THE MAC REVISITED AND UPDATED

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Abstract:
This paper presents a new model correlation technique based on an iteration of the MAC coefficient. The validation of a complex model such as a complete rotating machine can be very difficult. It depends upon correct localisation of modelling errors which few existing correction methods have succeeded. The MAC value has been used in model correlation, but it is not sufficient to locate errors. A subsequent step to investigate the correlation is to use the COMAC or enhanced COMAC. For a model which is not well correlated the results may suffer from difficulties in mode pairing. A newly developed ranking method based on the MAC iteration procedure and presented here shows a more robust behaviour than the MAC and COMAC. A major reason for this is that a normalisation and mode pairing is no longer necessary. Another advantage of this new tool is that several weighting functions can be built in. All these methods can also be used for modifying dynamic characteristics of a structure, and a combination of these methods provide a toolbox which can be used for judging alternative modifications. A case study using a complete automotive alternator as a complex model for the experimental and the analytical model is presented in this paper.

1.0 Introduction
Numerical models for the prediction of noise and vibration play an important role in the design and optimisation of (rotating) machines. The validity and reliability of these models can be drastically improved by the application of correlation and updating techniques. Model correlation and updating with experimental data aim for the verification of numerical models of dynamic structures. Extensive comparative studies reviewing model updating and correlation methods can be found in papers by Imregun et. al.[1, 2, 3, 41].

In the recent decades numerous papers were published on the verification of FEM models using measured data. Normal mode parameters are most commonly used as a comparison basis to describe the discrepancies between the FEM and the EMA model. Model updating procedures are conceived to adjust FEM models so that the updated modal parameters better fit the experimental modal results.

2.0 Techniques to Compare Dynamic Models
In the design and optimisation of rotating machines, numerical models for the prediction of the noise and vibration behaviour play an important role. The validity and reliability of these models can be drastically improved by comparison with an experimental model. But different comparisons gives different answers. Several methods and a newly developed one is listed in the following sections.

2.1 Direct Comparison
The most common method to compare two sets of data is to plot data set A against data set B. Even so, the method can be used to compare natural frequencies from two different models and to plot experimental values against analytical ones for the available modes. The points of the resulting curve should lie on a straight line of slope 1 for perfectly correlated data. A systematic deviation suggests a consistent error (e.g. in material properties) while large random scattering suggests poor correlation. The mode shapes can also be compared by plotting analytical mode shapes against experimental ones. For well correlated modes the points should lie on a straight line of slope ±1. The slope
of the best straight line through the data points of two correlated modes is described as the model scale factor MSF and is defined in [4] and the result is a real number even if $\phi$ is complex.

$$MSF(\phi_A, \phi_X) = \frac{\{\phi_A\}^T \{\phi_X\}^*}{\{\phi_A\}^T \{\phi_A\}^*},$$  \hspace{1cm} (1)

### 2.2 The Orthogonality Methods

Another method of comparison based on the property of modal orthogonality is the cross orthogonality check:

$$[COM(\phi_A, \phi_X)] = [\phi_A]^T [M_A] [\phi_X]$$  \hspace{1cm} (2)

and the mixed orthogonality check

$$[MOC(\phi_A, \phi_X)] = [\phi_X]^T [M_A] [\phi_X]$$  \hspace{1cm} (3)

techniques which are referred to in [5,6]

For perfect correlation, the leading diagonal elements of the orthogonality matrices must all be equal to 1 while the off-diagonal terms remain at 0.

### 2.3 The Modal Assurance Criterion (MAC)

The Modal Assurance Criterion was introduced at IMAC 1 as "A Correlation Coefficient For Modal Analysis" by R.J. Allemang and D.L. Brown [7]. The modal assurance criterion was built in this paper in analogy of the frequency response function and the coherence function with respect to noise contamination and error evaluation. The MAC is now used as one of the primary tools for correlating two sets of mode shapes in modern modal analysis software. The modal assurance criterion is a correlation of each single mode shape pair end is defined in [7] for test mode $j$ and analyzed mode $i$ as:

$$MAC(\phi_{iA}, \phi_{jX}) = \frac{\{\phi_A\}_i^T \{\phi_X\}_j^T \{\phi_A\}_i \{\phi_X\}_j}{\{\phi_A\}_i^T \{\phi_A\}_i \{\phi_X\}_j^T \{\phi_X\}_j^*}.$$  \hspace{1cm} (4)

A MAC value close to 1 suggests that the two modes are well correlated and a value close to 0 indicates uncorrelated modes. State-of-the-art is to compare only translational DOFs. By combining straight forward data sets with rotational and translational DOFs the MAC gives invalid results due to the incoherent units and normalization.

### 2.4 The Co-ordinate Modal Assurance Criterion (COMAC)

The Co-ordinate Modal Assurance Criterion (COMAC) seeks to identify those DOFs of the structure which demonstrate low degrees of correlation. The COMAC correlates to two sets of mode shapes, either from a test or finite element model, and then identifies those DOFs at which the mode pairs consistently disagree.

Before using the COMAC, other correlation tools have to be used to identify the "pre-correlated" mode pairs. After constructing a set of $L$ correlated mode pairs, a correlation value is calculated for each co-ordinate over all correlated mode pairs as follows [5]:

$$COMAC(i) = \frac{\left(\sum_{r=1}^{L} (\psi_A)_r (\psi_X)_r^*\right)^2}{\sum_{r=1}^{L} (\psi_A)_r^2 \sum_{r=1}^{L} (\psi_X)_r^{2*}}$$  \hspace{1cm} (5)

This correlation method weights all degrees of freedom equally regardless of whether or not they have large or small mode shape coefficients.

A slightly different formulation has been invented by SDRC (Structural Dynamic Research Corporation) which weights the relative difference between degrees of freedom [9].

$$COMAC(i) = 1 - \frac{\sum_{r=1}^{L} |(\psi_A)_r - (\psi_X)_r|}{2L}$$  \hspace{1cm} (6)

A difference at a degree of freedom which has relatively large amplitudes will have more a significant contribution to the final COMAC value than a difference at a degree of freedom with relatively small coefficients. Hence, discrepancies that might occur between mode shapes at points on the structure that exhibit very little motion will not heavily affect the calculation.

### 3.0 A New Iterative Correlation Method

The starting point for the new correlation method is the definition of the Modal Assurance Criterion (MAC). The MAC value as described in [4] gives the correlation factor for each mode pair and can also written as:

$$MAC(A, X) = \frac{\left(\sum_{j=1}^{n} (\phi_X)_j^T (\phi_A)_j\right)^2}{\sum_{j=1}^{n} (\phi_X)_j^T (\phi_X)_j \sum_{j=1}^{n} (\phi_A)_j^T (\phi_A)_j}$$  \hspace{1cm} (7)
By an iterative cancelling of a single DOF\( (k) \) in each calculation, the DOF which produce a poor correlation in each mode pair can be identified in each iteration \( I \).

\[
MAC_{1}(A, X) = \frac{\sum_{j = 1}^{n} (\phi_{X})_{j} (\phi_{A})_{j}^*}{\sum_{j = 1}^{n} (\phi_{X})_{j} (\phi_{A})_{j}^*}
\]

under the condition of

\[
MAC_{1}(A, X) - MAC_{1, i}(A, X) = \max
\]

The optimised result of the \( MAC_{1}(A, X) \) gives a different cancelled DOF for each mode pair. Therefore, the output after \( I \) iterations is a better MAC value but the list of cancelled DOFs differs from mode pair to mode pair. More detailed information is needed to investigate a systematic connection, why and where are the DOFs which leads to the best MAC value over all modes?

The accumulated improvement by leaving out a single DOF over all mode pairs and iterations gives a value which indicates the increase of the MAC value by cancelling one specific DOF, \( d \). The higher this value, the higher is the influence of the specific DOF. To focus only the MAC values that give reliably high MAC values (along the diagonal), the result is weighted by the starting MAC value. The product of this is called the High-Sensitivity-Point-Value (HPV) and is defined with following equations.

The first formula is the iteration without the specific DOF, \( d \).

\[
MAC_{1}(A, X) = \frac{\sum_{j = 1}^{n} (\phi_{X})_{j} (\phi_{A})_{j}^*}{\sum_{j = 1}^{n} (\phi_{X})_{j} (\phi_{A})_{j}^*}
\]

The idea is to identify the specific DOF or set of DOFs with the highest improvement by comparison of two models (e.g. analytical and experimental). The Eq. 10 gives a result of the MAC value without the specific DOF, \( d \). If, by cancelling an arbitrary DOF, the improvement is no better than the value without the DOF, \( d \), a discrimination value \( (DV) \) can be set. Where in \( MAC_{1}(A, X) \) the specific DOF, \( d \), is excluded and in \( MAC_{1d}(A, X) \), it is included.

\[
DV = \frac{1}{2} \left[ MAC_{1d}(A, X) - MAC_{1}(A, X) + MAC_{1d}(A, X) - MAC_{1}(A, X) \right]
\]

If there is no better improvement by cancelling the DOF, \( d \), the discrimination factor \( DV \) will be equal to 0.

Further, the high sensitive point value (HPV) is then defined as:

\[
HPV(d) = \sum_{i = 1}^{a} \sum_{j = 1}^{b} DV(MAC_{1d}(A, X) - MAC_{1,i}(A, X)) MAC_{1}(A, X)
\]

The advantages of the new method are:

1. normalisation of the mode shapes is not necessary,
2. there is no mode pairing required,
3. no wrong integration of false mode pairing.

As a result, this technique is more robust than the COMAC and the enhanced COMAC. In section 6, results are shown in a case study, where the efficiency of the new technique is demonstrated.

### 4.0 Case Study: A fully equipped Automotive Alternator for a Dynamic Test and Simulation in High Frequency Range

An experimental modal analysis and finite element analysis were made of a fully equipped automotive alternator under operating boundary conditions. The dynamic test and analysis were undertaken on the non rotating alternator.

For simulation software for the automotive alternator, the pre- and post-processor from I-DEAS was chosen, and for a FE solver, the software PERMAS was employed. After validation of each single part the major task is to couple the individual parts to simulate the complete machine for the purpose of predicting its sound.

#### 4.1 The Finite Element Model of an Automotive Alternator

The alternator FE model consists of six substructures and individual parts which are outlined in Fig. 1. The substructures consist of A-housing, B-housing, stator, cooling plate and mounting arm and brackets are fixed together by screws in the experimental structure. In the FE model all screws are idealised with volume elements. The nodes on the circumference are only coupled in the three transla-
tional degrees of freedom. The ball bearings are idealised with spring elements in the radial direction and, at the A-housing, additionally in axial direction. Also the poly-v-belt is idealised with simple spring elements. Finally the whole FE-model has the size of more than 17,000 Elements, 27,000 nodes and 81,000 DOFs.

4.2 Experimental Modal Analysis of an Automotive Alternator

One aim of an experimental modal analysis is to validate the finite element model over a frequency range up to 2000 Hz. Hence, there is a fine mesh distribution required to avoid aliasing. At the alternator the mesh has 137 nodes each with three degrees of freedom. The distributions of the nodes are as follows:

(i.) nodes on the rotor
(ii.) nodes on the A-housing
(iii.) nodes on the B-housing
(iv.) nodes on the cooling plate
(v.) nodes on the mounting arm
(vi.) nodes on the stator

Dynamic tests were carried out on the alternator using a test rig specially developed for non-linear structures under operating condition [11].

The extraction of the mode shapes from the measured FRFs was made using the software package I-DEAS (Integrated Design Engineering Analysis Software) from SDRC (Structural Dynamics Research Corporation) on an HP Workstation 7000 and for more complicated close modes a detailed analysis with ICATS Software (Imperial College Analysis and Test Software) was performed.

Previous studies [12] have shown that the natural frequencies of the alternator are very closely spaced and the damping is greater their spacing. To identify each single mode, the MDOF method is the adequate method.

5.0 Application of the New Correlation Method on a Large Modal Model

A primary difficulty in noise and vibration simulation of complex structures is that the frequency range of interest encompasses "dozens" of system modes in the noise-relevant frequency range.

In addition, the effect of the connection between each as assembled part and variations in construction makes it quite impossible to develop a finite element model that is accurate up to 2000 Hz. Furthermore, it is impossible to research the structure without a set of independent test methods.

The first task to validate an assembled structure is to validate each single part of the structure under free-free conditions. Despite the good correlation results of single design parts (diagonal MAC values >0.8), the correlation of the assembled structure gives a poor result with MAC values below than 0.3 (Fig. 2).

For example, the gapping effect (Fig. 3) could be correlated by eye, but the original MAC value of 0.35 was quite low.
This led to the idea that a subset of DOFs might pollute the MAC value. To improve the model, the developed method was employed. After 15 iterations and extraction of 15 LXX’s, the MAC-matrix showed a better quantitative result (Fig. 4).

The highest MAC value in the MAC in the Previous matrix (Fig. 3) was only 0.58. This means there is a poor correlation between the two models. After cancelling less than 4% of the DOFs the diagonal MAC values increased to 0.8. The further calculation by ranking the sensitive DOFs shows that the most sensitive DOFs were mostly in the Z-direction which is shown in Fig. 5.

6.0 A Comparison between the COMAC and the New Iteration Procedure in Situ

The large FE and experimental models of the alternator were used to test the correlation methods in practice on a real model. After this test, the differences are discussed.

The enhanced COMAC need the same normalisation of the mode shapes for a calculation. Usually the experimental and the analytical result came from different packages and it can happen that the normalisation of the modes is different (e.g. I-DEAS Test and Permas). Therefore, it has to be carefully checked, if there is a unique normalisation of the mode shapes, otherwise the result is wrong!

After normalisation of the mode shapes, the relevant vectors have to be paired. For models with a prominent diagonal line in the MAC-matrix this will be an easy task, but if there is no such prominent line the user must choose the mode pairing. In this example, the following mode orders were paired (Fig. 6).
The previous figures show clearly the influence between mode pairing and the COMAC results. In Fig. 8 completely different regions of the alternator are indicated than in the Fig. 7.

To give a more detailed look at the correlation values, the different methods are plotted separately against each degree of freedom (Figs. 9 - 11).
It can be seen in Figs. 9 - 11 that the values obtained from the iteration procedure are less sensitive to the mode order than are both the COMAC calculations. With the new ranking technique it was possible to identify the least uncorrelated DOFs in one run without mode pairing.

7.0 Discussion of the Results

The comparison of the correlation tools shows the power and the weakness of each. The most robust method is the HPV criterion. The main advantage to this method is that there is no mode pairing or normalisation necessary and, therefore, it is easier to use.

The enhanced COMAC comes very close to the result of the HPV-value when the correct mode pairs are used.

The normal COMAC does not weight the amplitudes of the DOFs. It can happen that in fixed structures DOFs with small amplitudes in the fixation have a great effect on the body motion. This effect can also be used as an indicator. A good example for this is also seen on the alternator. Only the COMAC indicates that in one case the bracket region is a prominent part (Fig. 8). Small effects in the bracket region can indeed influence the vibration in the upper region of the alternator.

The differences between and how to use the methods are outlined in Fig. 13 for the COMAC and enhanced COMAC and the more straightforward calculation route for the iteration procedure is outlined in Fig. 14.
9.0 Conclusions

With the iterative MAC method an additional tool for test analysis comparison and correlation is introduced. Correlation methods are also tools which can be used for judging design improvement.

For large and complex simulation models the interpretation of correlation is not an easy task and the results often have far-reaching consequences in the design process. Hence it is good to have an additional tool to look at the problem from a different angle. In this sense, the HPV factor can also easily be changed by adding a different weighting function. The usage of the different correlation techniques as tool box provides a great understanding of the structural-acoustic interaction of a product. Furthermore by an inverse application these techniques can be also used to improve the whole design of a rotating machine by answer the main question of an design improvement task:
(i.) Where/what is the most effective design change to reduce noise and vibration?
(ii.) Which component of the machine is mostly responsible for noise and vibration?

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References:

1 Imregun,M.; Ewins,D.J.; Hagiwara,l and Ichikawa,T.
   A comparison of Sensitivity and Response Function based on Updating techniques.
   Proc. IMAC XII, 1994, pp. 1390-1400

2 Visser,W.J.
   Updating Structural Dynamics Models Using Frequency Response Data

3 Visser,W.J. and Imregun,M.
   A Technique to Update Finite Element Models using Frequency Response Data
   Proc. IMAC IX 1991

4 Ewins, D. J.
   Modal Testing: Theory and Practice
   Research Study Press 1984

5 Mottershead, J.E.
   A Method for Improving Finite Element Models by Using Experimental Data

6 Targoff, W.P.
   Orthogonality Check and Correction of Measured Modes

7 Allemang,R.J. and Brown,D.L.
   A Correlation Coefficient for Modal Vector Analysis

8 Lieven,N.A.J. and Ewins,D.J.
   Spatial Correlation of Modes, the Co-ordinate Assurance Criterion (COMAC)
   Proc. IMAC VI, 1988, pp. 690-695

9 Chu, D.F.H.; Debroy, J.M.; Yang, J.
   Pitfalls Of Mass Orthogonality Check
   Proceedings IMAC VII, 1989

10 Brughmans, M.; Leuridan, J.; Blauwkamp, K.
    The Application of FEM-EMA Correlation and Validation Techniques on Body in white
    Proceeding Link Rev. 3, Leonberg, 1995

11 Blaschke, P.G.; Ewins, D. J.
    Vibro-Acoustic Model for the noise reduction of a cm alternator

12 Blaschke, P.G.
    Prediction of the Acoustic and Vibration Behaviour Of Technical Products
    Imperial College 12 Dynamic Section, Internal Report, 1994