ABSTRACT. The modal behavior of structures is governed by the mechanical characteristics of the constituent components. The dynamic mechanical properties of composite structures can be modified according to design requirements through hybridization. The objective of this research is to utilize hybridized pultruded composite materials for the dynamic modification and/or optimization of frames and space structures. Candidate materials used in this investigation include pultruded glass/epoxy, graphite/epoxy and various lay-up combinations of hybrid glass-graphite/epoxy composite beams. Experimental modal analysis (EMA) and finite element analysis (FE) were employed for the identification and validation of modal properties of the primary and modified frame structures. SMS STAR structural analysis software package and the super computer based PATRAN/P3 and ABAQUS finite element programs were used for modal identification. The structures were dynamically modified for design flexibility by coupling various combinations of glass/epoxy and graphite/epoxy flat beams. Both the EMA data and FEA predictions were found to be in good agreement. The results of this research indicate the advantages of Fiber reinforced Composites (FRC) in structural modifications, demonstrating their applicability and dynamic superiority.

NOMENCLATURE

- A: cross section area of beams
- [A]: extensional stiffness matrix
- [B]: coupling bending/extension stiffness matrix
- b: width of the beams
- [C]: damping matrix
- [D]: transverse stiffness matrix
- \( E_{11}, E_{22} \): longitudinal and transverse modulus
- \( G_{12} \): shear modulus
- \( t \): thickness of the laminate
- \( I_p \): polar moment of inertia
- \( L_p \): transverse stiffness matrix
- \( K_l, [\Delta K] \): stiffness matrix
- \( M_l, [\Delta M] \): mass matrix and change in mass matrix
- \( w \): displacement, velocity and acceleration
- \( w_t \): transverse deformation
- \( \varphi \): space
- \( \alpha \): mass proportional damping factor
- \( \delta_1, \delta_2 \): strain and displacement variation
- \( \rho \): material density
- \( \epsilon, \sigma \): strain and stress
- \( \Phi \): eigen vector
- \( \beta \): torsional displacement
- \( \omega \): natural frequency
- \( v_{12}, v_{21} \): major and minor Poisson’s ratio

1. INTRODUCTION

Recent literature emphasizes the efficiency, quality and economical aspects of structural analysis. The thrust of research in structural optimization and finite element idealization indicates the significance of SDM in the design of engineering components and structural systems. Application of modal analysis and structural dynamics modifications has become one of the most crucial aspects of structural design, both in research and industry.

In this study the dynamic behavior and performance characteristics of structural frames made with pultruded
composites have been investigated using both Experimental Modal Analysis (EMA) and Finite Element techniques (FE). The modal parameters of sub-structures and frames assembled using various combinations of pultruded products (shown in Table 1) are identified and compared.

<table>
<thead>
<tr>
<th>Cross-section view</th>
<th>Test I.D.</th>
<th>Material composition</th>
<th>Density ($\rho$) gm/cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>L</td>
<td>60% Graphite(Gr)</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>40% GI outer, 20% Gr inner</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>30% GI outer, 30% Gr inner</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>40% Gr outer, 20% GI inner</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>30% Gr outer, 30% GI inner</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>30% GI, 30% Gr random mix</td>
<td>1.83</td>
</tr>
</tbody>
</table>

Table 1
Specifications of materials used in modal analysis

Numerous experimental and analytical/numerical techniques have been proposed to facilitate the analysis of modal behavior of simple and complex dynamic structures [1-3]. Extensive software packages have been developed in relation to structural dynamics modifications to experimentally evaluate the dynamic characteristics of components, sub-structures and complex systems. On the other hand almost all the major finite element software packages provide access to dynamic analysis of engineering systems. Further more, experimental instrumentations and software packages are integrated to simultaneously analyze and update the dynamic behavior of systems.

In the present research, the application of Fiber Reinforced Structural Dynamics Modifications (FRSDM) in which hybridized pultruded composites with various material properties are utilized in the analysis and modification of dynamic performance of structures is introduced. The application of reinforced composites in complex structures not only reduces the number of sub-structures but also results in the associated benefits of weight savings, assembly, cost, and safety while increasing their performance. Cost-competitive advantages are expected to enable pultruded composites to become traditional materials alongside steel, wood and aluminum before the end of the 20th century [4]. For most of the applications, pultruded composites offer inherent technical advantages such as structural strength, improved physical properties and increased life when compared to the other composite manufacturing processes. The influence of various pultrusion process variables and the advantages of hybridizing both glass and graphite fibers in a common epoxy matrix on the dynamic behavior of flat beams made with such hybrid combinations were the subject of recent papers [5-6].

2. BASIC THEORY

Much of the theoretical basis of structural dynamics modification (SDM) utilizing conventional materials is long established. From a theoretical perspective, this is a situation in which a given change in the mass, stiffness, and damping would be associated with corresponding changes in the eigenvalues and eigenvectors of the dynamic system. Eigen modification of linear systems and their applications in engineering systems have been the focus of many researchers over the years [7-10].

The theoretical analysis of FRC materials, categorized as heterogeneous, is far more complex than their conventional counterparts. As the subject of this study, pultruded composites (unidirectional laminates) are governed by the behavior of individual laminae. In a unidirectional composite, longitudinal and transverse directions are the axes of symmetry. A unidirectional composite is an orthotropic material but has more than one axes of symmetry-the longitudinal direction and all other directions perpendicular to it [11], hence, transversely isotropic. In the analysis of these materials extensional [A], bending [D] and coupling [B] (coupling between the bending and extension) stiffness matrices are the governing influential terms. In the course of this investigation the [B] matrix is eliminated due to geometrically symmetric laminates (mid-plane symmetry) and the effects of [A] matrix is assumed to be negligible in the absence of in-plane forces.

The governing equations of motion for the uncoupled bending
and torsional vibrations and the associated orthotropic engineering constants [12-13]. Then take the form:

\[ b[D](\frac{\partial^2 w}{\partial x^4})+ \rho A(\frac{\partial^2 w}{\partial t^2})=0 \]

\[ [D]^\prime(\frac{\partial^2 \psi}{\partial x^4})-[D] \xi(\frac{\partial^2 \psi}{\partial x^4})- \rho I_A(\frac{\partial^2 \psi}{\partial t^2})=0 \]  

(1)

Where \([D]\) denotes the non-fully populated bending matrix. Matrices with superscripts \(C\) and \(C_t\) represent the torsional rigidity and warping rigidity, respectively, associated with torsional vibration. The contributing elements of \([D]\) matrix are represented as:

\[ D_{11}=\frac{E_{11} h^3}{12(1-\nu_{12} \nu_{21})} \]

\[ D_{12}=D_{11} \nu_{21} \]

\[ D_{22}=\frac{E_{22} h^3}{12(1-\nu_{12} \nu_{21})} \]

\[ D_{44}=\frac{G_{12} h^3}{12} \]  

(2)

With reference to the above assumptions and the functional relationship existing between the engineering constants, only four independent constants are required in the analysis (Table 2). The governing Equation of motion (1) is presented as mathematical eigen system in classical matrix notation.

\[ ([K]+\omega[C]-\omega^2[M])\Phi=0 \]  

(3)

The stiffness matrix \([K]\) for transversely orthotropic materials therefore contains non-zero elements of \([D]\) matrix.

Depending on the design requirements, SDM can be performed either on a single or simultaneous changes in the \([K]\) and \([M]\) matrices. The mechanics and mathematical complexity of FRSDM is far deeper, when compared with SDM. The orthotropic nature and contribution of more engineering constants into the analysis of FRSDM escalates the entire domain of structural dynamics. Replacement of one or more sub-structure(s) will result in modification of the contents of both \([D]\) and \([M]\) matrices. Unlike conventional SDM, however, a change in stiffness \([K]\) may not necessarily result in changing the mass \([M]\) of the original system.

If the stiffness and mass changes to the original system are \([AK]\), and \([AM]\), respectively, The modified eigen system takes the form:

\[ ([K]+\Delta K_{mn})+\omega^2([M]+\Delta M_{mn})\Phi=0 \]  

(4)

Table 2

<table>
<thead>
<tr>
<th>Material type</th>
<th>Engineering properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(E_{11})(Mpsi)*</td>
</tr>
<tr>
<td>K</td>
<td>16.81</td>
</tr>
<tr>
<td>L</td>
<td>6.67</td>
</tr>
<tr>
<td>ST</td>
<td>28</td>
</tr>
<tr>
<td>AL</td>
<td>10.3</td>
</tr>
</tbody>
</table>

3. FINITE ELEMENT ANALYSIS

The finite element formulation was based on the dynamic virtual work equation [15]:

\[ \int \rho \delta u \cdot \dot{u} dV + \int \rho A \delta u \cdot \dot{\delta u} dV + \int \delta e \cdot \dot{e} dV - \int \delta u \cdot T dS = 0 \]  

(5)

In which \(T, v\) and \(S\) represent the surface traction, volume and the surface bounding the volume, respectively. The discretized form of Eq. (5) can be shown as:

\[ \delta u^M \cdot [M]^M \delta u^M + (C_{MN}^M)^M \delta u^M + (C_{MN}^M)^M u^M + K_{MN}^M \delta u^M - P_{MN} \delta \Phi^M = 0 \]  

(6)

where terms from left to right represent mass matrix \([M]\), damping matrix \([C]\), stiffness matrix \([K]\) and the external load vector \([P]\), respectively.

The eigenvalue problem for natural modes of small vibration of the finite element models is:

\[ ([K]^{MN}+\omega[C]^{MN}-\omega^2[M]^{MN})\Phi^M=0 \]  

(7)

In the extraction of eigenmodes the associated damping matrix \([C]\) is neglected.

Triangular stress/displacement elements with five DOF(s) per node (3 displacement and 2 rotation components) were employed to model structures made with all material types.
shown in Table 1. The elements do not include transverse shear deformation (the shear modulus constants $G_{13}$ and $G_{23}$ are neglected).

Since the elastic material properties for laminated (layered) sections were different for each material type, numerical integration through the thickness was required to define and calculate the behavior of section properties. For each section thickness, number of integration points, material properties and the orientation of layers were specified.

It was neither computationally efficient nor practical to obtain convergency tests for all the FE models studied. However, a few models that were meshed for convergent solution resulted in sets of consistent natural frequencies. The models were generally heavily refined to generate high resolution 3-D mode shapes.

4. TEST MODELS, SET UP AND MODAL TESTING

For the purpose of this investigation, four different lay-up combinations of symmetric glass-graphite/epoxy hybrids (designated M, N, O and P) were pultruded (volume fractions of all the composites were maintained at 60% fiber and 40% epoxy). One random combination of glass and graphite fibers intermingled in the epoxy matrix was also produced (Q), to study the effects of uneven fiber distribution on the dynamic behavior. Also, for comparative purposes, some cold rolled steel (ST) and 6061 aluminum (AL) flat beams were mechanically bent to maintain joint continuity (STB and ALB) while forming the L-shaped and portal frames, and some others were joined using metallic angle brackets and rivets (STJ and ALJ). The pultruded flat beams are mechanically joined to form geometrically symmetric and non-distorted frame structures (Figure 1).

Various experimental procedures and testing configurations are proposed for characterizing the modal behavior of a structure [1-2]. Standard EMA Test set up and configurations were employed by means of utilizing instrumented modal hammer and appropriate accelerometers. Frequency Response Functions of each structure were estimated and transferred to the computer using the STAR MODAL software package. Appropriate curve fitting routines were used to estimate the unknown modal parameters from the FRFs.

Test models were studied under Fixed-Free configurations for the sub-structures and L-frames. But for other frame structures Fixed-Fixed boundary conditions were imposed.

In order to extract the experimental bending and torsional modes the model geometries shown in Fig. 1 were meshed 3 nodes across the width and 1 node/25.4 cm along the length.

5. RESULTS, COMPARISONS AND DISCUSSIONS

The natural frequencies of frames made with all the material types shown in Table 1 were obtained and traced with respect to their corresponding mode shapes. The results for both FE and EMA methods are presented.

The first four bending frequencies of L-frames in Fixed-Free condition are compared in Table 3. For this structure, the results seem to be with in acceptable agreement for most materials.

The small deviation of results in some modes for certain pultruded material types are attributed to inhomogeneity of fiber distribution across the thickness. Due to technological difficulties in manufacturing, the fiber volume fractions of the unidirectional pultruded hybrid composites may be concentrated in one region and deviate in another. Consequently, the cross sections at all points may not contain the same properties consistent with the models generated by FE analysis. This could influence the individual modes of vibrations (for example, the nodal location of individual modes will be disturbed due to local variation in the stiffness or mass).

<table>
<thead>
<tr>
<th>Material Type</th>
<th>$\omega_1$ (Hz)</th>
<th>$\omega_2$ (Hz)</th>
<th>$\omega_3$ (Hz)</th>
<th>$\omega_4$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STB FE</td>
<td>11.09</td>
<td>30.87</td>
<td>144.12</td>
<td>216.31</td>
</tr>
<tr>
<td>EXP.</td>
<td>10.58</td>
<td>29.73</td>
<td>140.08</td>
<td>212.13</td>
</tr>
<tr>
<td>ALB FE</td>
<td>12.04</td>
<td>31.33</td>
<td>143.80</td>
<td>217.44</td>
</tr>
<tr>
<td>EXP.</td>
<td>10.79</td>
<td>29.93</td>
<td>141.62</td>
<td>214.73</td>
</tr>
<tr>
<td>K FE</td>
<td>18.421</td>
<td>52.9</td>
<td>248.15</td>
<td>379.91</td>
</tr>
<tr>
<td>EXP.</td>
<td>16.99</td>
<td>50.31</td>
<td>244.41</td>
<td>368.85</td>
</tr>
<tr>
<td>L Fe</td>
<td>10.31</td>
<td>29.65</td>
<td>139.03</td>
<td>212.09</td>
</tr>
<tr>
<td>EXP.</td>
<td>8.6, 30.03</td>
<td>135.48</td>
<td>219.53</td>
<td></td>
</tr>
<tr>
<td>M FE</td>
<td>10.99</td>
<td>31.6</td>
<td>148.17</td>
<td>226.92</td>
</tr>
<tr>
<td>EXP.</td>
<td>10.59</td>
<td>33.46</td>
<td>152.1</td>
<td>244.5</td>
</tr>
<tr>
<td>N FE</td>
<td>11.92</td>
<td>34.19</td>
<td>160.33</td>
<td>245.53</td>
</tr>
<tr>
<td>EXP.</td>
<td>11.58</td>
<td>36.47</td>
<td>166.01</td>
<td>267.06</td>
</tr>
<tr>
<td>O FE</td>
<td>17.45</td>
<td>50.142</td>
<td>235.17</td>
<td>360.11</td>
</tr>
<tr>
<td>EXP.</td>
<td>15.38</td>
<td>45.08</td>
<td>216.44</td>
<td>339.51</td>
</tr>
<tr>
<td>P FE</td>
<td>16.64</td>
<td>47.8</td>
<td>224.18</td>
<td>343.17</td>
</tr>
<tr>
<td>EXP.</td>
<td>14.53</td>
<td>44.34</td>
<td>207.45</td>
<td>322</td>
</tr>
</tbody>
</table>

Comparison of FE and EMA for flexural natural frequencies of L-frames (fixed-free)
The flexural frequencies of portal frames are tabulated and compared in Table 4. The variation of natural frequencies for FE and EMA were consistent with the L-frame results above. It is evident from both Tables 3 and 4 that material types K and hybrids with graphite cm the outer side (0 and P) naturally behave stiffer than the other hybrids and conventional materials.

The diverse dynamic behavior of hybridized pultruded materials required special attention in the analysis of torsional modes. Depending upon the volume fraction and combination of graphite or glass on the outer or inner side of the cross section, the flexural and torsional modes are influenced. As shown in Table 5, material types M and N with glass outside and graphite inside dominated considerably (both for L-frame and portal frame) over the other hybridized test frames. This phenomenon could be used to advantage in the design or modification of structures.

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Natural Frequencies (Hz)</th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
<th>( \omega_3 )</th>
<th>( \omega_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>STB FE</td>
<td>27.06</td>
<td>117.00</td>
<td>169.39</td>
<td>204.92</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 27.66</td>
<td>116.32</td>
<td>166.22</td>
<td>203.23</td>
<td></td>
</tr>
<tr>
<td>ALB FE</td>
<td>28.13</td>
<td>122.11</td>
<td>176.04</td>
<td>215.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 26.95</td>
<td>114.16</td>
<td>166.82</td>
<td>202.00</td>
<td></td>
</tr>
<tr>
<td>K FE</td>
<td>46.01</td>
<td>200.54</td>
<td>287.88</td>
<td>351.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 40.01</td>
<td>199.32</td>
<td>277.34</td>
<td>358.99</td>
<td></td>
</tr>
<tr>
<td>L FE</td>
<td>25.79</td>
<td>112.34</td>
<td>161.40</td>
<td>197.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 23.90</td>
<td>111.71</td>
<td>164.13</td>
<td>219.68</td>
<td></td>
</tr>
<tr>
<td>M FE</td>
<td>27.49</td>
<td>119.73</td>
<td>172.00</td>
<td>210.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 27.20</td>
<td>124.18</td>
<td>182.75</td>
<td>245.05</td>
<td></td>
</tr>
<tr>
<td>N FE</td>
<td>29.74</td>
<td>129.56</td>
<td>186.09</td>
<td>227.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 27.95</td>
<td>136.36</td>
<td>195.88</td>
<td>263.71</td>
<td></td>
</tr>
<tr>
<td>O FE</td>
<td>43.59</td>
<td>190.02</td>
<td>272.79</td>
<td>333.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 35.78</td>
<td>180.27</td>
<td>252.06</td>
<td>331.99</td>
<td></td>
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<tr>
<td>P FE</td>
<td>41.56</td>
<td>181.17</td>
<td>260.09</td>
<td>317.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXP. 35.81</td>
<td>172.27</td>
<td>244.08</td>
<td>317.51</td>
<td></td>
</tr>
</tbody>
</table>

Table 5
Comparison of FE and EMA for torsional natural frequencies of frames (fixed-fixed). [ . . . modes do not contribute]

![Diagram](image)

Fig. 1. Test models
The modified portal frames are designated as KLK and LKL. The first and last letters correspond to the material types used for the left and right sub-structures, and the middle one represents that of top sub-structure.

The results of two separate but related cases shown in Table 6 indicate the potential of FRSMDM without modifying the geometries of the original frames. Flexural, torsional and out of plane modes are designated as (f), (t) and (op), respectively. It is observed that both the magnitude of natural frequencies and the natural occurrence of modes (i.e. flexural, torsional and/or out of plane) can be controlled. For example, when the top part of the frame made with material type K is replaced by a sub-structure with material type L, the fourth flexural mode becomes the second out of plane mode (in the case of KLK). It is also shown in Table 6 that for frame made with material type K modes 9 and 10, and for other cases modes 10 and II are identified as the first two torsional natural frequencies. It is noteworthy to examine the case LKL, that when the top part of the portal frame L is replaced with material type K, the order of natural modes are maintained but the magnitude of frequencies are modified.

<table>
<thead>
<tr>
<th>Mode</th>
<th>K</th>
<th>L</th>
<th>KLK</th>
<th>LKL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46 (f)</td>
<td>25.19 (f)</td>
<td>38.34 (f)</td>
<td>23.31 (f)</td>
</tr>
<tr>
<td>2</td>
<td>200.54 (f)</td>
<td>112.34 (f)</td>
<td>160.38 (f)</td>
<td>133.12 (f)</td>
</tr>
<tr>
<td>3</td>
<td>228.04 (op)</td>
<td>129.76 (op)</td>
<td>208.73 (op)</td>
<td>139.89 (op)</td>
</tr>
<tr>
<td>4</td>
<td>287.88 (f)</td>
<td>161.4 (f)</td>
<td>253.9 (f)</td>
<td>178.54 (f)</td>
</tr>
<tr>
<td>5</td>
<td>301.4 (op)</td>
<td>173.69 (op)</td>
<td>274.04 (f)</td>
<td>180.53 (op)</td>
</tr>
<tr>
<td>6</td>
<td>351.51 (f)</td>
<td>197.14 (f)</td>
<td>287.9, (op)</td>
<td>260.39 (f)</td>
</tr>
<tr>
<td>7</td>
<td>713.26 (f)</td>
<td>399.66 (f)</td>
<td>547.78 (f)</td>
<td>452.12 (f)</td>
</tr>
<tr>
<td>8</td>
<td>757.46 (f)</td>
<td>524.75 (f)</td>
<td>702.76 (f)</td>
<td>457.56 (f)</td>
</tr>
<tr>
<td>9</td>
<td>841.23 (f)</td>
<td>563.54 (f)</td>
<td>793.71 (f)</td>
<td>779.36 (f)</td>
</tr>
<tr>
<td>10</td>
<td>851.27 (f)</td>
<td>783.13 (f)</td>
<td>835.05 (f)</td>
<td>780.68 (f)</td>
</tr>
<tr>
<td>11</td>
<td>1443.50 (f)</td>
<td>792.37 (f)</td>
<td>845.64 (f)</td>
<td>785.65 (f)</td>
</tr>
</tbody>
</table>

Table 6
Comparison of natural frequencies (Hz) of original (K and L) and modified (KLK and LKL) portal frames. [(f) = Flexural, (t) = Torsional and (op) = out of plane modes]

Figure 2. Selected mode shapes of portal frame KLK in fixed. fixed configuration
The first ten transverse, torsional and out of plane mode shapes of frame with material combination KLK are presented in Fig.2. In comparing the frequency modifications discussed above (Table 6), the corresponding mode shapes should be traced.

6. CONCLUDING REMARKS

The natural dynamic behavior of frames made with pultruded composite materials in various geometric configurations have been presented. The application of FRSDM on structural frames has been introduced. It is shown that, if appropriately applied, FRC would be beneficial in modification and vibration control of engineering structures. FRSDM can be most appreciated in the design and dynamic analysis of systems that should retain their original geometries, due to design/operation limitations. This study also concludes that in order to fully study FRSDM, sophisticated experimental modal analysis tools (such as software packages that incorporate anisotropic nature of the structures) must be developed.

7. ACKNOWLEDGEMENTS

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