The Application of Stepped Sine and Normal Mode Testing to Automotive Structures

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Abstract

Shaker excitation techniques such as burst random, pure random, chirp, etc. are common methods that are used in experimental modal analysis to determine the “best” set of modal parameters. These modal parameters are generally considered to be a good linearized estimate of the dynamic properties of the structure.

Many structures do not have linear dynamic properties and may have high modal density in the frequency range of interest. Sine dwell test methods, such as digital stepped sine and normal mode tuning, are two excitation/measurement methods that allow the specific input amplitude control necessary to characterize system nonlinearities or measure response to a specific force condition in the nonlinear range.

This paper presents the results of applying stepped sine and normal mode tuning to a fully trimmed automotive body. The resulting modal properties, frequency response characteristics, and test setup procedures are compared with the more common burst random properties. The ability of the techniques to assess the nonlinear characteristics and to extract real normal modes of vibration will be presented.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>[M]</td>
<td>mass matrix</td>
</tr>
<tr>
<td>[C]</td>
<td>damping matrix</td>
</tr>
<tr>
<td>[K]</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>[B]</td>
<td>stiffness matrix (non-linear part)</td>
</tr>
<tr>
<td>{x}, {v}, {a}</td>
<td>acceleration, velocity, and displacement vectors</td>
</tr>
<tr>
<td>{F}</td>
<td>forcing function</td>
</tr>
<tr>
<td>{Φ}</td>
<td>matrix of modal vectors</td>
</tr>
<tr>
<td>[q]</td>
<td>modal coordinates</td>
</tr>
<tr>
<td>ω</td>
<td>frequency (rad/sec)</td>
</tr>
<tr>
<td>[F]</td>
<td>matrix of forcing functions</td>
</tr>
</tbody>
</table>

Introduction

Broadband random modal testing has long been the standard for a majority of modal testing, specifically in the automotive industry. Given the computing power presently available, this method yields quick results, making it possible to gain insight into a structure’s dynamic behavior in an efficient manner. One tradeoff with broadband testing however is the employment of random excitation, which causes the structure to respond in a nondeterministic fashion and thus limits the ability to study critical frequencies or nonlinearities in detail.

Stepped sine testing involves measuring the structure’s response at steady state to a sinusoidal forcing vector. For a linear system, the response of the structure at discrete frequency intervals would be the same as that arrived at by random testing. In reality, however, most structures of interest exhibit some nonlinearities as the relative displacements between points are varied (consider clearances) or changes in input force drive the structure into the range where nonlinearities are significant (as in the case of hardening springs). Deterministic sinusoidal excitation allows maximum control of excitation forces and/or responses permitting nonlinear phenomena to be better studied. As dynamic modeling of automotive structures becomes more sophisticated, it may be necessary to supplement the approximate linear model with details of nonlinear behavior to explain critical phenomena.

In normal mode tuning, the structure is spatially forced to vibrate in one of its natural modes. This shape is realized by applying a suitable forcing vector to the structure that ideally eliminates the contribution of all other modes to the total response. The resulting undamped normal mode shapes and natural frequencies correlate directly to finite element models and allow the tuned mode to be studied in greater detail (i.e. response to varying force levels, etc.).
**Stepped Sine Background**

Consider an example of a nonlinear system such as the hi-linear spring/mass/damper system below:

$$[M] \{x\} + [C] \{x\} + [K] \{x\} + [B] \{x^3\} = \{f\}$$  \hspace{1cm} \text{Eq.1}

The system response will be:
- Linear if [B] terms are 0
- Quasi-linear if [B] or \(x^2\) terms are small
- Nonlinear if [B] or \(x^3\) terms are significant

In the latter two cases, since [B] terms are a property of the structure, it is clear that \(\{x\}\) and consequently \(\{f\}\) must be controlled to characterize the properties of the system, which are now a function of the applied force \(\{f\}\).

Applying input with random amplitude and frequency content to a system such as Eq.1 inherently limits the ability to control the \(\{f\}\) and \(\{x\}\) that govern the nonlinear system response. If multiple averages are taken it is obvious that each average has the potential to exhibit different system properties. For this reason, random excitation applied to a nonlinear system is considered to yield the best fit, linearized response given an average input/output amplitude.

Stepped sine testing allows specific allocation of excitation frequency and amplitude, thereby allowing the exact response at a force condition to be studied.

An important consequence of accurate input control is the ability to build dynamic models corresponding to specific force input or displacement conditions. For an inherently nonlinear structure such as an automobile, most analysis is carried out upon or related to linearized modal models. Under actual operating conditions, the linear model may not sufficiently describe vehicle dynamics.

Multiple reference stepped sine excitation requires a number of frequency sweeps at least equal to the number of exciters factorial (n!). Each with different force vectors, to permit calculation of full ranked FRF's[8]. This, of course, can lead to large increases in time as the number of references increases. In the study of nonlinearities, it is important to ensure consistent system properties between averages [8].

**Normal Mode Tuning Background**

A linear, proportionally damped system of the form:

$$[M] \{\dot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = \{F\}$$  \hspace{1cm} \text{Eq.2}

consists of coupled differential equations that can be uncoupled by transformation into modal coordinates by:

$$\{x\} = [\Phi] \{q\}$$  \hspace{1cm} \text{Eq.3}

Where \([\Phi]\) is the matrix satisfying:

$$[K] - \omega^2 [M] \{\Phi\} = 0$$  \hspace{1cm} \text{Eq.4}

The eigenvalues and associated eigenvectors that satisfy Eq.4 are the undamped natural frequencies and normal modes of the system. For a proportionally damped system \([\Phi]\) will be composed of real elements.

Solution of the uncoupled equations in the modal coordinate, \(q\), yields the contribution of each made to the steady state response. Realizing the solutions will be of the form:

$$q_n = \frac{[\Phi]^T \{F\}_n}{(K_{nn} - M_{nn} \omega^2) + jC_{nn} \omega}$$  \hspace{1cm} \text{Eq. 5}

it is clear that the contribution of any mode to the steady state response will be negligible if the denominator of Eq.5 is sufficiently large for that mode or the numerator goes to zero. The former is likely to occur when the applied excitation frequency is significantly different from the natural frequency associated with the mode. These modes are considered to be nonparticipating modes. The contribution of other modes occurring in the neighborhood of the mode of interest must be eliminated by forcing the denominator to zero in Eq.5 by proper choice of the forcing vector \(\{f\}\).

**Suppression of Neighboring Modes**

Consider a n DOF system with r available excitation forces and m participating modes. This implies that m-1 modes must be suppressed to best approximate pure normal mode behavior. Each available excitation force is placed at an arbitrary measurement location to yield a forcing vector of the form:

$$\{F\} = \begin{bmatrix} f_1 & 0 & f_2 & 0 & \ldots & 0 & f_r \end{bmatrix}^T_{1 \times n}$$  \hspace{1cm} \text{Eq.6}

A basis describing all possible forcing vectors for these given force locations can be written as:

$$[F] = \begin{bmatrix} 1 & 0 & 0 & 0 \ldots & 0 & 0 \\ 0 & 1 & 0 & 0 \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix}_{r \times n}$$  \hspace{1cm} \text{Eq.7}

This implies that any possible forcing vector can be constructed by a linear combination of the vectors in \([F]\).
Now consider the right hand side of E4.5.
\[
[\Phi]_{n \times m}^T [F]_{n \times r} = [A]_{m \times r}
\]
Eq. 8

where the columns of [A] now correspond to the modal contributions of each individual exciter force.

In order to tune a normal mode, the modal contributions of the undesirable modes participating in the response must be combined in such a way as to eliminate one another. This corresponds to combing the modal contributions of each exciter, given by the columns of [A], in some linear fashion to give a new modal contribution vector void of the undesirable modes. These operations are equivalent to Gaussian reduction of [A] into an echelon matrix. The same linear operations carried out on [F] will give the corresponding force vector.

Since the columns of [A] will be linearly independent, the above discussion implies that each shaker is capable of eliminating at least one mode from the total response. This also implies that at least \( r-1 \) modes can be eliminated from the total response. An example serves to illustrate this concept.

**Example**

A compatible [F] matrix for a 3 DOF cantilever beam is:
\[
[\Phi] = \begin{bmatrix}
0.34 & 0.87 & -0.70 \\
-0.54 & 0.47 & 0.50 \\
0.76 & 0.14 & 0.51
\end{bmatrix}
\]

For 3 available exciters and 3 participating modes:
\[
[\Phi]_{3 \times 3}^T \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix} = [A] = \begin{bmatrix}0.34 & -0.54 & 0.76 \\ 0.87 & 0.47 & 0.14 \\ -0.70 & 0.50 & 0.51\end{bmatrix}
\]

By column operations, this can be reduced to
\[
[A] = \begin{bmatrix}
5.48 & -1.29 & 0.76 \\
0 & 0.33 & 0.14 \\
0 & 0 & 0
\end{bmatrix}
\]

The same operations carried out on [F] give
\[
[F] = \begin{bmatrix}1 & 0 & 0 \\ -3.18 & 1 & 0 \\ 4.44 & 0.97 & 0\end{bmatrix}
\]

Consequently, the second and third modes can be eliminated by choosing a force vector proportional to the first column of [F]:
\[
[F]_{3 \times 1} \begin{bmatrix}1 \\ -3.18 \\ 4.44\end{bmatrix} = \begin{bmatrix}5.48 \\ 0 \\ 0\end{bmatrix}
\]

If only two exciters were available such that
\[
[F] = \begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix}, \text{the forcing vector } [F] = \begin{bmatrix}1 \\ 0 \end{bmatrix}
\]
can be found in a similar manner to yield
\[
[\Phi]_{3 \times 2}^T [F] = \begin{bmatrix}1.39 \\ 1.04 \\ 0\end{bmatrix}. \text{Hence the third mode has been eliminated.}
\]

In some cases, it may be possible to eliminate a number of modes greater than the number of shakers when the modal matrix is highly symmetric. This is not discussed as it is not typically encountered in practice.

**Determination of Normal Forcing Vector**

Numerous methods have been proposed to appropriate a force vector that elicits response from only one mode (SDOF behavior). Asher's method [1],[2] appropriates a force vector that minimizes the in-phase response components. Multivariate mode indicator function methods seek to minimize/maximize the active/reactive energy with respect to the total energy [4],[5]. These methods are discussed in detail in the respective references.

**Practical Sine Dwell Test Considerations**

Contrary to expectations, the s/n ratio achieved with application of sinusoidal forcing vectors was very poor due to the low acceleration levels realized at the vehicle in the linear/quasi-linear ranges (despite the use of high sensitivity accelerometers). This is of added concern in sine dwell testing; random noise such as rattles are no longer averaged out of the measurement.

A major noise source proved to be the stinger attachment scheme used to transmit the force from the shakers to the structure. Eventually it was necessary to utilize a pre-load wire excitation setup to eliminate stinger rattles, resulting in tremendous improvement in data quality especially at the driving points.

The low signal output and relatively high electrical noise floor limited the lowest response level that could be effectively measured.

Sine dwell testing methods consequently proved to be very sensitive to setup. The added effort expended in
the elimination of rattles is significant when compared to random testing schemes.

Results

A fully trimmed automotive body was tested with all three previously discussed test methods and the measurements / modal models compared. From previous comprehensive tests, the region from 20-30 Hz. was chosen for further analysis and the shakers located to optimize excitation of the vertical modes in this region.

Stepped Sine

Preliminary tests indicated that the vehicle behaved in a nonlinear fashion as force levels were increased (Figure 1). I order to make meaningful random/sine comparisons, it was necessary to choose a compatible force level in the quasi-linear range. Compatibility was defined as levels that resulted in equivalent driving point FRF's. Compatible random vs. sine measurements are shown in Figure 2.

The modal modes corresponding to these measurements are compared in Table 1.

As shown by Table 1, nearly identical mode shape and frequency estimates were obtained for the well excited modes. The region from 25.5-W Hz. (which included dominant lateral modes) was difficult to estimate modal parameters, and it is in this region that the major differences between random and sine excitation are evident. Linear measurement ranges that are difficult to fit with a modal model are not uncommon in modal testing, reasons for which include high modal density, presence of highly damped modes, or low output modes coupled with dominant ones. In the latter case, stepped sine is better suited to optimize measurement of low output modes since the dynamic range is filled at each frequency line. With random excitation, it is necessary to fill the dynamic range with measurements dominated by the well excited modes, possibly leading to resolution problems in regions of low output. This may help explain why the stepped sine modal model in the 25.5-27 Hz. range is superior in terms of mode shape complexity.

Measurements resulting from two different sine force levels are shown in Figure 3. The 6N/6N test was chosen to approximate the linear range. The 12N/15N represents a specific forcing condition in nonlinear range.

NORMAL MODES

Using the vertical shaker orientation, isolation of the 24 Hz. vertical bending mode and 28.5 Hz. vertical bounce was attempted. The tuned condition for the 24 Hz. bending mode is shown in Figures 4 and 5.

Examination of individual measurements indicated that the significant in-phase response components present in the tuned 24 Hz. mode were attributable to lateral DOF's still being influenced by the neighboring lateral modes. This implies that it was not possible to eliminate the contribution of all neighboring modes with only four shakers.

To facilitate the control of the lateral modes, two shakers were re-oriented to provide significant lateral force components. This orientation did not lead to added control of these modes and only aggravated the tuning process by exciting them better.

The tuned mode shape and frequency for the 24 Hz. mode is in agreement with random and sine estimates (Table 2). Noticeable differences were evident between the 28.5 Hz. modes and previous estimates however (Table 3). It is significant to note that the damping associated with this mode was higher than previous modes. In the event that this damping was nonproportional, the discrepancies between the tuned normal mode shape and complex mode shapes derived form parameter estimation can be expected to increase. The tuned mode shape, void of the damping effects, corresponds to the true normal mode shape.

With the mode tuned, it was possible to study the effect of varying force level on the tuned condition. As the force level was increased, and hence the influence of nonlinearities, the quality of the tuned condition decreased. Mathematically speaking, a nonlinear system can no longer be represented by the normal mode model. This may explain the lack of a tuned condition in the nonlinear region. Mode shapes corresponding to the minimum in-phase energy condition obtainable at each force level are compared in Table 4. The preservation of mode shapes in the presence of moderate linearities is a simplifying assumption made in nonlinear analysis techniques such as [9] and is verified by Table 4.

Conclusions

Sine dwell test methods proved extremely sensitive to proper setup. Actual test times were increased ten fold with dual reference stepped sine testing compared to random testing.

In the linear range, stepped sine and random excitation techniques yielded nearly identical measurements and modal models for the well excited modes. Differences were noted in the region of poorly excited modes between 25.5-27 Hz., implying that the two methods handle difficult data differently.

The specific force control capability of stepped sine...
testing was well established. In the nonlinear range, measurements to specific force conditions differed significantly from the linear approximation.

Isolation of two normal modes was possible with normal mode toning. The process was subject to the same noise concerns of stepped sine testing and test time varied depending on the mode to be toned. Frequent movement of shakers may be necessary to isolate various modes or obtain the best toned condition possible for any given mode.

Tuned normal mode shapes for the lightly damped bending mode were equivalent to random and sine estimates. Significant differences were noted in the second bending under the influence of higher damping.

As face levels moved into the nonlinear range, it was impossible to isolate a normal mode as purely as in the low force, linear region. Despite the additional complexity, the linear mode shape was approximately preserved in nonlinear region.

References

Table 1: Modal Model Comparisons; Random vs. Sine

<table>
<thead>
<tr>
<th>Description</th>
<th>Freq</th>
<th>Freq.</th>
<th>Damping</th>
<th>Damping</th>
<th>Mass</th>
<th>Mass</th>
<th>MPC</th>
<th>MPC</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hz</td>
<td>Hz</td>
<td>%</td>
<td>%</td>
<td>kg</td>
<td>kg</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>Vertical bending</td>
<td>24.2</td>
<td>24.2</td>
<td>1.76</td>
<td>1.70</td>
<td>43.6</td>
<td>45.4</td>
<td>99</td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>Lat/Vert bending</td>
<td>25.5</td>
<td>25.5</td>
<td>90</td>
<td>1.03</td>
<td>79.7</td>
<td>111.0</td>
<td>95</td>
<td>91</td>
<td>96</td>
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<tr>
<td>Lateral bending</td>
<td>25.9</td>
<td>25.8</td>
<td>2.91</td>
<td>2.34</td>
<td>19.2</td>
<td>36.4</td>
<td>72</td>
<td>89</td>
<td>77</td>
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<tr>
<td>V bending2 s</td>
<td>26.3</td>
<td>26.2</td>
<td>2.66</td>
<td>2.17</td>
<td>78.3</td>
<td>114.0</td>
<td>72</td>
<td>67</td>
<td>26</td>
</tr>
<tr>
<td>v bending2 us</td>
<td>26.8</td>
<td>27.0</td>
<td>2.98</td>
<td>1.79</td>
<td>44.4</td>
<td>129</td>
<td>74</td>
<td>90</td>
<td>62</td>
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<tr>
<td>V bounce</td>
<td>28.5</td>
<td>28.6</td>
<td>2.31</td>
<td>2.51</td>
<td>68.6</td>
<td>59.9</td>
<td>97</td>
<td>94</td>
<td>99</td>
</tr>
<tr>
<td>Door/cabin</td>
<td>29.3</td>
<td>29.5</td>
<td>3.87</td>
<td>3.04</td>
<td>11.4</td>
<td>5.4</td>
<td>99</td>
<td>99</td>
<td>97</td>
</tr>
<tr>
<td>Door/cabin</td>
<td>30.4</td>
<td>30.4</td>
<td>1.30</td>
<td>1.73</td>
<td>51.2</td>
<td>77.1</td>
<td>97</td>
<td>92</td>
<td>96</td>
</tr>
</tbody>
</table>

a. Shaded columns indicate sine results.
### Table 2: Comparison of Mode Shapes (First Bending)

<table>
<thead>
<tr>
<th>MAC</th>
<th>Normal 24.3Hz.</th>
<th>Sine 24.2 Hz.</th>
<th>Random 24.21Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sine</td>
<td>99.4</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Random</td>
<td>99.4</td>
<td>99.9</td>
<td>100</td>
</tr>
</tbody>
</table>

### Table 3: Comparison of Mode Shapes (Vertical Bounce)

<table>
<thead>
<tr>
<th>MAC</th>
<th>Normal 28.6Hz.</th>
<th>Sine 28.6Hz.</th>
<th>Random 28.5Hz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sine</td>
<td>92.5</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Random</td>
<td>93.5</td>
<td>98.8</td>
<td>100</td>
</tr>
</tbody>
</table>

### Table 4: Comparison of Mode Shapes Under Varying Force Levels

<table>
<thead>
<tr>
<th>MAC</th>
<th>1N Force 24.3 Hz</th>
<th>4N Force 24.3 Hz</th>
<th>8N Force 24.2 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1N Force</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4N Force</td>
<td>99.9</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>8N Force</td>
<td>99.7</td>
<td>99.8</td>
<td>100</td>
</tr>
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</table>
Figure 1: System Response Due to Changes in Narrowband Force Level

Figure 2: Summary of Dual Reference Random vs. Sine Modal Measurements
Figure 3: **6N/6N** (Linear) vs. **12N/15N** Stepped Sine Measurements

![Graph showing stepped sine measurements for 6N/6N and 12N/15N.](image)

Figure 4: **Tuned** Condition for **24 Hz Bending** Mode

![Graph showing tuned condition for 24 Hz bending mode.](image)
Figure 5: Tuned Mode Shape of Trimmed Body (24.266 Hz).