ABSTRACT

Optimal structural dynamic modification (SDM) aims to improve dynamic behaviour of a structure by attempting to get a global optimum solution. Very little work is reported on application of genetic algorithms (GA) to structural dynamic modification area of dynamic design. Genetic Algorithms based on natural evolution are capable of finding the optimal solution.

In this paper it is proposed to use GA as a tool for optimisation of structural dynamic modifications. A simple Genetic Algorithm consists of three operators namely mutation, crossover and reproduction. The nature of these operators is such that each subsequent generation tends to have an average fitness level higher than the previous generation. In case of vibrating structures modal parameters of a structure are first obtained by finite element methods (FEM) or experimental modal testing followed by modal extraction. A fitness function and constraints are defined based on natural frequencies and mode shapes. Then GA is used to search for optimal structure by applying the three operators to satisfy the fitness function that defines the improved dynamic characteristics. A case study illustrating the above-mentioned optimal SDM of an 'F' structure using genetic algorithms is also presented.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>M</td>
<td>Mass matrix of the structure</td>
</tr>
<tr>
<td>K</td>
<td>Stiffness matrix</td>
</tr>
<tr>
<td>dM</td>
<td>Changes in mass matrix</td>
</tr>
<tr>
<td>dK</td>
<td>Changes in stiffness matrix</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Receptance</td>
</tr>
<tr>
<td>c</td>
<td>Viscous damping</td>
</tr>
<tr>
<td>x</td>
<td>Translational degree of freedom</td>
</tr>
<tr>
<td>i</td>
<td>(\sqrt{-1})</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Eigenvector</td>
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\[ \phi \] Mass normalised eigenvector matrix

\[ \omega \] Frequency of vibration

\[ p_c \] Probability of crossover

\[ p_m \] Probability of mutation

\[ FRF \] Frequency response function

\[ DOF(s) \] Degree(s) of freedom

1 INTRODUCTION

Structural dynamic modification is an area of study that deals with the effects of physical parameter changes on the dynamic properties of a structural system. These physical parameters are related to the mass, stiffness and damping properties of the system, or a combination of them. Structural modification usually comprises two different approaches. Firstly, given prescribed dynamic characteristics such as a new resonance, seek what, where and how much structural modifications will best accomplish it. Secondly for suggested structural modifications, determine which dynamic characteristic changes will occur. Sensitivity analysis is often used in structural modification to predict the optimal location for modification.

2 GENETIC ALGORITHMS FOR STRUCTURAL DYNAMIC MODIFICATION

Inspired by Darwin’s theory of evolution and natural law of survival of the fittest, GA is a global search procedure for gradually improving the solution in succeeding populations using operations that mimic those of the natural evolution, such as reproduction, crossover and mutation. Genetic algorithm flow chart is shown in figure 1.
2.1 GA Operators

The simplest form of genetic algorithm involves three operators: reproduction, crossover and mutation. **Reproduction** selects chromosomes in the population for further production. The fitter the chromosome, more the times it is likely to be selected to reproduce. **Crossover** randomly chooses a locus and exchanges the subsequences before and after that locus between two chromosomes to create two offspring. **Mutation** randomly flips some of the bits in a chromosome. Mutation can occur at each bit position in a string with some probability.

2.2 A simple genetic algorithm

Given a defined problem to be solved and a bit string representation, a simple genetic algorithm works as:

1. Start with a randomly generated population of n 1-bit chromosomes.
2. Calculate the fitness of each chromosome in the population.
3. Repeat the following steps until n offspring have been created.

The genetic algorithm is initialized with a pool of chromosomes. The next generation is then achieved by the key operations: selection, crossover and mutation. A number of the previous generation's chromosomes are selected such that those with greater fitness have a higher probability of selection. Some of these chromosomes are then "mated" in pairs; two mating chromosomes swap information beyond a crossover point which is randomly selected. The average fitness of the generation successively increases and the process is halted after a number of generations by a suitable convergence criterion. Normally, the best solution encountered through the entire optimization is taken as the result. In the elitist strategy, the best so far solution is guaranteed to survive into the next generation. Goldberg [4] analyzed the underlying nature of the algorithm using schemata to represent common patterns within the strings (a subset of the search space). The schemata with higher fitness experience, on an average exponentially increasing trials in subsequent generations. The basis towards particular schema, representing a number of solutions, implies an implicit parallelism so that the search space is sampled diversely and efficiently.

3 SINGLE DEGREE OF FREEDOM SYSTEM

For a single degree of freedom system as shown in figure 2 the equation of motion is given as (1)

\[ m \ddot{x} + cx + kx = f \]  

(1)

![Figure 2: single dof system](image-url)
The receptance frequency response function (FRF) is of the form (2)

\[ \alpha(\omega) = \frac{1}{(k - \omega^2m) + iwc} \]  

For \( m = 1200 \text{ kg} \)
\( k = 70000 \text{ Nm}^{-1} \)
\( c = 5000 \text{ Ns}^{-1} \)

The magnitude of frequency response function is as per figure 3.

4 TWO DEGREE OF FREEDOM SYSTEM

For 2 dof system:

\[ m_1x_1 + (k_1 + k_3)x_1 - (k_1x_3) x_3 = f_1 \]  
\[ m_2x_2 + (k_2 + k_3)x_2 - (k_2)x_1 = f_2 \]  

In matrix this can be expressed as equation 5.

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
= \begin{bmatrix}
  (k_1 + k_3) & (-k_2) \\
  (-k_1) & (k_2 + k_3)
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
= \begin{bmatrix}
  f_1 \\
  f_2
\end{bmatrix}
\]  

Figure 4: fitness statistics verses no. of generations
Considering a case of 2 dof system as shown in figure 5

For given values of mass and stiffness for a two degree of freedom system
\( m_1 = m_2 = 1 \text{ kg} \)
\( k_1 = k_3 = 0.4 \text{ MN/m} \)
\( k_2 = 0.8 \text{ MN/m} \)

\[
\begin{bmatrix}
1 & 0 \\ 0 & 1 \\
\end{bmatrix}
\quad \begin{bmatrix}
1.2 & -0.8 \\ -0.8 & 1.2 \\
\end{bmatrix}
\text{(MN/m)}
\]

Receptance plot \((\alpha_{12})\) is given as per fig.6

5 'F' TYPE STRUCTURE

The F- letter type structure is somewhat geometrically closer to that of a machine tool structure particularly that of a milling machine or a drilling machine (figure 7).

The spatial model of undamped vibrating structure consists of its mass and stiffness matrices, \([M]\) and \([K]\) respectively. They are the product of finite element modelling of the structure from the design data. Natural frequency and mode shapes are derived from
\[
[K] - \omega^2[M] = 0
\]

The response model of the vibrating structure comprises its frequency response function (FRF). The receptance FRF matrix \((7)\) of the structure is related to the special data by
\[
[K] - \omega^2[M] = [\alpha(\omega)]
\]
The structural modifications change parameter matrices of the special model of the structure as per equation (8) and (9).

\[
([M] + [dM]) \{x\} + ([K] + [dK]) \{x\} = \{f\} \quad (8)
\]

\[
([K] + [dK] - \lambda ([M] + [dM])) \{x\} = 0 \quad (9)
\]

Element mass matrix of frame type F Structure is given by Equation (10)

\[
m' = \begin{bmatrix}
2a & 0 & 0 & 0 \\
156b & 22lb & 0 & 54b \\
4lb & 0 & 0 & -13lb \\
2a & 0 & 0 & 0 \\
156b & -22lb & 0 & 4lb \\
\end{bmatrix}
\]

(10)

where \( a = \frac{\rho Ae}{6}, \quad b = \frac{\rho Ae l e}{420} \)

\( \rho Ae \) = area of element , \( l e \) = length of element , \( \rho \) = density of element mass

The elements have two displacements and a rotational deformation for each node. The nodal displacement vector is given by equation (11)

\[
q = \begin{bmatrix} q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6 \end{bmatrix}^T \quad (11)
\]

\( q' \) is transformed into (12) with \( L \) matrix

\[
q' = L q \quad (12)
\]

where \( L \) has 1 and m direction cosines corresponding to each node (13)

\[
L = \begin{bmatrix}
1 & m & 0 & 0 & 0 & 0 \\
-m & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & m & 0 \\
0 & 0 & 0 & 0 & -m & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(13)

Element stiffness matrix of 'f' structure is given by matrix (14)

\[
k'' = \begin{bmatrix}
EA & 0 & 0 & -EA & 0 & 0 \\
0 & 12EI & 6EI & 0 & -12EI & 6EI \\
0 & 6EI & 4EI & 0 & -6EI & 2EI \\
0 & -12EI & -6EI & 0 & 12EI & -6EI \\
0 & 6EI & 2EI & 0 & -6EI & 4EI \\
\end{bmatrix}
\]

(14)

where \( E \) = Young's modulus and \( A \) = Area of cross section

5.1 lumped mass representation of f type structure

Figure 8: lumped mass representation of f type structure
5.2 Mass Modification of 'f' structure by GA

Effects of SDM using GA on dynamic characteristics of the F type of structure has been studied. F type structure consists of four beams each 0.3x0.025x0.025 m. Young's modulus value of each beam is 0.207e+12. Density is 7806 kg/m³. The first natural frequency of the unmodified structure is 172 rad/sec. GA is applied with the objective function of minimisation of mass of beam 1 of the structure to obtain desired frequency shift. Simple GA is applied with following parameters

- Population size = 30
- Probability of crossover $p_c = .9$
- Probability of mutation $p_m = .03$
- Max. no. of generation = 100
- Length of chromosome = 16
- Value of lower bond for freq. = 172
- Value of upper bond for freq. = 220

A reduction of 2.8 kg of mass for a max value of frequency shift of 220 rad./sec is observed.

6 CONCLUSIONS

GA applied to mass modification of 'f' structure searches for optimal mass to get the desired frequency range. On similar lines it is possible to use GA for stiffness modification or combination of both mass and stiffness. The convergence of search is at global optimal.

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REFERENCES


