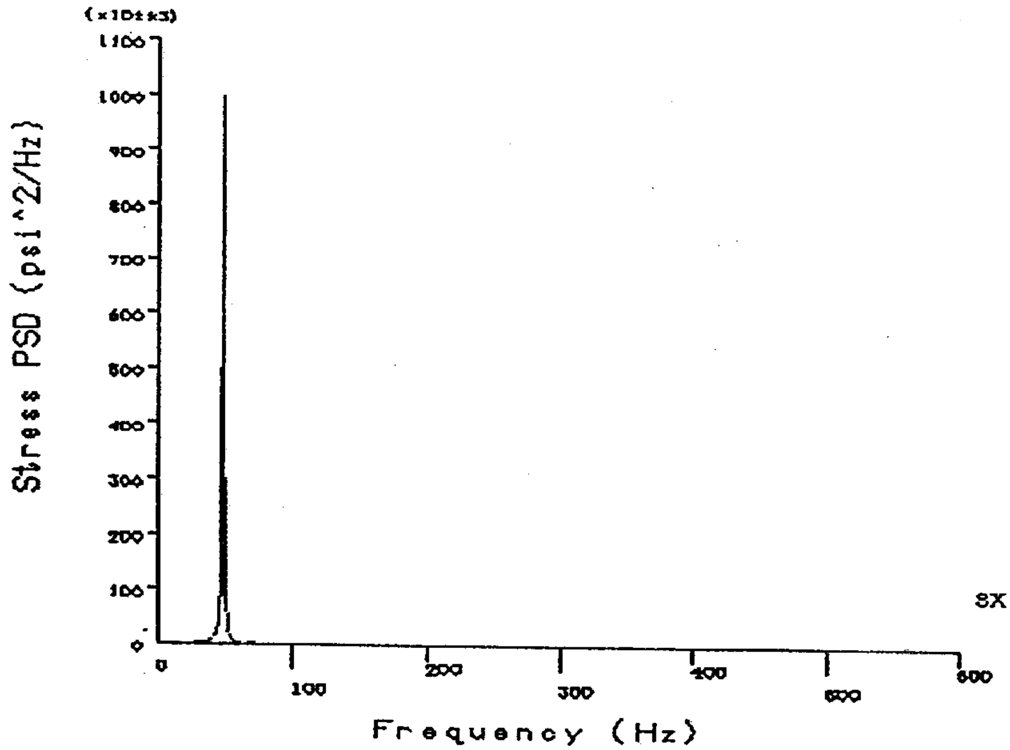

Figure 3. Stress PSD

Method	Theoretical fatigue life in min
Dirlik	35.11
Narrow band	32.83
Tunna	40.9
Wirsching	53.46
Hancock	33.56
Kam & Dover	33.56
Steinberg	44.35

TABLE 2. Theoretical Fatigue Life Computed by Various Theories

Run No	Fatigue life observed in min
1	53
2	36
3	40
4	51
5	39
6	39.38
7	34.16
8	39.16
9	22.35
10	41.66

TABLE 3. Fatigue Life by Experiment