SAMPLING TECHNIQUE IN WAVELET ANALYSIS OF VIBRATING SIGNALS OF ROTATING MACHINERY*

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ABSTRACT

A main feature of wavelet analysis is localizable characteristic in time-domain and frequency-domain. However, localizable characteristic is formed from time-frequency windows with elasticity. Sampling interval of wavelet transform can be automatically adjusted in the second wavelet sampling, therefore wavelet transform can focus to arbitrary details of observed signal.

Sample is the important task of wavelet analysis. Sample of wavelet analysis and sample of Fourier analysis have great difference. Sample interval of Fourier analysis is a constant in time and frequency domains. Sampling intervals of wavelet analysis are gradually becoming fine with increasing frequency. On the other hand, after the first sampling of the signals (gotten by A/D converter) and sampling of wavelet base, wavelet transform needs the second sample of the signals and the wavelet base. In signal analysis Shannon theorem on sampling must be satisfied.

In this paper, sampling principle and technology of orthogonal wavelet transform of signals are deeply researched. Moreover, edge effect of wavelet analysis of signals is discussed.

In this paper, the wavelet analysis of singular signal and vibrating of rotating machinery is discussed, and some application examples are given.

1 INTRODUCTION

In recent ten years, wavelet analysis with good localization characteristics in domain-time and domain-frequency has been developed. It is different from Fourier analysis. Its time-frequency windows is elastic. Sampling interval of wavelet analysis is automatically adjusted in the second sampling. Wavelet analysis can zoom arbitrary details of objects.

Wavelet sampling is the most important technology of wavelet analysis. If the problem of wavelet sampling can not be completely solved, wavelet transform and decomposition of signals can not be implemented, and in turn, wavelet decomposition of signals in engineering is useless. Unfortunately, the research of wavelet sampling is very dreary recently in the application research area of wavelet analysis. It is difficult to find a paper specifically discussing wavelet sampling technology and application. In this paper, research on wavelet sampling and application technique was introduced in details to break this dreariness. We expect to throw out a minnow to catch a whale and hope to see a great extent papers on wavelet sampling in the near future, to stimulate the application of wavelet analysis in engineering.

To more efficiently use of the localization characteristics of wavelet analysis and its ability of analyzing singular signals in the treatment of engineering signals, we choose the Mexican hat function as wavelet mother function, analyze and deduce sample principle and technology of wavelet analysis as well as other relevant questions.

2 DYADIC DISCRETE WAVELET ANALYSIS

2.1 Discrete wavelet bases

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Mexican hat mother wavelet is

$$\psi(t) = \frac{2}{\sqrt{3}} \pi^{-1} (1-t^2) \exp\left(-0.5t^2\right), \ t \in \mathbb{R}$$ (1)

We dilate and translate Eq. (1), dyadic wavelet bases series in $$L^2(\mathbb{R})$$ are created

$$\psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m} t - n)$$

$$\psi_{m,n}(t) = \frac{2}{\sqrt{3}} \pi^{-1} 2^{-m/2} \left[1 - (2^{-m} t - n)^2\right] \exp\left[-0.5(2^{-m} t - n)^2\right]$$ (2)

Wavelet bases $$\psi_{m,n}(t)$$ can form a space $$\{W_m, \ m \in \mathbb{Z}\}$$

Furthermore $$\bigcup_{m=-\infty}^{\infty} W_m = L^2(\mathbb{R})$$, it is perfect.

where $$W_m = \text{span}\{\psi_{m,n}(t), \ n \in \mathbb{Z}\}$$.

which is norm and orthogonal (more precisely, Mexican hat wavelet is similarly orthogonal).

$$\psi_{m,n}(t)$$ has window properties of short-time Fourier transform.

When $$n = 0$$, Fourier transform of $$\psi_{m,0}(t)$$ is

$$\hat{\psi}_{m,0}(\omega) = \frac{2\sqrt{2}}{\sqrt{3}} \pi^{0.25} 2^{m/2} (2^m \omega)^2 \exp\left(-0.5(2^m \omega)^2\right)$$ (3)

Wavelet bases $$\psi_{m,0}(t)$$ and its frequency property are shown in Figure 1, its ordinate value is non-dimensional.

2.2 Relationships of scale parameter $$m$$ with frequency $$f$$ and time parameter $$n$$ with time $$t$$

Observe and study band limit signal with sampling frequency $$f_s$$

$$X(t) = \begin{cases} x(t) & t \in [0, T_s] \\ 0 & \text{others} \end{cases}$$ (4)

Sample is the first task of wavelet analysis. Wavelet analysis sample and Fourier analysis sample have great difference. Fourier analysis sampling interval is a constant in time and frequency domains, wavelet analysis sampling intervals are gradually becoming fine with increasing frequency; on the other hand, after the first sample of the signals (gotten by A/D converter) and wavelet base sample, wavelet transform needs the second sample of the signals and the wavelet base. In signal analysis Shannon theorem on sampling must be satisfied.

3.1 Sample principle and technology of the first sample

1. First sample of signal

In the condition of signal satisfying sample theorem, we take
sample interval of the signal $X(t)$ as

$$
\Delta t = \frac{1}{2B_\infty} \quad (\beta \geq 1 \text{ and } \beta \in \mathbb{Z}) .
$$

If $T_x$ is length of the signal, the point number of the discrete signal is $N_x = \lfloor T_x / \Delta t \rfloor + 1$, the value of discrete time is $\Delta t^x = \Delta t^x K^x, K^x = 0, 1, \ldots, N_x$, and the sample value of the signal is $\hat{X}(K^x) = X(\Delta t^x)$.

2. Sample principle of wavelet basis

If sampling frequency of wavelet basis $\psi_{w,n}(t)$ is $f^w_n$, then

$$f^w_n = \frac{\alpha}{2^{m}} 2^{-m} \; \text{ where } \alpha \text{ is appropriately closed natural number and related to length of nonzero interval of wavelet basis.}
$$

Actually, when $m = 0$, the value of $\alpha$ satisfies

$$\alpha = \lfloor 2 f^w_n \rfloor + 1 \quad (7)
$$

where $f^w_n$ is truncated frequency of mother wavelet.

In fact, $\alpha$ may appropriately be large than the value given by Eq. (7): calculating indicates $\alpha$ may be two or four times of the value given by Eq. (7).

If nonzero interval length of mother wavelet is $\gamma$, the nonzero interval length of wavelet basis is $T_w = \gamma 2^m$. In the follows, calculating method of nonzero interval length of Mexican hat wavelet basis was given out.

2. $m$ series wavelet bases Eq. (2), created with $\psi(t)$ in $L^2(\mathbb{R})$ space, can determine three extreme value points by calculating extreme value of the function $t_a = 2^m (25 + n)$, $t_b = 2^m n$ and $t_c = 2^m (1.5 + n)$. We let $t_1 = 2^m (25 + n)$ and $t_2 = 2^m (5 + n)$ respectively as left and right breakpoints of wavelet basis. The maximal absolute value of the truncated parts of $\psi(t)$ is $1$ million. The relative error of truncated parts of wavelet basis is very small. If mother wavelet is compactly supported, the both ends of wavelet basis can precisely take as compactly supported points. If mother wavelet is exponentially attenuated, left and right breakpoints of wavelet basis may be gained by above-mentioned method.

3. Range of $m$

To successfully use wavelet transform, $m$ must satisfy $T_w = t_1 - t_0 = 5 \times 2^{-m-1} < T_x$, so, the range of $m$ is determined by Eq. (6) and (8).

4. Sampling of wavelet bases

To satisfy signals sample theorem, we take one-40th of interval $[t_0', t_0]$ as sampling interval of wavelet basis which is $\Delta w = 2^{m+1} \times 5 / 40$, sampling point number of wavelet basis is $N_w = 41$, and sampling time value of wavelet basis is $\Delta w K_w = 2^{-m} (-5 + n) + \Delta w K_w$, and sample value of wavelet base is

$$
\psi_{w,n}(K_w) = (2 / \sqrt{3}) 2^{-0.5} . \exp[-0.5(2^{-m} \Delta w (K_w) - n)^2]
$$

where $K_w$ satisfies $K_w = 0, 1, \ldots, N_w$.

5. First sample of wavelet transform

Let $M$ as the upper band of $m$, which satisfies Eq. (8). To correctly use wavelet transform, the first sample interval $\Delta w^M$ must satisfy $\Delta w^M = \min(\Delta w^M, \Delta M)$, where $\Delta w^M$ is first sample interval of wavelet basis.

In the first sample, the sampling points of signals are $N_w^M = \lfloor T_w / \Delta w^M \rfloor + 1$, the sampling points of wavelet basis are $N_{w,n}^M = \lfloor T_w / \Delta w^M \rfloor + 1$; if we replace respectively variables of Eq. (8) and (9) by $\Delta w^M$, $N_w^M$ and $N_{w,n}^M$, the first sampling value of signal and wavelet basis were gained.

3.2 Sample principle and technology of the second sample

1. Second sample

Along with different signal wavelet decomposition range, signal and wavelet basis sample interval will be changed with $m$, therefore there is the second sample in signal wavelet transform. With respect to $m$ the second sampling interval of wavelet basis is $\Delta w^M$, i.e., $2^{m-M}$ times of $\Delta w^M$, so is signals; If $\Delta w$ is sampling interval of the second transform, it satisfies $\Delta w = \min(\Delta w, \Delta y)$, sampling points of signal are $N_{w,x} = \lfloor T_w / \Delta w \rfloor + 1$, and sampling points of wavelet are $N_{w,v} = \lfloor T_w / \Delta w \rfloor + 1$.

2. Edge effect and the range of $n$

There is the edge effect in wavelet transform of signals. First, signal near the two ends can not be completely modulated by wavelet basis; see Figure 2. Furthermore, when
the signals cannot be completely decomposed (or distortion), which is referred as the edge effect of wavelet transform. Second, two ends of signal is singular. In signal wavelet decomposition magnanimous useful information of signal will be drowned. Other singular points of signal can be found by finding extreme value method, but it is by far not sufficient. On the other hand, signal can not be extended by symmetric, antisymmetric and periodic extension, because extension point is a extreme point or turn point, which is a singular point. The information of other singular point or breakdown point will be drowned, so the purpose of signal analysis can not be finished. Meanwhile edge effect domain will increase with the increasing of frequency parameter \( m \) or the decreasing of analyzed frequency.

\[ n \in \left[ 0, \text{int} \left( \frac{N_{\psi}}{2} \right) + 1 \right] \quad \text{and} \quad n \in \left[ N_{\psi} - \text{int} \left( \frac{N_{\psi}}{2} \right) + 1, N_{\psi} \right], \]

In the wavelet transform, the range of \( n \) of \( \psi_{m,n}(t) \) is
\[ n \in \left[ \text{int} \left( \frac{N_{\psi}}{2} \right) + 1, N_{\psi} - \text{int} \left( \frac{N_{\psi}}{2} \right) + 1 \right] = \Delta T \quad (10) \]

Time value of the second signal sample is
\[ \Delta_{\psi T} = \Delta_{\psi} \cdot K_{\psi T}, \quad K_{\psi T} \in \Delta T \]
the time value of the second wavelet basis sample is
\[ \Delta_{\psi V} = 2^{n} \Delta_{\psi} \cdot K_{\psi V}, \quad K_{\psi V} = 0, 1, \ldots, N_{\psi V} \]
We replace \( \Delta_{\psi T} \) and \( \Delta_{\psi V} \) by \( \Delta_{x T} \) and \( \Delta_{x V} \) of Eq. (8) and (9), which is the second sampling value of signal and wavelet basis.

In implementing above steps, if \( \Delta_{\psi} = \Delta_{\psi}, \) signal must be interpolated; if \( \Delta_{\psi} = \Delta_{x}, \) wavelet basis must be interpolated. In fact, after the condition \( \Delta_{\psi} = \Delta_{\psi} \) was satisfied, signal is sampled, so the error of signal interpolation will be decreased in signal wavelet analysis.

Sampling process of wavelet transform are shown in Figure 3. The units of ordinate and abscissa are Voltage (V) and Second (s) in Figure 3, respectively.

4 SIGNAL DISCRETE WAVELET TRANSFORM

4.1 Time-frequency spectrum of wavelet transform
If signal is \( X(t) \) wavelet basis is \( \psi_{m,n}(t), \) time-frequency spectrum of orthogonal wavelet transform is
\[ W_{m,n} = \langle X(t), \psi_{m,n}(t) \rangle = \sum_{k} X(l) \psi_{m,n}(k) \delta_{l} \quad (11) \]

4.2 Signal wavelet decomposition
With frequency parameter \( m, \) wavelet orthogonal decomposition of signal \( X(t) \) is
\[ X_{m}(t) = \sum_{n} (W_{m,n}) \psi_{m,n}(t) \quad (12) \]
Eq. (12) may be written in discrete form
\[ X_{m}(l) = \sum_{n=-N_{\psi}}^{N_{\psi}} W_{m,n} \psi_{m,n}(l) \quad (13) \]
where \( 2N + 1 = N_{\psi V}, \ l \in \Delta T, \ l \in \mathbb{Z} . \)

The modulating processes of wavelet transform are shown in Figure 4. The units of ordinate and abscissa are Voltage (V) and Second (s) in Figure 4, respectively.
5 Applications examples of wavelet analysis for experimental signals & vibrating signals of rotating machinery

5.1 Wavelet analysis of experimental signals

Eq. (14) expresses a singular signal hiding in sinusoidal signal

\[
X(t) = \begin{cases} 
\text{Sin}20\pi t + 0.001\delta(t, t_0), \\
\text{Sin}20\pi t + 0.001\delta(t, t_0), \\
152.58189ms < t_0 < 152.893060ms & \text{when } t = t_0 \\
213.623046ms < t_0 < 213.928222ms & \text{when } t \neq t_0 
\end{cases}
\]

where \(\delta(t, t_0) = \begin{cases} 
1 \\
0 
\end{cases}\)

In processing signal \(X(t)\) with type QLWT-1 wavelet transform analyzer, a wavelet base of Mexican hat form is selected to carry out the transform, the processed result is shown in Figure 5, the wavelet composition curve of \(X(t)\) is shown in Figure 5(a) which discovers the singular signals hiding in the signal observed via wavelet transform, the transient impulses shown in Figure 5(a) are the singular signal discovered and the three-dimensional time-frequency spectrum is shown in Figure 5(b).
5.2 Analysis of vibration signals of inductive motor

A wavelet analysis test carried out for the vibration signals generated from a rotary inductive motor with power of 3.5KW and speed of 1500rpm before and after balance adjusting. Vertical and horizontal vibration transducer is mounted on the outer rings of the rotor bearings to pick up the vibration signals of the two directions. The wavelet analysis is make with Daubechies the first wavelet base at [-2,3] and test result is shown in Figure 6, Figure 6(a), (b) and Figure 6(c), (d) shown respectively the wavelet transform decomposition curves and three-dimensional spectrum of vibration signal of the motor before and after dynamic balance adjusting. It is shown that the vibration status has been improved obviously.

Figure 6: Analysis of vibration signal of inductive motor

FFT for original signals and all decomposition can be made conveniently with type QLWT-1 wavelet analyzer, it is shown clearly the information result from wavelet transform contains all and more perfect information than that of FFT.

6. CONCLUSIONS

The good localization property of wavelet decides particularity of its sampling, twice sampling must be done in signal orthogonal decomposition; in the second sampling, analyzed frequency range is automatically adjusted along with frequency parameter $m$. Wavelet transform can detect singular points of signals (include non-continuous points), so singularity can be found at signal ends, that is edge effect; Wavelet transform has edge effect whose range varies with the change of frequency extracted, i.e., becomes narrow with the increase of the frequency.

REFERENCES


