FREQUENCY-DOMAIN MAXIMUM LIKELIHOOD METHOD AND ITS APPLICATION IN THE IDENTIFICATION OF AIRCRAFT EQUIVALENT FLYING QUALITY PARAMETERS


* Northwestern Polytechnical University
Aircraft Engineering Department
P.O. Box 120. Xi’an. 710072
P. R. of China

ABSTRACT In this paper, a new method for parameter identification — Frequency Domain Maximum Likelihood Method (FDMLM) is proposed. As an example, it’s applied to identify equivalent flying quality parameters of a jet fighter with digital flight control system. The computing results show that the present method is efficient, can identify aircraft flying quality parameters from many characteristic response modes of aircraft and overcome all the shortcomings of traditional Bode-Plot Optimum Matching Method. It is perhaps worthwhile to point out that the authors’ method is applicable to other parameter identification problems.

NOMENCLATURE

$X(t)$ : "Xi state vector
$Y(t)$ : $m \times 1$ observation vector
$U(t)$ : $r \times 1$ control vector
$\theta$ : $s \times 1$ unknown parameter vector
$Y(j\omega)$ : $m \times 1$ frequency spectrum vector of $Y(t)$
$U(j\omega)$ : $r \times 1$ frequency spectrum vector of $U(t)$
$V(j\omega)$ : $m \times 1$ Gaussian white noise vector with zero mean and covariance $R$ in frequency $\omega$
$L$ : Likelihood function
$J$ : the negative logarithm of $L$
$\frac{\partial Y(j\omega)}{\partial \omega}$ : Sensitivity matrix
$K_q$ : Equivalent gain
$\tau_q$ : Equivalent time delay
$\xi_q$ : Equivalent damping ratio of short period modal
$\omega_n$ : Equivalent natural frequency of short period modal
$T$ : Equivalent numerator time constant
$*: Conjugate transposition of complex number
Re : Real part of complex number

1. INTRODUCTION

The increased use of augmentation in aircraft flight control system has made comparison of these aircraft with MIL-F-8785C (Military Specification-Flying Qualities of Piloted Aircraft) increasingly difficult, because highly augmented aircraft can have many characteristic response modes. Some aircraft designers have examined longitudinal equations of order as high as 80th, 90th, even 100th. How can such characteristics be checked for compliance with the specification? One solution is by using a lower-order equivalent system (LOES) of the complete aircraft high-order system (HOS). At present, Bode-Plot
Optimum Matching Method (BPOMM) is widely used to determine equivalent flying quality parameters. Although the principle of this method is simple, it has shortcomings below: (1) It’s very difficult for users to choose weight factors. (2) The convergence property is poor. (3) The initial values are very important to this method. If an arbitrary initial values are given to the optimum program, it may converge to a totally meaningless final match, wasting users and computer time.

In this paper, a new method of parameter identification – Frequency Domain Maximum Likelihood Method (FDMLM) – is proposed. Measured information which this method needs includes discrete input/output signals’ frequency spectrums and their statistical properties. As an application example, this method is used to identify some military aircraft longitudinal equivalent flying quality parameters. The results show that this method can overcome all the shortcomings of traditional optimum matching method and its application in the identification of aircraft equivalent flying quality parameters is very successful. Finally, it should be pointed that this method can also be used to solve other parameter identification problems.

2. THEORY

Consider the equations of linear steady system

\[ X(t) = A(\theta)X(t) + B(\theta)U(t) \]  
(1)

\[ Y(t) = C(\theta)X(t) \]  
(2)

where \( X(t) \) is \( nx1 \) state vector, \( Y(t) \) is \( mx1 \) observation vector, \( U(t) \) is \( rx1 \) control vector, \( \theta \) is \( sx1 \) unknown parameter vector, \( A(\theta) \) is \( nxn \) matrix about \( \theta \), \( B(\theta) \) is \( nxr \) matrix about \( \theta \) and \( C(\theta) \) is \( mxn \) matrix about \( \theta \). By Fourier transform, equations (1) and (2) can be written as

\[ j\omega X(j\omega) = A(\theta)X(j\omega) + B(\theta)U(j\omega) \]  
(3)

\[ Y(j\omega) = C(\theta)X(j\omega) \]  
(4)

Then

\[ Y(j\omega) = C(\theta)[j\omegaI - A(S)]^{-1}B(\theta)U(j\omega) \]

\[ = G(j\omega,\theta)U(j\omega) \]  
(5)

where \( I \) is \( nxn \) unit matrix, \( Y(j\omega) \) is \( mx1 \) frequency spectrum vector of \( Y(t) \), \( U(j\omega) \) is \( rx1 \) frequency spectrum vector of \( U(t) \) and \( G(j\omega,\theta) \) is \( mXr \) complex matrix. The measurement equation in frequency domain can be written as

\[ Y_m(j\omega_k) = Y(j\omega_k) + V(j\omega_k) \]  
(6)

where \( Y_m(j\omega_k) \) is \( mx1 \) complex measured value vector of frequency spectrum of \( Y(t) \) in frequency \( \omega_k \), \( V(j\omega_k) \) is \( mx1 \) complex Gaussian white noise vector with zero mean and covariance \( R \) in frequency \( \omega_k \).

According to statistic properties of \( V(j\omega_k) \), if a sequence of \( Y_m(j\omega_k), Y_n(j\omega_{k1}), \ldots, Y_n(j\omega_{kn}) \) is a sequence of measured value vector of frequency spectrum of \( Y(t) \), the maximum likelihood estimate of \( \theta \) is given by

\[ \hat{\theta} = \max_\theta L \]  
(7)

where \( L \) is the likelihood function. It and its negative logarithm can be written as respectively

\[ L = \left[ (2\pi)^m \cdot R \right]^{-n/2} \cdot \exp \left( \sum_{k=1}^{N} \frac{1}{2} \left[ Y_m(j\omega_k) - Y(j\omega_k) \right]^T R^{-1} \left[ Y_m(j\omega_k) - Y(j\omega_k) \right] \right) \]  
(8)

\[ J = -\ln L = \frac{1}{2} \text{Re} \left\{ \sum_{k=1}^{N} \left[ Y_m(j\omega_k) - Y(j\omega_k) \right]^T R^{-1} \left[ Y_m(j\omega_k) - Y(j\omega_k) \right] \right) \]  
(9)

The unknown parameter vector \( \theta \) must satisfy likelihood equation \( \partial J/\partial \theta = 0 \), i.e.

\[ -Re \sum_{k=1}^{N} \left( \frac{\partial Y(j\omega_k)}{\partial \theta} \right)^T R^{-1} \left[ Y_m(j\omega_k) - Y(j\omega_k) \right] = 0 \]  
(10)

(10) is a group of nonlinear equation about \( \theta \). Its root can be found by a Newton Raphson iteration. Assuming that \( \theta_0 \) is the initial of \( \theta \), then the first order Taylor expansion for \( Y(j\omega_k) \) can be written as

\[ Y(j\omega_k) = Y(j\omega_k) \bigg|_{\theta_0} + \frac{\partial Y(j\omega_k)}{\partial \theta} \bigg|_{\theta_0} \Delta \theta \]  
(11)

An expression for \( \Delta \theta \) can be derived from equations
(10) and (11) as
\[ \theta(t) = \left[ \text{Re} \sum_{k=1}^{N} \left( \frac{\partial Y(j\omega_k)}{\partial \theta^t} \right) \right] \cdot R^{-1} \left( \frac{\partial Y(j\omega_k)}{\partial \theta^t} \right) \int_{\theta}^{\theta(t)} \] 
\[ \left[ \text{Re} \sum_{k=1}^{N} \left( \frac{\partial Y(j\omega_k)}{\partial \theta^t} \right) \right] \cdot R^{-1} \left[ Y_n(j\omega_k) + i^{*}(j\omega_k) \right] \int_{\theta}^{\theta(t)} \] 
\[ (12) \]

The update equation can be obtained from equation (12): 
\[ \theta_{t+1} = \theta_t + \left[ \text{Re} \sum_{k=1}^{N} \left( \frac{\partial Y(j\omega_k)}{\partial \theta^t} \right) \right] \cdot R^{-1} \left( \frac{\partial Y(j\omega_k)}{\partial \theta^t} \right) \int_{\theta}^{\theta(t)} \] 
\[ \left[ \text{Re} \sum_{k=1}^{N} \left( \frac{\partial Y(j\omega_k)}{\partial \theta^t} \right) \right] \cdot R^{-1} \left[ Y_n(j\omega_k) - Y(j\omega_k) \right] \int_{\theta}^{\theta(t)} \] 
\[ (13) \]
where \( \frac{\partial Y(j\omega_k)}{\partial \theta^t} \) is sensitivity matrix, which can be written as
\[ \frac{\partial Y(j\omega_k)}{\partial \theta^t} = \begin{bmatrix} \frac{\partial Y_1(j\omega_k)}{\partial \theta_1} & \frac{\partial Y_1(j\omega_k)}{\partial \theta_2} & \cdots & \frac{\partial Y_1(j\omega_k)}{\partial \theta_{t}} \\ \frac{\partial Y_2(j\omega_k)}{\partial \theta_1} & \frac{\partial Y_2(j\omega_k)}{\partial \theta_2} & \cdots & \frac{\partial Y_2(j\omega_k)}{\partial \theta_{t}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Y_{t}(j\omega_k)}{\partial \theta_1} & \frac{\partial Y_{t}(j\omega_k)}{\partial \theta_2} & \cdots & \frac{\partial Y_{t}(j\omega_k)}{\partial \theta_{t}} \end{bmatrix} \] 
\[ (14) \]

Measurement noise covariance \( R \) must satisfy:
\[ \frac{\partial J}{\partial R} = 0 \] 
\[ (15) \]
An expression for \( R \) can be derived from equations (9) and (15) as
\[ R = \frac{1}{N} \sum_{k=1}^{N} \left[ Y_n(j\omega_k) - Y(j\omega_k) \right] \left[ Y_n^*(j\omega_k) - Y^*(j\omega_k) \right] \] 
\[ (16) \]
with the equations (13) and (16), we can obtain the maximum likelihood estimation \( \hat{\theta} \) of \( \theta \).

Cost function can be written as
\[ F_i = \frac{1}{2} \text{Re} \sum_{k=1}^{N} \left[ Y_n(j\omega_k) - Y(j\omega_k) \right] \left[ Y_n(j\omega_k) - Y(j\omega_k) \right] \] 
\[ (17) \]
If \( |F_i| \leq \xi \left| F_i \right| \), the Newton Raphson iteration is terminated and \( \hat{\theta} \) is regarded as the maximum likelihood estimate \( \hat{\theta} \) of \( \theta \). 

3. EXAMPLE

As an example, the present method is applied to identify equivalent flying quality parameters of a jet fighter equipped with digital flight control system which has many characteristic response modes.

3.1 Mathematical model for longitudinal equivalent system of aircraft

The following mathematical model for longitudinal equivalent system of aircraft is often adopted:[8]
\[ q(s) = \frac{K_s (s + \frac{1}{T})}{s^2 + 2\zeta \omega_n s + \omega_n^2 e^{-r \tau}} \] 
\[ (18) \]
where \( q(s) \) is the Laplace transform of pitch angle rate \( q(t) \) of aircraft, \( F_s(s) \) is the Laplace transform of longitudinal stick input \( F_s(t) \), \( T \) is equivalent numerator time constant, \( K_s \) is equivalent gain, \( \tau \) is equivalent time delay, \( \zeta \) is equivalent damping ratio and \( \omega_n \) is equivalent natural frequency of short period modal.

3.2 Realization of FDMLM

Let \( S = j\omega \) in equation (18), then observation equation in frequency domain can be written as
\[ q(j\omega) = \frac{K_s (j\omega + \frac{1}{T})}{(j\omega)^2 + 2\zeta \omega_n (j\omega) + \omega_n^2 e^{-r \tau}} \] 
\[ (19) \]
where \( e^{-r \tau} \) is time delay item, which can be written in the Euler's formula as
\[ e^{-r \tau} = \cos(\omega_r \tau) - j \sin(\omega_r \tau) \] 
\[ (20) \]
The following final observation equation from equations (19) and (20) can be obtained:
\[ q_n(j\omega) = G(j\omega, \theta) F_s(j\omega) \] 
\[ (21) \]
where unknown parameter vector \( \theta = (K_s, T, \omega_n, \zeta, \tau) \).

Measurement equation corresponding to equation (21) can be written as
\[ q_n(j\omega_k) = q(j\omega_k) + V(j\omega_k) \] 
\[ (22) \]
If discrete frequency spectra \( F_s(j\omega_k) \) and \( q_n(j\omega_k) \) \( (K = 1, 2, \ldots N) \) are given, FDMLM can identify un-
known parameter vector 0.

Because ground platform test data of aircraft were given in the form of Bode-Plot (i.e., frequency characteristic for \( q(t) \) to \( f_s(t) \)). Bode-Plot data must be transformed into discrete frequency spectra \( f_s(j\omega_k) \) and \( q_m(j\omega_k) \) (\( k = 1, 2, \ldots, N \)). The process for the transformation is:

1. Designing a kind of input signal \( f_s(t) \) and computing its frequency spectrum \( f_s(j\omega_k) \) by FFT.
2. Expressing Bode-Plot data in the form of complex, i.e., writing Bode-plot data as \( A(\omega_k) = B(\omega_k) + jC(\omega_k) \) where \( A(\omega_k) \) is frequency characteristic of \( q(t) \) to \( f_s(t) \).
3. Computing \( q_m(j\omega_k) \) by multiplying \( A(\omega_k) \) by \( f_s(j\omega_k) \).

### 3. Results

In this paper, FDMLM and BPOMM are applied to identify unknown parameter vector \( \theta = (K_a, T, \omega_{hop}, \bar{\omega}_p, \tau_e) \), respectively. The part of computing results are listed in Table 1. The detailed computing results see reference [3].

<table>
<thead>
<tr>
<th>State</th>
<th>Method</th>
<th>( \theta )</th>
<th>( K_a )</th>
<th>( T )</th>
<th>( \omega_{hop} )</th>
<th>( \bar{\omega}_p )</th>
<th>( \tau_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h = 3 \text{km} )</td>
<td>BPOMM</td>
<td>6.27</td>
<td>1.55</td>
<td>2.42</td>
<td>0.79</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>( M = 0.6 )</td>
<td>FDMLM</td>
<td>6.26</td>
<td>1.56</td>
<td>2.38</td>
<td>0.78</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>( h = 1 \text{km} )</td>
<td>BPOMM</td>
<td>3.90</td>
<td>2.44</td>
<td>1.68</td>
<td>0.44</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>( M = 0.6 )</td>
<td>FDMLM</td>
<td>3.82</td>
<td>2.44</td>
<td>1.68</td>
<td>0.45</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

Table 1

A comparison of results of BPOMM and FDMLM

### 4. CONCLUDING REMARKS

(1) The application of FDMLM in the identification of aircraft equivalent flying quality parameters is successful. FDMLM can obtain the same results as BPOMM.

(II) Because the frequency domain mathematical model is used in FDMLM and digital integration can be avoided, computing efficiency of FDMLM is higher than BPOMM.

(III) It is perhaps worthwhile to point out that FDMLM is applicable to other parameter identification problems.

### 5. REFERENCES

[1] Ai Jianliang and Deng Jianhua
Optimization method for Identifying Aircraft Equivalent Flying Quality Parameters

[2] Hodgkinson
Equivalent System Criteria for Handling Qualities of Military Aircraft
AGARD-CP-333, April 1982

A New method for Identifying Aircraft Equivalent Flying Quality Parameters.