ABSTRACT
This paper describes the estimation of the sound power radiated from a modified structure using an aluminum plate as a simple example. The modal parameters of the original, unmodified aluminum plate are first determined by the experimental modal analysis. Second, those of the modified aluminum plate are calculated by the Modal Synthesis Method. Using these obtained modal parameters, the three factors for estimation - total loss factor, radiation loss factor, and driving point mobility - are calculated for the original and modified aluminum plates, respectively. The sound powers radiated from the original and modified aluminum plates are estimated by giving an exciting force. Comparison of the estimated sound power changes with the measured changes obtained by the sound intensity method shows that the tendencies of the changes caused by the modification are well estimated. The Modal Synthesis Method is hence effective to estimate the sound power after structural modification.

1. INTRODUCTION
We are researching the estimation method of the sound power radiated from a vibrating structure after structural modification by using the Modal Synthesis Method and the experimental modal analysis. Firstly, the modal parameters of an unmodified flat aluminum plate were determined by the experimental modal analysis: the absolute frequency response of the sound power radiated from the aluminum plate was then estimated based on the proposed estimation method of sound radiation power. Secondly, the modal parameters of the modified aluminum plate, which was reinforced by adhering a small plate of same thickness were calculated by the Modal Synthesis Method; the absolute frequency response of the sound power radiated from the modified aluminum plate was then estimated based on the proposed estimation method. Finally, the estimated change of the sound power radiation were compared with the changes determined from the sound radiation powers measured before and after the structural modification by the sound intensity method.

2. BASIC THEORY FOR THE ESTIMATION OF SOUND RADIATION POWER

2.1 EQUATION FOR THE ESTIMATION OF SOUND RADIATION POWER
Equation (1), which was derived based on the definition of loss factor, is the basic equation for the estimation of sound radiation power.

\[
W_{\text{rad}}(\omega) = \frac{\eta_{\text{rad}}(\omega)}{\eta_{\text{dis}}(\omega)} \cdot \frac{P_0(\omega)^2}{2} \text{Re}[Y(\omega)]
\]  

(1)

Total loss factor, \( \eta_{\text{rad}}(\omega) \), is the ratio of the total consumption energy to the vibration energy of a vibrating
structure. Radiation loss factor, \( \eta_{\text{rad}}(\omega) \), is the ratio of the acoustic consumption energy to the vibration energy of the vibrating structure. Driving point mobility, \( Y(\omega) \), is the quantity representing the sensitivity of vibration of the structure. Calculating the frequency response of the three factors with the modal parameters—natural frequencies, modal masses, modal dampings, and mode vectors—of the vibrating structure, we can estimate the sound power radiated from the structure by giving the external exciting force, \( P_0 \).

2.2 TOTAL LOSS FACTOR

Total loss factor is calculated with Eq. (2)

\[
\eta_{\text{dis}}(\omega) = \frac{\left( \sum \eta_{\text{dis},m} \cdot \omega_m \cdot m_m \cdot F_{0,m} \right) \omega}{\sum \omega_m^2 \cdot m_m \cdot F_{0,m}}
\]

where:

- \( \eta_{\text{dis}}(\omega) \): total loss factor
- \( \omega_m \): the \( m \)-th angular natural frequency
- \( m_m \): the \( m \)-th modal mass
- \( F_{0,m} \): the \( m \)-th modal displacement

The \( m \)-th modal displacement is calculated by the following equation.

\[
F_{0,m} = \frac{P_0 \cdot \Phi_{m,0}}{m_m \cdot \omega_m \sqrt{1-(\omega/\omega_m)^2 + (2 \zeta_m \cdot \omega/\omega_m)^2}}
\]

where:

- \( \Phi_{m,0} \): the value of the \( m \)-th modal vector on the driving point

2.3 DRIVING POINT MOBILITY

The driving point mobility is the ratio of the vibration velocity to the exciting force. Equation (4) is the transfer function between the displacement vibration and the exciting force.

\[
H_{ii} = \frac{(\Phi_{mi} \Phi_{mi})}{(-\omega^2 + \omega m^2 + j2 \zeta_m \omega \cdot m)}
\]

\( \Phi_{mi} \): the value of the \( m \)-th modal vector at the node \( i \)

The driving point mobility is given by the first derivative of Eq. (4).

2.4 RADIATION LOSS FACTOR

Radiation loss factor is calculated by Eq. (5).

\[
\eta_{\text{rad}}(\omega) = \frac{W_{\text{rad}}}{\omega \cdot E_p}
\]

where:

- \( W_{\text{rad}} \): sound radiation power
- \( \omega \): angular excitation frequency
- \( E_p \): vibration energy

\( W_{\text{rad}} \) is given by the following volume integral calculus of sound intensity.

\[
W_{\text{rad}} = \int_0^\pi \int_0^{2\pi} \frac{P^2}{\rho_{\text{air}} \cdot V_{\text{air}}} R^2 \cdot \sin \phi \cdot d\phi \cdot d\theta
\]

where:

- \( P \): the rms value of sound pressure
- \( \rho_{\text{air}} \): air density
- \( V_{\text{air}} \): sound velocity
- \( R \): the radius of a semi-sphere, on which sound pressure is measured

The rms value of sound pressure is given by the following equation considering the phase difference of vibration and the time delay due to the propagation of vibration from a point of the structure to the measuring point on a semi-sphere.
\[ P = \frac{\rho \cdot \omega^2}{2\pi r} \left[ \int \int \left( \sum_{mode} \omega \cdot d \rho \right)^2 \right] + \frac{\rho \cdot \omega^2}{2\pi r} \left[ \int \int \left( \sum_{mode} \sin(\phi) \cdot d \rho \right)^2 \right] \]

\( U \) : vibration displacement
\( \Sigma \) : summation with respect to mode
\( \phi \) : phase of vibration
\( d_\rho \) : distance between a point of a vibrating structure to the measuring point of sound pressure on a semi-sphere

3. ESTIMATION AND MEASUREMENT

3.1 EXPERIMENTAL MODAL ANALYSIS

Figure 1 shows an aluminum plate 400mm \( \times \) 200mm in size and 5mm thick, one shorter end of which is fixed on a frame composed of L-shaped steels. The experimental modal analysis was carried out by exciting the exciting point and measuring the accelerations on forty measuring points of the aluminum plate. Those points are shown in Fig 2. From the results fifteen natural modes of flexural vibration were determined; the obtained natural frequencies, modal dampings, and mode shapes are summarized in Table 1.

![Fig. 1 Experimental Object](image1)

![Fig. 2 Tested Aluminum Plate](image2)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural Frequency (Hz)</th>
<th>Modal Damping Ratio (Hz)</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>16.6</td>
<td>3.75</td>
<td>1st Bending</td>
</tr>
<tr>
<td>2nd</td>
<td>108</td>
<td>0.727</td>
<td>1st Torsional</td>
</tr>
<tr>
<td>3rd</td>
<td>121</td>
<td>0.581</td>
<td>2nd Bending</td>
</tr>
<tr>
<td>4th</td>
<td>290</td>
<td>0.481</td>
<td>2nd Bending</td>
</tr>
<tr>
<td>5th</td>
<td>323</td>
<td>0.505</td>
<td>2nd Torsional</td>
</tr>
<tr>
<td>6th</td>
<td>336</td>
<td>0.573</td>
<td>2nd Torsional</td>
</tr>
<tr>
<td>7th</td>
<td>408</td>
<td>0.379</td>
<td>2nd Torsional</td>
</tr>
<tr>
<td>8th</td>
<td>460</td>
<td>0.804</td>
<td>2nd Torsional</td>
</tr>
<tr>
<td>9th</td>
<td>474</td>
<td>0.606</td>
<td>3rd Bending</td>
</tr>
<tr>
<td>10th</td>
<td>485</td>
<td>0.467</td>
<td>3rd Bending</td>
</tr>
<tr>
<td>11th</td>
<td>518</td>
<td>0.753</td>
<td>3rd Bending</td>
</tr>
<tr>
<td>12th</td>
<td>670</td>
<td>0.421</td>
<td>3rd Bending</td>
</tr>
<tr>
<td>13th</td>
<td>703</td>
<td>0.273</td>
<td>Longitudinal Bending</td>
</tr>
<tr>
<td>14th</td>
<td>733</td>
<td>0.328</td>
<td>3rd Torsional</td>
</tr>
<tr>
<td>15th</td>
<td>768</td>
<td>0.587</td>
<td>4th Bending</td>
</tr>
</tbody>
</table>
3.2 ESTIMATION OF SOUND RADIATION POWER

The sound radiation power of the original, unmodified aluminum plate was estimated using the modal parameters of fifteen modes determined by the experimental modal analysis. Figure 3 shows the frequency response of the exciting force, the total loss factor, and the radiation loss factor of the plate. Small circles in the figure show the modal total loss factors which were measured by the experimental modal analysis; those values are twice as large as the corresponding modal damping ratios. Radiation loss factor is from 1/10 to 1/100 times as large as the obtained total loss factor. From comparison of the radiation loss factor shown in Fig. 3 with the natural modes listed in Table 2, it is noticed that the frequency range where the radiation loss factor is lower than in the other frequency range almost corresponds the frequency range where the torsional modes of flexural vibration exist.

Figure 4 shows the estimated and experimentally determined driving point mobilities. The estimated peaks coincided well with the measured peaks of driving point mobility. Since this driving point mobility has a large influence on the estimation of sound radiation power, it is required to fit a curve as precisely as possible in the experimental modal analysis.

Figure 5 shows the comparison of estimated sound radiation power with the measured values: As seen in the figure, the estimated peaks coincided well with the measured peaks, though there were small differences between them.
3.3 ESTIMATION OF THE SOUND RADIATION POWER AFTER STRUCTURAL MODIFICATION

A reinforcement of aluminum plate 50mm × 200mm in size and 5mm thick was adhered on the original aluminum plate as shown in Fig. 6. To determine the modal model, namely the modal parameters of the reinforced aluminum plate, the modal model of the reinforcement itself was first determined by the FEM modal analysis. Then, the modal model of the reinforced aluminum plate was determined by the Modal Synthesis Method combining the modal parameters of the reinforcement and the original unmodified aluminum plate.

The sound power radiation after the structural modification was estimated using the obtained modal parameters. One example of the results is shown in Fig. 7.

From the figure, the followings are noticed on the remarkably high four peaks.
1) The peak which exists near 120Hz is reduced by 6 dB.
2) The peak which exists near 290Hz is reduced by 2 dB.
3) The peak which exists near 500Hz is reduced by 5 dB.
4) The peak which exists near 700Hz is shifted to approximately 830Hz, and the level is increased by 3 dB.

To confirm these changes of the four peaks due to structural modification, the sound radiation powers were measured before and after the modification by the sound intensity method. Fig. 8 shows the reinforced aluminum plate. The measured frequency response of sound radiation power were shown in Fig. 9. From the figure, the changes of sound power are summarized for the four peaks mentioned above as follows:
1) The peak which exists near 120Hz is reduced by 10 dB.
2) The peak which exists near 290Hz is reduced by 5 dB.
3) The peak which exists near 500Hz is reduced by 3 dB.
4) The peak which exists near 700Hz is shifted to approximately 840Hz, and the level is increased by 2 dS.

From the comparison of estimated and measured changes of sound radiation power, it is clear that the tendencies of the sound radiation power changes are well estimated by the Modal Synthesis Method with the proposed estimation theory of sound power.

4. CONCLUSION
A new estimation method of sound power has been proposed based on the experimental modal analysis and the Modal Synthesis Method to estimate the changes of sound power before and after a structure is modified. From some experiments and estimations of sound radiation power using an aluminum plate as a simple example, it has been cleared that the proposed new method is effective since the tendencies of sound power changes after structural modification were well estimated.

REFERENCE