ABSTRACT. Principal Components Analysis of nonlinear systems is based on the singular value decomposition of a collection of response time-histories. The principal components are analogous to the modal response time-histories of linear structural analysis, except that the singular values are related to energy rather than frequency. This paper presents a theoretical basis for Principal Components Analysis, including the derivation of modal metrics for use in nonlinear model correlation, updating and uncertainty evaluation. A numerical example based on current experience will be presented to illustrate application to nonlinear model validation and verification.

1. INTRODUCTION
Validation and verification (V&V) of structural dynamic models is an iterative process which includes the correlation of model predictions with available test results, qualitative adjustment of model physics to match observed behavior, parameter estimation to minimize the quantitative differences between predicted and measured response, and uncertainty analysis to estimate the predictive accuracy of the model. Previously developed tools for model-test correlation, parameter estimation, and predictive accuracy evaluation are based on the linearity of the equations of motion, the ability to characterize the structure in terms of its normal modes, and evaluate structural response to arbitrary excitation by modal superposition [1, 2, 3].

These tools are not directly applicable to nonlinear models. In principle it may be possible to discretize the response time-history, generate a response surface at each temporal point, interpolate these response surfaces in the time domain, and apply some systematic tools. If the functional dependencies are simple enough to permit such an approach, the number of curve fits is directly proportional to the temporal resolution and can quickly become untenable. The usual approach involves either ad hoc comparisons or generation of response surfaces for certain response variables in terms of the physical parameters of the model. At best, only a few response variables (for example: initial velocity, peak displacement, time to peak, etc.) are treated systematically. This does not permit rigorous characterization of the correlation of the effects of the physical parameters on the response parameters or of the residual modeling error.

An approach which overcomes these limitations is sought. Principal Component Analysis provides such an approach. This method effectively separates the temporal dependency from the spatial and parameter dependency and yields a set of "modes" which facilitates the extension of parameter estimation and uncertainty analysis tools to nonlinear models.

2. PRINCIPAL COMPONENTS ANALYSIS
Principal Components Analysis is based on the singular value decomposition (SVD) of a collection of time-histories. Let \( x(t) \) denote a response time-history, where \( x \) may be displacement, velocity, or any time-dependent quantity of interest. A response matrix, \( X \), is a collection of discretized time-histories,

\[
X = \begin{bmatrix}
x_1(t_1) & \cdots & x_1(t_s) \\
\vdots & \ddots & \vdots \\
x_n(t_1) & \cdots & x_n(t_s)
\end{bmatrix}
\]

(1)

where each row corresponds to either a different measurement location or set of physical parameters, and each column corresponds to response at a specific time. Typically, \( m \neq n \), so \( X \) is generally rectangular. The SVD of \( X \) may be written

\[
X = U \Sigma V^T
\]

(2)

where \( U \) is an orthonormal \( m \times m \) matrix whose columns are the left singular vectors of \( X \), \( \Sigma \) is an \( m \times n \) matrix containing the singular values of \( X \) along the main diagonal and zeros elsewhere, and \( V \) is an \( n \times n \) orthonormal matrix whose columns correspond to the right singular vectors of \( X \). The
left singular vectors and the singular values are the
eigenvectors and square roots of the eigenvalues of $XX^T$, respectively.

The matrices on the right hand side of (2) may be partitioned
so that

$$X = \phi D \eta$$

where $D$ is the diagonal matrix of nonzero singular values, $d_i$ ($i = 1, ..., p \leq \min(m,n)$), $\phi$ and $\eta$ are the matrices of left and right singular vectors, respectively, corresponding to the nonzero singular values, and $\eta$ span the orthogonal complements of the respective subspaces spanned by $\phi$ and $\eta$. By (3),

$$X = \phi D \eta$$

The columns (rows) of $\phi$ ($\eta$) are called the left (right)
principal vectors of $X$ and are pairwise orthonormal, i.e.,

$$\phi^T \phi = \eta \eta^T = I_p$$

where $I_p$ is the $p$-dimensional identity matrix.

The factorization given by (4) is called the Principal Components Decomposition (PCD) of the response matrix. By (4) and (5),

$$XX^T = \phi D^2 \phi^T$$

Note that the trace of (6) is given by

$$\sum_{i=1}^{m} \sum_{k=1}^{n} x_i^2(t_k) = \sum_{i=1}^{n} d_i^2$$

Thus, the sum of the squared singular values is $mn\Delta t$ times the average mean square value of the response, where $\Delta t$ is the discretization interval (assumed constant).

3. APPLICATIONS TO MODEL V&V

3.1 Model Reduction

The PCD furnishes a compact representation of the response of a nonlinear model. The right principal row vectors, $\eta_i$, represent the normalized basic shapes comprising the response time-histories. The squared singular values, $d_i^2$, correspond to the energy content of these shape functions. Each row of the left principal vector matrix, $\phi$, denotes the specific linear combination of the shape functions which reproduces the response time-history at the corresponding value of the parameter vector, $r$, and spatial location of the response.

For simplicity assume that each row of the response matrix corresponds to the response at a single location and a unique set of parameters. Then each left singular vector can be considered as a function of the parameter vector only. If the left singular vectors are considered as functions of the parameter vector, $r$, then $X$ may be approximated by

$$\hat{x}(r) = \hat{\phi}(r) D \eta$$

where the columns of $\hat{\phi}(r), \hat{\eta}(r)$, are represented by

individual response surfaces.

3.2 Parameter Estimation

Principal components methods are useful for parameter estimation in nonlinear models for the same reason that modal properties in linear structural dynamics are useful in comparing analysis and test results, and estimating the parameters of linear models. In linear structural dynamics, the structural modes are conveniently associated with resonant frequencies. This is generally not the case with nonlinear problems. The interpretation of modal properties in general depends on the selection of response data included in $X(t)$. Nevertheless, there are certain properties of the SVD that can be exploited for purposes of parameter estimation. These properties are suggested by the following equations:

$$\psi = \hat{\phi}^T \phi \enspace (Left \ principal \ vector \ cross-orthogonality) \enspace (9a)$$

$$\Delta D = D^{\phi} - D^{\eta} \enspace (Difference \ in \ singular \ values) \enspace (9b)$$

$$\nu^T = \hat{\eta} \eta^T \enspace (Right \ principal \ vector \ cross-orthogonality) \enspace (9c)$$

where $\phi, D, \phi$ and $\eta$ represent modal parameters derived from analysis for comparison with the corresponding modal parameters $\phi, D, \eta$ derived from experimental data.

In parameter estimation, an optimization problem is formulated by defining an objective function to be minimized or maximized. For example, one can define an objective function to minimize the differences between analysis and test results in terms of the three sets of modal properties derived from an SVD. When the principal vectors, $\phi$, can be approximated as a linear combination of the analysis principal vectors, $\phi$ (the degree of approximation is easily verified), then

$$\phi = \hat{\phi} \psi$$

and

$$\Delta \phi = \phi - \hat{\phi} \psi = 0 \phi (\psi - I) = \phi \Delta \psi$$

Agreement between analysis and test modes implies $\phi = \phi$, in which case $\psi = I$ and $\Delta \psi = 0$. 

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The same argument can be applied to the right eigenvectors. In this case, when the eigenvectors, \( \eta \), can be approximated as a linear combination of the analysis eigenvectors, \( ^0\eta \), then

\[
\eta = v^0\eta
\]  

(12)

Since the individual eigenvectors in \( \eta \) are row vectors instead of column vectors as they are in \( \phi \). Then

\[
\Delta \eta = (\eta - 0) = (v - I)^0\eta = \Delta v^0\eta
\]  

(13)

Agreement between analysis and test modes implies \( ^0\eta = \eta \), in which case \( v = I \) and \( \Delta v = 0 \).

Clearly, one may define an objective function where \( \Delta \phi \), \( \Delta D \), and \( \Delta \eta \) are to be minimized. Such an objective function may take the form

\[
J = \sum_j \left( \Delta \psi_j \right)^2 + \sum_j \left( \Delta D_{ij} \right)^2 + \sum_k \left( \Delta v_{jk} \right)^2
\]  

(14)

where \( J \) is a scalar,

\[
\Delta \psi_j = \psi_j - \delta_j
\]  

(15)

\[
\Delta D_{ij} = (D_{ij} - 0) = \psi_j
\]  

(16)

\[
\Delta v_{jk} = v_{jk} - \delta_{jk}
\]  

(17)

and where

\[
\psi = ^0\phi^T \phi,
\]  

(18)

\[
v = \eta^T \eta^T.
\]  

(19)

The matrices \( \psi \) and \( v \) are seen to be the cross-orthogonality matrices between analysis and test eigenvectors (both left and right eigenvectors, respectively).

### 3.3 Uncertainty Analysis

When the Principal Components approach is used to reduce a nonlinear model, the parameters are \( \phi \), \( D \), and \( \eta \), and modeling uncertainty is defined in terms of these modal parameters. With this approach, one is not limited to considering one response variable at a time, and may in fact consider many response variables simultaneously, where all are functions of the same model parameters (in the case modal parameters). Once a covariance matrix of the modal parameters is obtained, it can be transformed to obtain a covariance matrix of the response variables. The predictive accuracy of the model is thereby determined. The following development closely parallels that described in References [1, 2, 3, 4] for linear structural dynamic analysis.

The PC representation of dynamic response, was given in (4) as:

\[
X(t) = \phi D \phi^T
\]

An element of the response matrix, \( X \), is

\[
X_{ij} = e_j^T \phi D \phi^T
\]  

(20)

where the vector, \( e_j \), is the \( j \)th column of the identity matrix, and \( \eta_j \) is the \( j \)th column of the matrix, \( \eta \). Substitution of Equations (10) and (12) into (20) gives

\[
X_{ij} = e_j^T \phi \psi D v^0 \eta_j
\]  

(21)

The parameter, \( r \), is taken to represent any of the elements of the matrices \( \psi \), \( D \), or \( v \). To first order approximation,

\[
\Delta X_{ij} = \sum_k \frac{\partial X_{ij}}{\partial \eta_k} \Delta \eta_k
\]  

(22)

where \( N_{mp} \) denotes the number of modal parameters collectively contained in \( \psi \), \( D \), and \( v \). In order to perform statistical analysis on the modal parameters, \( r \), they must first be normalized. Since \( \psi \) and \( v \) are already orthonormal, it is only necessary to normalize the diagonal matrix of singular values, \( D \). This is done by dividing the elements, \( D_{ij} \), of \( D \) by the corresponding analytical values, \( ^0D_{ii} \). The normalized singular value matrix, \( \tilde{D} \), is simply

\[
\tilde{D} = D^{1/2} D^{-1}
\]  

(24)

With this change of variables, Equation (22) becomes

\[
\frac{\partial X_{ij}}{\partial \eta_k} = e_j^T \phi \left( \frac{\partial \psi_{ij}}{\partial \eta_k} + \frac{\partial \tilde{D}}{\partial \eta_k} + \frac{\partial D}{\partial \eta_k} \frac{\partial \bar{v}}{\partial \eta_k} \right) \eta_j
\]  

(25)

and Equation (23) becomes

\[
\Delta X_{ij} = \sum_k \frac{\partial X_{ij}}{\partial \eta_k} \Delta \eta_k
\]  

(26)

Where \( \Delta \eta_k \) is an element of the vector \( \Delta \eta \).
\[ \Delta \vec{r} = \begin{bmatrix} \text{vec}(\Delta \psi) \\ \text{vec}(\Delta D) \\ \text{vec}(\Delta \nu) \end{bmatrix} \]  

(27)

The vectorized matrices, \( \text{vec}(\Delta \psi) \), \( \text{vec}(\Delta D) \), and \( \text{vec}(\Delta \nu) \) are formed by placing the columns of each matrix sequentially in one long vector. For example,

\[ \text{vec}(\Delta \psi) = \begin{bmatrix} \Delta \psi_{11} \\ \vdots \\ \Delta \psi_{mm} \end{bmatrix} \]  

(28)

where \( m \) is the number of modes in the modal expansion of Equation (8). Since the matrix, \( \Delta D \), is diagonal, the vector, \( \text{vec}(\Delta D) \), has only \( m \) elements instead of \( m^2 \), so that

\[ \text{vec}(\Delta D) = \begin{bmatrix} \Delta D_{11} \\ \vdots \\ \Delta D_{mm} \end{bmatrix} \]  

(29)

The vector, \( \text{vec}(\Delta \nu) \), is similar in form to Equation (28).

Equation (26) has a particularly simple form when the matrix, \( \Delta \psi \), is also vectorized as

\[ \Delta \psi = \text{vec}(\Delta \psi) \]  

(30)

Then Equation (26) becomes

\[ \Delta \psi = T_{\psi} \Delta \vec{r} \]  

(31)

where the elements of \( T_{\psi} \) are populated by the scalar values

\[ \frac{\partial X_k}{\partial \psi_m} = e^\phi D_m e^\eta, \]  

(32a)

\[ \frac{\partial X_k}{\partial D_{nn}} = e^\phi D_{nn} e^\eta, \]  

(32b)

\[ \frac{\partial X_k}{\partial \psi_m} = e^\phi D_m e^\eta. \]  

(32c)

The covariance matrix of the vector \( \vec{r} \) is given by

\[ S_{rr} = \text{E}[\Delta \vec{r} \Delta \vec{r}^T] = \frac{1}{N} \sum_{i=1}^{N} \Delta \vec{r}_i \Delta \vec{r}_i^T \]  

(33)

where the index, \( i \), is on a particular data set consisting of corresponding analysis-test pairs, \( N \) is the total number of data sets in the sample, and the analytical model is assumed to predict mean response. When the analytical model contains bias-type error, then

\[ S_{rr} = \text{E}[(\Delta \vec{r} - \mu_{s_i})(\Delta \vec{r} - \mu_{s_i})^T] = \frac{1}{N-1} \sum_{i=1}^{N} (\Delta \vec{r}_i - \mu_{s_i})(\Delta \vec{r}_i - \mu_{s_i})^T \]  

(34)

where

\[ \mu_{s_i} = \frac{1}{N} \sum_{i=1}^{N} \Delta \vec{r}_i \]  

(35)

In this case, the analytical model is first adjusted by \( \mu_{s_i} \), which is obtained from \( \mu_{s_i} \) by rescaling that portion of \( \mu_{s_i} \) corresponding to \( \Delta D \), i.e.,

\[ \Delta D = \mu_{s_i} D \Delta D \]  

(36)

consistent with (24). One must keep in mind that the original scaling on the singular values is done with the analytical values corresponding to the data in the generic database, i.e. past experience. When rescaling, the analytical singular values corresponding to the new application are used.

Finally, it is recognized that \( S_{rr} \) will be of the form

\[ S_{rr} = \begin{bmatrix} S_{rr} & S_{\psi \phi} & S_{\psi \phi} \\ S_{\phi \psi} & S_{\phi \phi} & S_{\phi \phi} \\ S_{\psi \psi} & S_{\phi \phi} & S_{\psi \psi} \end{bmatrix} \]  

(37)

\( S_{rr} \) represents the generic modeling uncertainty inherent in analytical predictions of the response matrix, \( X \), based on normalized comparisons of previous analysis and test data. In order to evaluate the predictive accuracy of a new response prediction, Equation (31) is used, with the understanding that \( T_{\psi} \) is evaluated with respect to the new model, i.e., the values of \( e^\phi \), \( e^\psi \), and \( e^\eta \) now represent the modal parameters of the new model, rather than those of the models that have been correlated with previous test data. Then

\[ S_{xx} = \text{E}[(\Delta \psi \Delta \psi^T] = T_{\psi} S_{rr} T_{\psi}^T \]  

(38)

Clearly, the foregoing analysis depends on the small perturbation assumptions where
When those conditions are not satisfied, the analytical model must first be adjusted to bring it into closer agreement with experimental data. Then Equations (39) will be satisfied. In fact, Equations (39) should be the criterion for model refinement in the sense that the objective function for model identification is defined so as to minimize $\Delta \psi_i$, $\Delta D_j$, and $\Delta v_j$ for all $1 \leq i \leq m$ and $1 \leq j \leq m$, where $m$ is the number of modes in the modal model. See Eqn. (14).

4. NUMERICAL EXAMPLE

4.1 Pretest Predictive Accuracy

Figure 1a shows a three-dimensional finite element model (FEM) of a test structure. A series of tests was conducted to evaluate the accuracy of first principle, high fidelity physics based (HFPB) models [6]. Figure 1b shows a finite element model of an interior wall of this structure. Internal blast tests were conducted by exploding a charge in the center of the center room, and measuring blast overpressures inside the room and accelerations of the two interior walls. The accelerations were double-integrated to obtain the "measured" displacements. Two accelerometers were located at the center of each wall. The resulting displacements are plotted in Figure 2a along with the displacement predicted by DYNA3D analysis of the FEM in Figure 1b. Figure 2a also shows $\pm \sigma$ and $\pm 2\sigma$ uncertainty bands on the predicted response. These uncertainty bands were derived by the procedure described in Section 3.3, where the entire difference between predicted and measured response was treated as random. A companion plot in Figure 2b shows the predicted response corrected for bias, along with an unbiased estimate of modeling uncertainty.

4.2 Model Validation and Verification

Modeling uncertainty can be reduced and predictive accuracy improved by a systematic process of model validation and verification. In this context, verification of the numerical algorithms in a computational code is assumed.

Model validation is the qualitative process of confirming the validity of a model, in the sense that it represents the essential physics or qualities of a system. The process of validation may include the investigation of alternative models (constitutive models for example) and the selection of one capable of representing essential physics.

Model verification, on the other hand, is the quantitative part of model V&V. It includes updating, i.e. "calibration" or "tuning," in the sense that the parameter values of a validated model are estimated from physical measurements, and the estimates themselves are analyzed for statistical significance and consistency. Statistical parameter estimation (such as Bayesian estimation) provides statistical information including (a) verification that the parameter changes are statistically consistent with the initial estimates of parameter uncertainty, (b) a comparison of revised parameter uncertainties with initial parameter uncertainties to evaluate the statistical significance of the estimates, i.e. how much additional information has been extracted from the data, and (c) the correlation structure of the estimates which is an indication of how much information has been
gained about individual parameters as opposed to correlated sets of parameters; the less the estimates are correlated, the more knowledge one has about the individual parameters. In addition to statistical parameter estimation, model verification includes an evaluation of residual modeling uncertainty and the resulting predictive accuracy of the model.

1.5
Test data
+1-2a uncertainty bounds
+1-<J uncertainty bounds
Nominal analysis

Time (ms)

(a) Predictive accuracy of DYNA3D model

(b) Predictive accuracy of bias-corrected DYNA3D model

Figure 2. Comparison of analysis and test data.

To illustrate model verification for a typical HFPB model, Bayesian estimation was applied to the FEM shown in Figure 3b. A sensitivity analysis was first performed to identify the parameters most affecting center-of-wall response. These parameters are:

1. Concrete strength, \( f'_c \)
2. Charge weight, \( w \)
3. Concrete tensile rate enhancement parameters
   - initial slope, \( m_1 \)
   - knee point, \( k \)
4. Cold joint friction parameters
   - static friction, \( \mu_s \)
   - kinetic friction, \( \mu_k \)
   - decay factor, \( \beta \)
5. Concrete dilatancy fraction, \( \omega \)

The two concrete tensile rate enhancement parameters and the three cold joint friction parameters were assumed to be perfectly correlated, so that only five parameters were estimated.

In order to perform the parameters estimation efficiently, a reduced fast running Principal Components (PC) model was generated. A PC model is similar to a response surface model, except that it uses singular value decomposition (SVD) to separate time-dependency from the rest of the model, resulting in a response surface-type model that produces response time-histories, i.e. vector instead of scalar response surfaces. This enables parameter estimation to be based on time-history data rather than scalar response variables such as peak displacement or velocity.

Bayesian estimation based on this type of reduced model produced the results shown in Table 1. The first column of the table (following the parameter name) lists the initial parameter values, \( r_i \). The second column lists the initial standard deviations, \( \sigma_i \), of these estimates. The third column lists the revised parameters estimates, \( r'_i \). The fourth and fifth columns, respectively, list the ratio of parameter changes to the initial standard deviation, \( \Delta r_i / \sigma_i \), and the ratio of revised-to-initial standard deviations, \( \sigma'_i / \sigma_i \). With Bayesian estimation, the latter ratio is always less than or equal to unity. The remaining portion of Table 1 lists the correlation coefficients, \( \rho_{ij} \), of the updated parameter estimates.

Table 1. Statistical analysis of parameter estimates.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( r_i )</th>
<th>( \sigma_i )</th>
<th>( r'_i )</th>
<th>( \frac{\Delta r_i}{\sigma_i} )</th>
<th>( \frac{\sigma'_i}{\sigma_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete strength</td>
<td>6100</td>
<td>364.0</td>
<td>6117</td>
<td>0.047</td>
<td>0.927</td>
</tr>
<tr>
<td>Charge weight</td>
<td>3.600</td>
<td>0.545</td>
<td>2.930</td>
<td>-1.229</td>
<td>0.175</td>
</tr>
<tr>
<td>Rate enhancement</td>
<td>0.029</td>
<td>0.012</td>
<td>0.010</td>
<td>-1.524</td>
<td>0.717</td>
</tr>
<tr>
<td>Cold joint friction</td>
<td>1.000</td>
<td>0.242</td>
<td>2.040</td>
<td>4.298</td>
<td>0.532</td>
</tr>
<tr>
<td>Concrete dilatancy fraction</td>
<td>0.500</td>
<td>0.121</td>
<td>0.221</td>
<td>-2.306</td>
<td>0.574</td>
</tr>
</tbody>
</table>

One of the criticisms of "model calibration" is that it often amounts to no more than curve fitting in the sense that the model is arbitrarily adjusted to match a given set of data, and there is no guarantee that the model has been
improved. The concern is that the adjusted model actually may be worse. Of course, "the proof is in the pudding," and ultimately one must demonstrate that the calibrated model does a better job of matching other data (not used in the calibration) than the original model. This is only possible when sufficient data exist.

As an alternative, the statistics produced by Bayesian parameter estimation can be used to evaluate the quality of the estimates, i.e. their statistical significance and consistency. The ratios listed in Table 1 are used for this purpose. Figure 3a presents a graphic interpretation of these data. The quantity \( \frac{\sigma_i}{\sigma_i^*} - 1 \times 100\% \) is plotted as a function of \( \Delta r / \sigma_i \) for each of the five parameters. The vertical axis represents the statistical significance of the estimates, while the horizontal axis represents statistical consistency [5]. An increase in confidence of 100\%, for example, means that the information gain of a parameter is 100\%, i.e., the information content doubled. Conversely a gain of 0\% means that the parameter gained no new information from the data. If a parameter (point) falls in the white zone centered on \( \Delta r / \sigma_i = 0 \), the prior estimate is confirmed. If it falls within the second set of white zones, a statistically significant change is indicated. If it falls in the third set of white zones, the estimate is considered statistically inconsistent, i.e. the change is inconsistent with the initially assumed uncertainty. If it falls within a gray zone, it's a judgment call. In this example, the cold joint friction parameter is inconsistent with the assigned uncertainty. In retrospect, insufficient data were included in \( \chi \) to estimate this parameter; wall displacement between the center of the wall and the cold joint is needed. Work in this area is continuing.

The foregoing procedure has proven to be useful in practical applications in that it provides valuable guidance in the qualification of statistical parameter estimates. Figure 3b shows the center-of-wall response of the revised DYNA3D model relative to the mean of the measured time histories, and the original DYNA3D model.

4.3 Posttest Predictive Accuracy

Figure 4 shows the response of the updated model relative to all four test measurements, with \( \pm \sigma \) and \( \pm 2\sigma \) uncertainty bands on the residual differences between the model and the data. The predictive accuracy of the updated model shown in Figure 4 may be compared with that of the original model shown in Figure 2. Figure 4a shows reduced uncertainty bands on the updated model, since the residual error between the updated model and the test data is smaller. Figure 4b shows the uncertainty bands to be further reduced by adding a bias correction to the updated model. The predictive accuracy of the model is seen to progressively improve with parameter estimation, and bias correction.

5. CONCLUSIONS

This paper presents the basic theory of Principal Components Analysis for nonlinear model-test correlation, model updating, residual uncertainty evaluation and predictive accuracy assessment. Application of the theory to nonlinear models closely parallels and extends the theory previously developed for linear models. Modal parameters are used in both cases to reduce, update and quantify the uncertainty of the models. In the case of nonlinear models, the system model cannot be characterized independently of the forcing function, so that superposition of modal responses cannot be used to evaluate the response of the system to arbitrary excitation. However, the response can be separated into time-dependent and non time-dependent
parts, allowing for computationally efficient function approximations.

![Graph](image)

(a) Updated model

![Graph](image)

(b) Updated model with bias correction

Figure 4. Improvement of predictive accuracy through model updating.

The results presented in this paper illustrate the use of statistical parameter estimation and modeling uncertainty analysis in a systematic process of model validation and verification. Modeling uncertainty derived from a Principal Components (PC) analysis of the pretest model and test measurements was used first to quantify the predictive accuracy of the pretest model. Statistical parameter estimation was used next to update the model. Residual modeling uncertainty was then derived from a PC analysis of the updated model and test results. Finally, this residual modeling uncertainty was used to evaluate the predictive accuracy of the updated model.

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