IMPROVED VIBRATION-BASED DAMAGE DETECTION ALGORITHM FOR STRUCTURES

Jeong-Tae Kim and Yeon-Sun Ryu
Department of Ocean Engineering, Pukyong National University, Pusan 608-737, KOREA

Norris Stubbs
Department of Civil Engineering, Texas A&M University, TX 77843, USA

ABSTRACT

A vibration-based damage detection algorithm is newly derived and its accuracy in damage prediction is evaluated for structures with limited modal parameters. In the first step, existing damage detection algorithms are reviewed and then a new algorithm is developed to improve the accuracy of damage localization and severity estimation by eliminating erratic assumptions and limits in the existing algorithms. The accuracy of the algorithms are assessed when applied to a two-span continuous beam for which pre-damage and post-damage modal parameters are available for only a few modes of vibration. Compared to the existing damage detection algorithms, the newly-derived algorithm improved the accuracy of damage localization and severity estimation results in the test beam.

1. INTRODUCTION

This paper deals with the general problem of utilizing changes in dynamic modal parameters of structures to nondestructively detect, locate, and estimate the severity of damage in these structures. Structural damage may be defined as any deviation of a geometric or material property defining a structure that may result in an unwanted response of the structure. A solution to this problem is important for at least two reasons. Firstly, damage localization and severity estimation are the first two steps in the broader category of damage assessment. Secondly, a timely damage assessment could produce desirable consequences such as saving of lives, reduction of human suffering, protection of property, increased reliability, increased productivity of operations, and reduction in maintenance costs.

During the past decade, a significant amount of research has been conducted in the area of damage detection using the dynamic response of a structure. Research efforts have been made to detect structural damage directly from dynamic response measurements in the time domain, e.g., the Random Decrement technique (1-2), or from Frequency Response Functions (FRF) (3-4). Also, many research studies have been conducted in the area of nondestructive damage detection (NDD) attempting to use changes in modal parameters. Research studies have focused on the possibility of using the vibration characteristics of structures as an indication of structural damage. Since 1988, studies on the topic appear to be accelerating. Attempts have been made to monitor structural integrity of bridges (5-12), to investigate feasibility of damage detection in large space structures using changes in modal parameters (13-14), and to localize damage in beam-type structures using changes in mode shapes characteristics (15-17).

Despite these research efforts, however, many problems related to vibration-based damage detection remain unsolved today. Outstanding needs remain to locate and estimate the severity of damage: (a) in structures with only few available modes, (b) in structures with many members, (c) in structures for which baseline modal responses are not available, and (d) in an environment of uncertainty associated with modeling, measurement, and processing errors.

In this paper, we present an improved vibration-based NDD algorithm to locate and estimate severity of damage in structures. The proposed methodology is presented here in two parts. In the first part, we outline vibration-based NDD algorithms. We first review existing NDD algorithms proposed by Kim and Stubbs (16-17). Then we formulate a new NDD algorithm to improve its accuracy in damage localization and severity estimation by eliminating erratic
assumptions and limits in the existing NDD algorithms. In the second part, we demonstrate the feasibility of the newly-derived NDD algorithm using numerical examples. The new NDD algorithm and two existing ones are evaluated by predicting damage locations and estimating severities of damage in a two-span continuous beam for which limited modal parameters are available for a few modes of vibration. The performance of each NDD algorithm is assessed by quantifying the accuracy of damage localization and severity estimation results.

2. EXISTING DAMAGE DETECTION ALGORITHM

For a linear, undamaged, skeletal structure with \(n_e\) elements and \(n\) nodes, the \(\alpha\)th modal stiffness of the arbitrary structure is given by

\[ K_{i\alpha} = \Phi_{i\alpha}^T C \Phi_{i\alpha} \]  

where \(\Phi_{i\alpha}\) is the \(\alpha\)th modal vector and \(C\) is the system stiffness matrix. The contribution of \(\alpha\)th member to \(\alpha\)th modal stiffness, \(K_{i\alpha}^\alpha\), is given by

\[ K_{i\alpha}^\alpha = \Phi_{i\alpha}^T C_j \Phi_{i\alpha} \]  

where \(C_j\) is the contribution of \(j\)th member to the system stiffness matrix. Then, the fraction of modal energy (i.e., the undamaged modal sensitivity) of the \(\alpha\)th mode and the \(j\)th member is defined as

\[ F_{ij}^\alpha = K_{i\alpha}^\alpha / K_{i\alpha} \]  

Let the corresponding modal parameters in Eqs. 1 to 3 associated with a subsequently damaged structure be characterized by asterisks. Then for the damaged structure, the damaged sensitivity of the \(\alpha\)th mode and the \(j\)th member is defined as

\[ F_{ij}^{\alpha*} = K_{i\alpha}^{\alpha*} / K_{i\alpha}^\alpha \]  

in which the quantities \(K_{i\alpha}^{\alpha*}\) and \(K_{i\alpha}^\alpha\) are given by

\[ K_{i\alpha}^{\alpha*} = \Phi_{i\alpha}^{\alpha*} C_j \Phi_{i\alpha}^\alpha \]  

\[ K_{i\alpha}^\alpha = \Phi_{i\alpha}^T C_{j\alpha} \Phi_{i\alpha}^\alpha \]  

The quantities \(C_j\) and \(C_{j\alpha}^\alpha\) in Eq. 2 and Eq. 4 may be written as follows:

\[ C_j = E_j C_{j\alpha} \]  

\[ C_{j\alpha}^\alpha = E_j^\alpha C_{j\alpha} \]  

2.1 Damage Index A - First Approximation

Suppose we make an approximation that the modal sensitivities for the \(\alpha\)th mode and the \(j\)th location is the same for both undamaged and damaged structure (i.e., \(K_{i\alpha}^{\alpha*} = F_{ij}^\alpha\)). Then Eqs. 3 and 4 are combined and reduced to the following expression:

\[ F_{ij}^{\alpha*} / F_{ij}^\alpha = (K_{i\alpha}^{\alpha*} / K_{i\alpha}^\alpha) = 1 \]  

On substituting Eqs. 1, 2, 5, and 6 into Eq. 7 and rearranging, a damage index \(\beta_j\) of \(j\)th member (and for \(nm\) vibrational modes involved) is obtained by

\[ \beta_j = \frac{E_j}{E_j^\alpha} = \frac{\sum_{i=1}^{nm} \gamma_{i\alpha} K_{i\alpha}^{\alpha*}}{\sum_{i=1}^{nm} \gamma_{i\alpha} K_{i\alpha}^\alpha} \]  

in which \(\gamma_{i\alpha} = \Phi_{i\alpha}^T C_{j\alpha}^\alpha \Phi_{i\alpha}^\alpha\) and \(\gamma_{i\alpha}^* = \Phi_{i\alpha}^{\alpha*} C_{j\alpha}^\alpha \Phi_{i\alpha}^\alpha\) and damage is indicated at \(j\)th member if \(\beta_j > 1\).

The severity of damage in the \(j\)th member is estimated as follows. Let the fractional change in the stiffness of the \(j\)th member be given by the severity estimator, \(\alpha_j\), then

\[ E_j^{\alpha*} = E_j \left(1 + \frac{\Delta E_j}{E_j}\right) = E_j(1 + \alpha_j) \]  

Combining Eq. 8 and Eq. 9 yields

\[ \alpha_j = \frac{\sum_{i=1}^{nm} \gamma_{i\alpha} K_{i\alpha}^{\alpha*}}{\sum_{i=1}^{nm} \gamma_{i\alpha} K_{i\alpha}^\alpha} - 1, \quad \alpha_j \geq -1 \]  

where damage severity is indicated as the reduction in stiffness in the \(j\)th member if \(\alpha_j < 0\).

2.2 Damage Index B - Second Approximation

From Eq.8, damage is indicated at \(j\)th member if \(\beta_j > 1\). However, Eq. 8 becomes singular if the denominator goes zero. This will occur when simultaneously, the element size approaches zero and the element is located at a node of a vibrational mode. To overcome this limitation (i.e., the division by zero difficulty), an approximation is made such that the axis of reference for the modal sensitivities is shifted by a value of \(1.0\) (i.e., \(F_{ij}^\alpha \rightarrow F_{ij}^\alpha + 1\) and \(F_{ij}^{\alpha*} \rightarrow F_{ij}^{\alpha*} + 1\)). Adding unity to both the numerator and the denominator of Eq. 7 yields

\[ \frac{(F_{ij}^\alpha + 1)}{(F_{ij}^{\alpha*} + 1)} = \frac{(K_{i\alpha}^{1} + K_{i\alpha}^{\alpha*}) K_{i\alpha} \sqrt{(K_{i\alpha} + K_{i\alpha}) K_{i\alpha}^{\alpha*}} = 1 \]  

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On substituting Eqs. 1, 2, 5, and 6 into Eq. 11 and rearranging, a damage index \( \beta_j \) of the system (and for \( nm \) modes) is obtained by (17)

\[
\beta_j = \frac{E_j}{E_j^*} = \frac{\sum_{k=1}^{m} \left( \gamma_{y}^{k} + \sum_{k=1}^{n} \gamma_{yi}^{k} \right) K_{ij}^*}{\sum_{k=1}^{m} \left( \gamma_{y}^{k} + \sum_{k=1}^{n} \gamma_{yi}^{k} \right) K_i}
\]

where damage is indicated at the \( j \)-th location if \( \beta_j > 1 \).

Once damage is located at the \( j \)-th member, damage severity is estimated by combining Eq. 12 and Eq. 9.

\[
\alpha_j = \frac{\sum_{k=1}^{m} \left( \gamma_{y}^{k} + \sum_{k=1}^{n} \gamma_{yi}^{k} \right) K_{ij}^*}{\sum_{k=1}^{m} \left( \gamma_{y}^{k} + \sum_{k=1}^{n} \gamma_{yi}^{k} \right) K_i}
\]

where damage severity is indicated as the reduction in stiffness in the \( j \)-th member if \( \alpha_j < 0 \).

3. NEW DAMAGE DETECTION ALGORITHM: Damage Index C

Let \( \lambda_j \) and \( \lambda_j^* \) are the \( j \)-th eigenvalues of pre-damage and post-damage mdof structural systems, respectively. Then the \( j \)-th eigenvalues can be related to the following forms:

\[
\lambda_j^* = \lambda_j + d\lambda_j = \left( K_j + dK_j \right) / \left( M_j + dM_j \right)
\]

in which \( K_j \) and \( M_j \) are the \( j \)-th modal stiffness and the \( j \)-th modal mass of the undamaged system, respectively. Also, \( d\lambda_j \), \( dK_j \), and \( dM_j \) are the change in the \( j \)-th eigenvalue, the change in the \( j \)-th modal stiffness, and the change in the \( j \)-th modal mass in the system.

On expanding and rearranging Eq. 14, we obtain

\[
\frac{dK_j}{K_j} = \frac{d\lambda_j}{\lambda_j} + \frac{dM_j}{M_j} \left( 1 + \frac{d\lambda_j}{\lambda_j} \right)
\]

where \( dK_j / K_j \) represents the fractional change in the \( j \)-th modal stiffness and all the terms in the right hand side of the above equation can be determined directly or via experimental measurements.

For the \( j \)-th mode and the \( j \)-th location, the undamaged and damaged modal sensitivities, \( F_{\gamma_j} \) and \( F_{\gamma_j}^* \), are related by the equation:

\[
F_{\gamma_j}^* = F_{\gamma_j} + dF_{\gamma_j}
\]

where \( dF_{\gamma_j} \) represents the change in the fraction of modal energy at the \( j \)-th mode and for the \( j \)-th mode. On differentiating Eqs. 3 and 16, the quantity \( dF_{\gamma_j} \) can be obtained from the expression:

\[
dF_{\gamma_j} = \frac{dK_{y_j}}{K_j} \left[ \frac{dK_{y_j}}{K_j} - \frac{dK_j}{K_j} \right]
\]

where \( dK_{y_j} \) represents the fractional change in \( K_{y_j} \). Also, by noticing \( K_j \gg K_{y_j} \), Eq. 17 can be reduced to the following form:

\[
dF_{\gamma_j} = \frac{dK_{y_j}}{K_j}
\]

Next, combining Eq. 2 and Eq. 6 and also Eq. 5 and Eq. 6, respectively, gives

\[
K_{y_j} = \gamma_{y_j} E_j, \quad K_{y_j}^* = K_{y_j} + dK_{y_j} = \gamma_{y_j}^* E_j
\]

in which \( \gamma_{y_j} = \Phi_j^T C_{y_j} \Phi_j \) and \( \gamma_{y_j}^* = \Phi_j^T C_{y_j} \Phi_j^* \). Also from Eq. 19, \( dK_{y_j} \) can be rewritten by

\[
dK_{y_j} = \gamma_j(E_j + dE_j) - \gamma_j E_j
\]

On dividing both sides of Eq. 20 by \( K_j \), substituting into Eq. 19, and only solving for the fractional change in the \( j \)-th member’s stiffness, we obtain

\[
\frac{E_j}{E_j + dE_j} = \left( \frac{\gamma_j}{K_j} \right) / \left( \frac{dK_j}{K_j} + \frac{\gamma_j}{K_j} \right)
\]

Assuming the structure is damaged at a single location and the resulting change in \( K_{y_j} \) is only the function of \( E_j \), a first approximation of \( dK_{y_j} \) can be obtained from the expression:

\[
\frac{dK_{y_j}}{K_j} = \left( \frac{\partial K_j}{\partial E_j} + \frac{\partial K_j}{\partial \gamma_j} \frac{d\gamma_j}{\partial E_j} \right) \frac{dE_j}{K_j}
\]

where \( \partial K_j / \partial E_j = \gamma_j \), \( \partial K_j / \partial \gamma_j = \gamma_j \).

On substituting Eq. 23 into Eq. 22 and further approximation gives

\[
\frac{dK_{y_j}}{K_j} = \frac{\gamma_j}{\gamma_j} \frac{dE_j}{E_j} + \frac{d\gamma_j}{\gamma_j}
\]

and

\[
d\gamma_j = \gamma_j - \gamma_j \cdot \gamma_j = \sum_{k=1}^{m} \Phi_j^T C_{k_j} \Phi_j
\]
Since we have assumed that the structure is damaged in a single location, it follows readily that \( dK_v = dK_j \) (note that \( dK_v \approx dK_j/nd \) if the structure is damaged in \( nd \) multiple locations, in which the \( nd \) locations can be predicted). Then by substituting Eq. 15 into Eq. 24, the fractional changes in modal stiffness can be approximately related to the fractional changes in modal properties.

\[
\frac{dK_v}{K_j} \approx g_i(\lambda_i, \Phi) = \frac{d\lambda_i}{\lambda_i} + \frac{dM_j}{M_j} \left[ 1 + \frac{d\lambda_i}{\lambda_i} \right]
\]

(26)

in which \( g_i(\lambda_i, \Phi) \) is the dimensionless factor representing the systematic change in modal parameters of the \( i \)th mode due to the damage.

By applying Eqs. 22-26 to Eq. 21, a new damage index for \( i \)th mode and \( j \)th location is given by

\[
\beta_i = \frac{E_i}{E_j} = \frac{\gamma_i}{\gamma_j} g_i(\lambda_i, \Phi) + \gamma_i = \frac{\text{Num}}{\text{Den}}
\]

(27)

For \( nm \) vibrational modes, a damage index \( \beta_i \) for the \( j \)th location is obtained by

\[
\beta_j = \sum_{i=1}^{nm} \frac{\text{Num}}{\text{Den}}
\]

(28)

Once damage is located at the \( j \)th member, damage severity of the \( j \)th member is estimated directly from Eqs. 21, 27 and 28.

\[
\alpha_j = dE_j/E_j = 1/\beta_j - 1 \quad \alpha_j \geq 1
\]

(29)

where damage severity is indicated as the reduction in stiffness in the \( j \)th member if \( \alpha_j < 0 \).

The method described above yields information on the location and severity of damage directly from changes in mode shapes of structures. The appealing features of this method include the following: (1) damage can be located and sized using a few modes; (2) damage can be located and sized without solving a system of equations; and (3) damage can be located and sized in structures containing many members.

4. NUMERICAL VALIDATION OF THEORY

The objective here is to evaluate the feasibility of the proposed algorithm to localize and estimate the severity of damage in a numerical model of a structure when only data on a few modes of vibration are available. We meet this objective in four steps: firstly, a test structure is defined and modal responses of the test structure are generated using the software package ABAQUS; secondly, a damage detection model of the test structure is selected; thirdly, the existing NDD algorithms (Damage Index A and Damage Index B) and the proposed algorithm (Damage Index C) are used to locate and estimate the severity of simulated damage in the test structure; and finally, the accuracy of NDD algorithm is evaluated by quantifying the damage prediction results. Here, by damage detection model we mean a mathematical representation of a structure with degrees of freedom corresponding to actual sensor readings or interpolated readings based on sensor readings at nearby locations.

4.1 Test Structure - Two Span Continuous Beam

The test structure selected here is a theoretical model of a two-span continuous beam. As shown in Fig. 1, the main structural subsystems of the theoretical model consisted of three element groups: (1) 50 beam members modeling the two-span continuous beam section; (2) two linear axial springs (Spring 1) modeling two outside supports; and (3) a linear axial spring (Spring 2) modeling a middle support. A typical arrangement of the test beam corresponding to 51 nodal points is schematized in Fig. 1. In this hypothetical example we assume that only vertical motion is measured at each nodal point.

Values for the material properties of the beam elements and springs were assigned as follows: (1) the elastic modulus \( E = 70 \text{ Gpa} \); (2) Poisson's ratio \( \nu = 0.33 \); and (3) the linear mass density \( \rho = 2710 \text{ kg/m}^3 \). Values for the geometric properties were assigned as follows: (1) for beam elements, the cross-sectional area \( A = 10.5 \times 10^{-3} \text{m}^2 \) and the second moment of area \( I = 5.0 \times 10^{-7} \text{m}^4 \); (2) for Spring 1 member, \( A = 4.96 \times 10^{-4} \text{m}^2 \) and \( I = 0 \); and (3) for Spring 2 member, \( I = 8.4 \times 10^{-5} \text{m}^2 \) and \( I = 0 \).

Next, we measured, via numerical simulation, the pre-damage and post-damage modal responses of the test structure. Here ten damage cases are investigated, as summarized in Table 1. The first eight damage cases are limited to the model damaged only at a single location. Cases 9-10 focus on Element 39 in which three magnitude levels of damage are simulated. The last two damage cases (Cases 9 and 10) consider the model damaged in two locations. In all cases, damage was simulated in the structure by reducing the elastic modulus of the appropriate elements. Typical numerically generated mode shapes and frequencies of the first three modes are shown in Fig. 2 and Table 1.

4.2 Damage Localization and Severity Estimation

We predict locations and severities of damage in the test structure using both the existing NDD algorithms (i.e., Damage Index A and Damage Index B) and the proposed NDD algorithm (i.e., Damage Index C). For each NDD algorithm involved, we perform the damage localization and severity estimation in five steps. In Step One, pre-damage and post-damage modal parameters of the first three modes (as shown in Fig. 2 and listed in Table 1) were obtained from modal analysis of the test structure.
In Step Two, we selected the Euler-Bernoulli beam as the damage detection model on the basis of the fact that the test model is a one-dimensional beam with only vertical motions are available. From the mode shape of the modal vector \( \phi(x) \), we generated a third order spline function, \( w(x) \), for the beam using the 51 nodal displacements. Using the spline approximation of the mode shape, \( \phi(x) = w(x) \) at the 51 modes of the test model. Then equivalent expressions for \( \gamma \), \( \gamma^* \), and \( \gamma_i \) in the damage index equations (e.g., Eqs. 8, 12, and 27) are computed by

\[
\gamma = \int_{x_k}^{x_{k+1}} (\phi'(x))^2 \, dx, \quad \gamma^* = \int_{x_k}^{x_{k+1}} (\phi''(x))^2 \, dx, \quad \gamma_i = \int_{x_k}^{x_{k+1}} (\phi(x))^2 \, dx
\]

in which \( x_k \) and \( x_k + \Delta x \) correspond to two nodal locations of an element \( j \) for the beam model.

In Step Three, we established the classification criterion for damage localization. For a given set of modes, the locations of damage are selected on the basis of a rejection of hypotheses in the statistical sense. Firstly, the values of the damage indicator \( \beta_j \) \( (j = 1, 2, 3, \ldots, n) \) are normalized according to the rule

\[
Z_j = (\beta_j - \bar{\beta}_j) / \sigma_j
\]

in which \( \bar{\beta}_j \) and \( \sigma_j \) are mean and standard deviation of the collection of indicators of \( \beta_j \) values. Next, the member is assigned to damage class via a statistical-pattern-recognition technique that utilizes hypothesis testing. The null hypothesis (i.e., \( H_0 \)) is that the structure is not damaged at the \( j \)th location. The alternate hypothesis (i.e., \( H_1 \)) is that the structure is damaged at the \( j \)th location. We define the decision rule as follows: (1) select \( H_0 \) (i.e., no damage exists at member \( j \) if \( Z_j < 2 \)) and (2) select the alternate \( H_1 \) if \( Z_j \geq 2 \). This criterion corresponds to a one-tailed test at a significance level of 0.023 (97.7\% confidence level).

For Damage Index A, the damage indicator, Eq. 8, and the above criterion were used to select potential damage location. For Damage Index B, we repeated the exercises using the damage indicator, Eq. 12. Finally, for Damage Index C, we repeated the same process using Eq. 28. The predicted results of damage localization for the three NDD algorithms are listed in Table 2.

In Step Four, damage severities were estimated for the predicted damage locations. For Damage Indices A, B, and C, we estimated damage severities using Eq. 9, Eq. 13, and Eq. 29, respectively. The estimated results of damage severities are listed in Table 2.
Table 2. Damage Prediction Results of Two-Span Continuous Beam (*Severity = (E' - E)/E × 100)

<table>
<thead>
<tr>
<th>Damage</th>
<th>Simulated Damage</th>
<th>Predicted Damage (Damage Index A)</th>
<th>Predicted Damage (Damage Index B)</th>
<th>Predicted Damage (Damage Index C)</th>
</tr>
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<tbody>
<tr>
<td>Case</td>
<td>Location</td>
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</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>9</td>
<td>9,34</td>
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<td>9,34</td>
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<tr>
<td>10</td>
<td>14,39</td>
<td>-1.11</td>
<td>-1.11</td>
<td>14,39</td>
</tr>
</tbody>
</table>

4.3 Quantification of Damage Prediction Accuracy

The accuracy of damage prediction results was quantified by measuring both metrical errors and the common errors used in tests of hypotheses. As the first NDD accuracy measure, we selected a mean localization error (mle) which is defined as

$$mle = \frac{1}{N} \sum_{i=1}^{N} |x_i^t - x_i|^2 / L, \quad 0 \leq mle \leq 1$$

where $N$ is the number of damage cases, $x_i^t$ and $x_i$ are the true location and the predicted location of the $i$th damage case, respectively, and $L$ is a characteristic distance. As the second NDD accuracy measure, we selected a detection missing error (dme) which is defined as

$$dme = \frac{1}{NT} \sum_{i=1}^{N} |TI_i|, \quad 0 \leq dme \leq 1$$

where $NT$ is the number of true damage locations, $TI_i$ is the number of Type I errors (i.e., fail-in-detection of true damage locations) for the number of true damage locations. The dme measures false negative errors such that true damage locations are not predicted. As the third NDD accuracy measure, we selected a false alarm error (fae) which is defined as

$$fae = \frac{1}{NF} \sum_{i=1}^{N} |TI_i|, \quad 0 \leq fae < \infty$$

where $NF$ is the number of the predicted locations, $TI_i$ is the number of Type II errors (i.e., prediction of locations that are not damaged). The fae measures false-positive errors such that predicted locations are not the true damage locations. As the last NDD accuracy measure, we selected a mean sizing error (mse) which is defined as

$$mse = \frac{1}{NF} \sum_{i=1}^{N} \left| \frac{\alpha_i^t - \alpha_i^p}{\alpha_i^t} \right|, \quad 0 \leq mse \leq \infty$$

where $\alpha_i^t$ and $\alpha_i^p$ are, respectively, a true damage severity and a predicted damage severity for $i$th location. The mse measures the NDD algorithm’s accuracy in severity estimation and the value close to zero means that the error is close to zero.

We implemented the four NDD accuracy measures given by Eqs. 32-35 to the damage localization and severity estimation results of each NDD algorithm. Then the accuracy of each NDD algorithm was quantified as listed in Table 3.

For Damage Index A, the accuracy measures are analyzed as follows: (1) a dme of 0.085 indicates that eleven out of twelve true damage locations can be predicted; (2) a fae of 0.57 indicates that about six out of ten predicted locations can be false-positive; (3) a mse of 0.133 indicates that damage can be located within about a distance of 13 percent of span length from the correct location of damage; and (4) a mse of 0.75 indicates that the estimated severities show an average 75 percent error and it consistently overestimates severity levels by about 1.75 times of the true damage sizes.

For Damage Index B, the accuracy measures are interpreted as follows: (1) all localization error measures (dme, fae, and mle) are zero and (2) a mse of 0.853 indicates that the estimated severities show an average 85.3 percent error and it consistently underestimates severity levels by about 0.15 times of the true damage sizes.

For Damage Index C, the accuracy measures are interpreted as follows: (1) all localization error measures are zero and (2) a mse of 0.077 indicates that the estimated severities show an average 7.7 percent error. As listed in Table 2, the predicted severities are very close to the true damage sizes. Compared to two other NDD algorithm, Damage Index C enhanced the accuracy of the damage localization and severity estimation results.

Table 3. Quantification of Damage Prediction Accuracy

<table>
<thead>
<tr>
<th>Damage Detection</th>
<th>Measures of Damage Prediction Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm Type</td>
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</tr>
<tr>
<td>Damage Index A</td>
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</tr>
<tr>
<td>Damage Index B</td>
<td>0</td>
</tr>
<tr>
<td>Damage Index C</td>
<td>0</td>
</tr>
</tbody>
</table>
5. SUMMARY AND CONCLUSIONS

The objective of this paper was to present an improved vibration-based damage detection algorithm which was newly-derived and to evaluate the accuracy of the algorithm when applied to a two-span continuous beam. This objective was achieved in two parts. In the first part, we reviewed existing damage detection algorithms and their limits in the accuracy of damage detection. Then we formulated a new damage detection algorithm which overcomes the limits of the existing algorithms and improved its accuracy in damage localization and severity estimation. In the second part, two existing algorithms and the new algorithm were evaluated by predicting damage location and severity estimation in a theoretical model of a two-span continuous beam. Each algorithm was assessed by quantifying the accuracy of damage localization and severity estimation results.

By applying the approach to the numerical example, we obtained the following relationships between the algorithms and their accuracy in damage prediction. First, the use of Damage Index A for the damage prediction exercises resulted in (1) relatively small Type I error (false detection of true damage locations), (2) small localization error, (3) relatively high Type II error (prediction of locations that are not damaged), and (4) high severity estimation error. It consistently overestimated severities of damage by about 1.75 times of the true damage sizes. Second, the use of Damage Index B resulted in no error related to damage localization but high severity estimation error. It consistently underestimated severities by about 0.15 times of the true damage sizes. Finally, the use of Damage Index C resulted in no error related to damage localization and very small severity estimation error. Compared to two other algorithms, Damage Index C enhanced the accuracy of the damage localization and severity estimation results.

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