THE ADAPTIVE NOTCH FILTER FOR ACTIVE NOISE CONTROL : AN EQUIVELENT TRANSFER FUNCTION APPROACH

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ABSTRACT : In this paper, stability studies of an active noise control system are approximated in terms of equivalent transfer function. By considering the presence of the physical plant, an adaptive notch filter with filtered-x LMS algorithm is used with a specified plant model, which is defined as a rotation matrix for the frequency response of the plant at a fixed frequency. It has ken shown that the system behaves as a linear time-invariant system with a slow adaptation and a limited error in plant estimation. And the stability of the active control system, under consideration is investigated.

NOMENCLATURE

C	: A FIR model of the acoustical channel
$\hat{C}$	: estimated version of C
d(k)	: desired signal
D(z)	: z-transformation of d(k)
e(k)	: error signal
E(z)	: z-transformation of e(k)
G(z)	: transfer function between Y(z) and E(z)
H(z)	: transfer function between E(z) and D(z)
r(k)	: filtered reference signal
$\hat{r}(k)$	: estimated version of r(k)
w(k)	: weighting function of adaptive FIR filter
x(k)	: reference signal
y(k)	: cancellation signal
α	: convergence coefficient
θ	: phase of the acoustic channel
$\hat{\theta}$	: estimated version of θ
$\Delta\theta$	: phase error : $\Delta\theta = \theta - \hat{\theta}$
ω_0	: frequency of a sinusoidal signal

1. INTRODUCTION

The active control of sound and vibration has progressed in recent years from laboratory curiosity to industrial application [I]. One form of primary sound or vibration field which is of particular importance in practice is that produced by rotating or reciprocating machines. The wave form of the primary field in these case is nearly periodic, and the fundamental frequency of the excitation is generally known. The adaptive notch filter introduced by Glover and Widrow [2], [3] could be used to cancel each harmonic. However, it cannot be used directely because of the presence of the physical plant. A compensation strategy or so called filtere-x LMS [3] method must be used to adjust the phase of reference signal. In practice, only an estimated version of plant model could be used in adaptive filtering. The errors in the plant model will influence the behavior of the control system and even cause unstability [4]. Considerable effort has been dedicated recently to the analysis of the behavior of the LMS algorithms. All of the work mentioned so far has been concerned with the analysis of random signals, and very little work has been published to date on the problem of deterministic inputs [2], [3]. A notable exception to this is the work of Glover [2] who considered the adaptive noise canceller with sinusoidal input. More recently Clarkson and White [5] developed an approximation to the analysis of the LMS algorithm in terms of linear tansfer function for a more general class of inputs. In this paper, the application of adaptive notch filter in active noise control is studied. A rotation matrix is used as a model of plant under control to adjust the phase of reference signal. Analysis of the adaptive notch filter is approximated by transfer function. And the stability of the active noise control system under consideration is investagited.

2. DERIVATION OF THE ALGORITHM

Consider the adaptive filtering system depicted in Fig. 1.

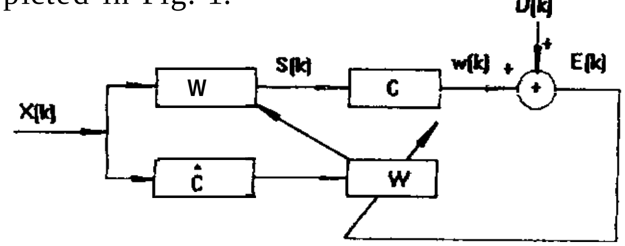

Fig. 1 Block diagram of an active noise control system

The reference vector x(k) , which is defined as

$$x(k)=\begin{bmatrix}\cos(\omega_0 k)\\ \sin(\omega_0 k)\end{bmatrix} \qquad (1)$$

is fed through an adaptive filter with two weights to generate the signal s(k) which drives the physical plant to generate the out put y(k) to the error microphone. If a filtered reference signal is defined by passing the reference vector through a perfect model of the plant response

$$r(k)=c\ x(k) \qquad (2)$$

The error signal in Fig. 1 can be expressed as d (k) , the " desired" primary signal, which is to be controlled, plus the filtered reference vector passing through the notch filter w, assuming w is time invariant at this moment.

$$e(k)=d(k)+r'(k)\ w \qquad (3)$$

The presence of the physical plant c prevents the conventional LMS algorithm from being used to update w and a compensation strategy, saying filterd-x LMS , must be used to adjust the phase of reference vector. The filtered-x LMS algorithm is given by [1]

$$w(k+1)=w(k)-\alpha\hat{r}(k)\ e(k) \qquad (4)$$

where

$$\hat{r}(k)=\hat{c}\ x(k) \qquad (5)$$

and the cap stands for the estimation version.

Frequently the physical plant is represented as a Jth order FIR filter [1]. In the case of a notch filter, only the cancelation of sinusoidal interference is under consideration, the physical plant model can be made up by a simple rotation matrix , and the relation between input $x(n)$ and output r(k) of the plant can be modeled as

$$r(k)=\begin{bmatrix}\cos(\omega_0 k+\theta)\\ \sin(\omega_0 k+\theta)\end{bmatrix} \quad (6)$$

and

$$\begin{aligned} r(k)&=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}x(k)\\ &=cx(k) \end{aligned} \quad (7)$$

The matrix c defined in (7) can be used as plant frequency response of a linear system model, which can be represented by matrices via the correspondence.

$$\begin{aligned} &\mathrm{Re}[c(\omega_0)]+j\,\mathrm{Im}[c(\omega_0)]\\ &\sim\begin{bmatrix}\mathrm{Re}[c(\omega_0)] & -\mathrm{Im}[c(\omega_0)]\\ \mathrm{Im}[c(\omega_0)] & \mathrm{Re}[c(\omega_0)]\end{bmatrix} \end{aligned} \quad (8)$$

The matrix c defined in (7) equeles to (8) by assuming that the magnitude of the frequency response is unit. And this representation is useful in doing complex arithmetic on a digital computer.

3. THE EQUIVALENT TRANSFER FUNCTION APPROACH

In this section a simplified analysis of the notch filter is approximated using an equivalent transfer function approach. Recalling the weight update equation (4) , Repeated application (4) gives (assuming $w(0)\equiv 0$)

$$w(k)=-\alpha\sum_{i=0}^{k-1}\hat{r}(i) \quad e(i) \quad (9)$$

from (3) we have

$$e(k)=d(k)-\alpha\sum_{i=0}^{k-1}e(i)\ r'(i)\ r(i) \quad (10)$$

Equation (10) is a relation between the desired input and the output error in terms of a purely recursive difference equation. In general, the coefficients of this difference equation are time varying and thus it is difficult to solve (10) directly. However, for sinusoidal reference vector x(k) as defined in (1). It is easy to verify that

$$e(k)=d(k)-\alpha\sum_{i=0}^{k-1}e(i)\ \Phi(i-k) \quad (11)$$

Where

$$\Phi(n)=\cos(\omega_0 n+\Delta\theta) \quad (12)$$

and $\Delta\theta=\theta-\hat{\theta}$

The result is now a constant coefficient difference equation.

Equation (10) can be transformed directly to give

$$E(z)=D(z)-\alpha[\Phi(z)-\Phi(0)]E(z) \quad (13)$$

Where E(z), D(z) and $\Phi(z)$ are the ztransform of e(k), d(k) and $\Phi(k)$ respectively and

$$\Phi(z)=\frac{z^2\cos(\Delta\theta)-z\cos(\omega_0-\Delta\theta)}{z^2-2z\cos(\omega_0)+1} \tag{14}$$

and

$$\Phi(0)=\cos(\Delta\theta) \tag{15}$$

Let $G(z)$ be defined as

$$G(z)=\alpha[\Phi(z)-\Phi(0)] \tag{16}$$

we have

$$\frac{E(z)}{D(z)}=\frac{1}{1+G(z)}=H(z) \tag{17}$$

where

$$H(z)=\frac{U(z)}{L(z)} \tag{18}$$

$$U(z)=z^2-2z\cos(\omega_0)+1$$

$$L(z)=z^2-z[2\cos(\omega_0)-\alpha\cos(\omega_0-\Delta\theta)]+1-\alpha\cos(\Delta\theta)$$

It is clear that the equation (18) is equivalent to a feedback control system as shown in Fig. 2.

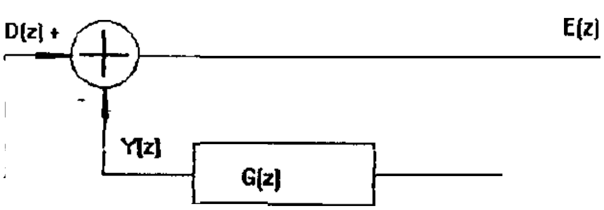

Fig. 2 Equivalent feed back control system

4. THE PERFORMANCE OF THE ADAPTIVE NOTCH FILTER

It is shown that if the reference input is a vector of sinusoidal signal, as defined in (1), then the system shown in Fig. 1, from primary input to the output behaves like a sharp, linear, time-invariant, notch filter. This discovery was first exploited by Glover [2]. The analysis in this paper shows that with the presence of physical plant, the behavior of the noise cacelling system is still a linear, time-invariant, notch filter, the dynamic of plant will not influence the zeros of system, but the location of poles is a function of the estimation error of plant phase and a. The zeros of the transfer function are located in the Z-plane at

$$z=\exp(\pm j\omega_0) \tag{19}$$

and are precisely on the unit circle at angles of $\pm\omega_0$ rad. If we chose a small positive constant a, the poles will be located at

$$z=[\cos(\omega_0)-\alpha\cos(\omega_0-\Delta\theta)/2] \pm j\{1-\alpha\cos(\Delta\theta)-[\cos(\omega_0)-\alpha\cos(\omega_0-\Delta\theta)/2]^2\}^{1/2} \tag{20}$$

The poles are at ratio distance $[1-\alpha\cos(\Delta\theta)]^{1/2}$ from the origin at angles of

$$\pm\arccos\frac{\cos(\omega_0)-\alpha\cos(\omega_0-\Delta\theta)/2}{[1-\alpha\cos(\Delta\theta)]^{1/2}} \tag{21}$$

Since a is assumed positive and small, the distance of poles from the origin can only be greater than one if $\cos(\Delta\theta)$ is negative, so the stability condition must be

$$\cos(\Delta\theta) > 0$$

therefore

$$-90° < \Delta\theta < 90° \qquad (22)$$

Example 1,. Let $\omega_0 = \frac{\pi}{4}$, a= 0. 25, and $\Delta\theta$ vary between $\pm$ 180”. The root loci of the poles are shown in figure 3. The root loci appear to be two circles symmetrical to the real axis, with the radius of a, and their origins are on the unit circle at the angle phase of $\pm\omega_0$. When the phase error varies within $\pm 90°$, the poles are located inside the unit circle, and the system keeps stable. When the phase error is beyond $\pm 90°$, the poles are outside the unit circle, and the system is unstable.

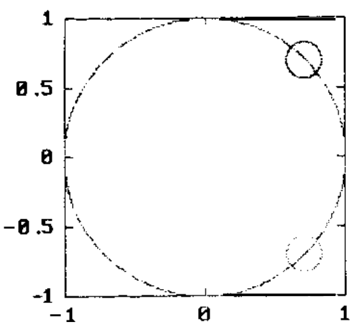

Fig. 3 The pole plots of the adaptive notch filter ($\alpha = 0.25, \omega_0 = \frac{\pi}{4}, -180° < \Delta\theta < 180°$)

Example 2. Let $\omega_0 = \frac{\pi}{4}$, $\Delta\theta = 0$, a vary from zero to infinitive. The root loci of the poles are shown in figure 4. when a equals to zero, the poles lie on the unit circle at the angle phase of $\pm\omega_0$. With the increase of α from zero to $2\sqrt{2}-1$, the two poles move into unit circle and then meet at the positive real axis with the position of $\sqrt{2}-1$ and then break away from each other at the same critical value of the convergence coefficient. They then migrate outside the unit circle causing instability at the value of convergence coefficient larger than 2.

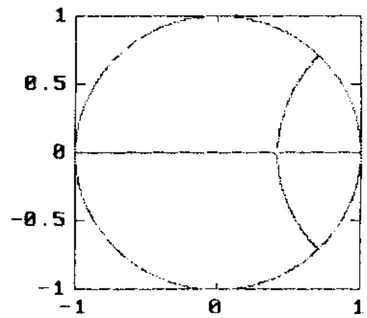

Fig. 4 The pole plots of the adaptive notch filter ($\Delta\theta = 0, \omega_0 = \frac{\pi}{4}, 0 \leqslant \alpha \leqslant \infty$)

5. CONCLUSION

It has been shown that the adaptive notch filter can be used in an active noise control system by consideration of the presence of secondary acoustic path or physical plant. The filtered-x LMS algorithm will still converge with slow adaptive, so long as the phase error in the plant model is less than 90”. The method can be extended to multichannel active noise control system. A part of these works can be found in [12]

6. ACKNOWLEDGMENT

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