Periodic Spectral Analysis of Diesel Vibration Data

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ABSTRACT

This work addresses the problem of trying to take advantage of the nominally periodic nature of a diesel engine running at constant speed to obtain a time-varying spectral description of vibration data over the shaft period. It requires addressing a number of issues, most notably period estimation and removal of tonal components whose presence in a time-varying spectrum is redundant and undesirable. Use of a recently designed method to identify sinusoids, in conjunction with an adaptive tracking algorithm, results in significant improvement in simulations, and reasonably good improvement in the case of the diesel vibration data, especially when considering the complexity of the stochastic structure of this data.

1. INTRODUCTION

Periodic phenomena are found in a wide variety of disciplines. Examples include noise and vibration associated with rotating machinery, communication signals, pulsar signals, and species extinction rates. In view of the periodic nature of such phenomena, it is natural to assume that related signals may exhibit a periodic structure. For this reason the frequency domain has been the predominant medium for attempting to describe the information contained in these types of signals. However, traditional frequency domain analysis assumes the signal to be wide sense stationary (wss). If the signal is indeed periodic and nonrandom, then this type of analysis will reveal the magnitudes of its Fourier series coefficients. At the other extreme, if it is periodic and entirely random (take for example modulated white noise), then this type of analysis will reveal a flat energy spectrum. In both cases, the temporal description of the signal information over a period of the phenomena is lost.

The goal of this work is to investigate the potential for extracting periodic time-varying spectral information from real world data; that is, data associated with a nominally periodic phenomenon, and whose stochastic description is entirely unknown. For this purpose, vibration data obtained from an automotive diesel engine run at constant speed will be the focus of this work. This data is believed to be suitably complex as to be representative of data associated with a wide variety of rotating machinery. It will provide the motivation to address a number of real issues, most notably time-varying period tracking and tone rejection. The signal processing tools needed to address these issues are described, and their performance is demonstrated in the case of the diesel data.
2. DIESEL DATA COLLECTION

Vibration data from a 6.2L 8-cylinder automotive diesel engine was collected while the vehicle was operated at a speed of 55 mph on a flat highway with negligible wind conditions. Cruise control was used to attempt to maintain speed as constant as possible. An accelerometer was magnetically mounted to the valve cover above the number 1 cylinder. The analog signal was run through a 5th order Bessel anti-aliasing filter with a cutoff frequency of 500 Hz. Digitization was performed by an RC Electronics A/D board at a rate of approximately 2000 samples/second. This board was housed in a portable Dollch 486 computer. A total of 32,768 samples were collected continuously over a period of approximately 16 seconds.

3. TRADITIONAL SPECTRAL ANALYSIS

A discrete Fourier transform (DFT) spectral estimate of the diesel data is shown in Figure 1. It was obtained by averaging 32 individual (modulus-squared) DFTs of data records of size 1024. The presence of peaks spaced apart an amount equal to the engine shaft frequency of approximately 13.2 Hz (the bin width is ~1 Hz) is characteristic of DFT spectra derived from rotating machinery data. The first large peak at ~104 Hz corresponds to the first cylinder harmonic. Analysis of this vibration spectrum suggests that a broad resonance just above 400 Hz is being excited by the 4th cylinder harmonic and neighboring shaft harmonics, while a narrow resonance is being excited primarily by the 6th cylinder harmonic. Such an analysis however, requires qualification. Specifically, are these resonances (if they are indeed resonances) being excited primarily by sinusoids (termed tones in this work) or by random narrowband energy sources. The problem is to determine which peaks are tones and which are not. One possible solution to this problem is to compute a family of DFT spectra indexed by the number of autocorrelation lags, n, used. Alternatively, a family of autoregressive (AR) spectra might be utilized, since it is well known that they can offer notably higher tone resolving properties than the DFT for the same number of lags. Let $x_t = [x_{t-1}, ..., x_{t-N}]^T$ and let $a = [a_1, ..., a_n]$ be the vector of linear prediction parameters which minimize the expected squared prediction error $\sigma_e^2 \triangleq E(X_t - aX_t)^2$. Then the corresponding AR(n) spectral estimate is:

$$AR_n(\omega) \triangleq |\sigma_e/\{1 - a^T w(\omega)\}|^2$$

where $w(\omega) \triangleq \{1, e^{-i\omega}, ..., e^{-in\omega}\}^T$. The diesel data AR(n) spectral estimates for $n=20$, 40, 80, and 160 are shown in Figure 2. One can use any one of these spectra as a spectral energy estimate (note the similarities between the AR(160) and DFT spectra). One could also use all of them to determine, from their behavior as a function of $n$, which frequencies correspond to tones. For example, the peaks corresponding to the 1st, 2nd and 4th cylinder harmonics all appear to increase as a function of $n$, while the one at the 6th harmonic (622 Hz) appears to have little dependence on $n$. From this, one might conclude that of these harmonics, only the 6th corresponds to a narrowband energy, as opposed to a tone source. We now propose the use of a different family of spectra for this purpose.
4. TONE AND PERIOD ESTIMATION

In DFT-based spectral analysis the only difference between an energy versus a power spectral estimate is a factor of $1/2N$. If this factor is included inside the limit operation in (1), then if $X_t$ is the above mentioned tone, (1) will converge to $A^2/4$ at the tone frequency and zero elsewhere. If $X_t$ is white noise (or any random process with a purely continuous energy spectrum) then (1) will converge to zero everywhere. In the last section the use of a family of spectral estimates, specifically AR($n$) spectra, indexed by $n$, the number of autocorrelation lags utilized, was suggested as a means of identifying tones. The problem with using AR($n$), as well as associated DFT spectra, for this purpose is that the growth rate of a peak as a function of $n$ can be quite erratic, making it difficult to draw conclusions regarding convergence. The following family of minimum variance (MV) spectra overcome this difficulty to a notable extent. The MV($n$) spectrum for a scalar process $X_t$ is

$$MV_n(\omega) = \left( \sum_{k=0}^{n} AR_k(\omega)^{-1} \right)^{-1} \quad (5)$$

The MV($n$) spectrum was introduced as a power spectrum estimate by Capon in 1969 [1]. Only recently, however, have its convergence properties as a function of $n$ been exploited for identification of tones [2],[3],[4]. The diesel data MV($n$) spectra for $n = 20, 40, 80$ and $160$ are shown in Figure 3. Not only do the MV($n$) spectra converge monotonically downward to the process power spectrum, but at each frequency where there is no point spectrum they drop at an asymptotic rate of 3 dB per doubling of $n$ [3]. With this in mind, a visual inspection of Figure 3 suggests that the peak near the 6th cylinder harmonic contains a tone component. It also suggests the possibility of other tonal sources in the region of the 4th cylinder harmonic, in particular at a distance of two shaft harmonics to either side of the 4th cylinder harmonic. These tonal components need to be removed from the data prior to performing any type of time-varying spectral analysis. Otherwise they will contribute time-invariant spectral peaks which could dominate any time-varying spectra in neighboring frequency regions; behavior which will be demonstrated in the next section.

The shaft period can be estimated from any number of the tones identified from the MV($n$) analysis. However, given the strong convergence tendency at the 6th cylinder harmonic, this peak frequency was chosen for this purpose. The MV(160) spectrum identified this harmonic in the interval $[622.3, 623.5]$ Hz, while the corresponding AR(160) peak was identified in the interval $[621.8, 622.8]$. These interval widths are controlled by the frequency bin spacing used in computing these spectra, as well as by speed variations over the observation period. Rather than trying to increase the accuracy of the peak frequency estimate by computing the spectra at more closely spaced frequencies, it was decided to estimate the shaft frequency as $f_{sh} = 622.5/48 = 12.97$ Hz. This decision was based on the belief that the vehicle speed may well vary by as much as 0.1 mph over the 16 second observation period. This much speed variation would correspond to variation of the 6th cylinder harmonic on the order of $\pm 1$ Hz.

5. PERIODIC SPECTRAL ANALYSIS

This section begins with some basic definitions and results for periodic processes. We then present periodic spectral estimates for the diesel data. Their time-invariant behavior is noted, and is hypothesized to be due to an extent by period bias. We present a simple scheme to accommodate the bias between a known true period which is not an integer number of samples, and an integer-valued computation period. To begin, a random process $X_t$ is said to be a (weakly) periodic process if both its mean $E(X_t) = \mu(t)$ and autocorrelation $E(X_t X_{t+\tau}) = R_x(t, \tau)$ are periodic functions of the variable $t$. Such processes are also termed wide sense cyclostationary (WSS), and we will refer to them as such in this work. There has been a notable amount of attention given to such processes (see [5] for references). The majority of it has concerned representation theory, while a lesser amount has focused on data-related issues. A fundamental result along these latter lines is that the autocorrelation function estimator given by [6]:

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converges in the mean square sense to
\[ R_x(t, t + \tau) \] as \( N \to \infty \) when \( X_t \) is a regular \( w.s.s \) process with period \( T \). By regular we mean, loosely, that \( X_t \) has only continuous spectrum for each \( t \) in \([0, T]\). The definition of a time-varying spectrum used in this work is:

\[ S_x(t; \omega) = \int_{-\infty}^{\infty} R_x(t - \frac{\tau}{2}, t + \frac{\tau}{2}) e^{-i\omega \tau} d\tau. \]  

Our first time-varying spectral analysis of the diesel data utilized the above shaft frequency estimate to obtain an estimated shaft period of \( 1/f_{\text{shaft}} = 0.077 \) seconds. At a sample rate of 2000 Hz, this corresponds to a true period \( T_{\text{true}} = 154.2 \) points. The closest integer valued computation period is \( T = 154 \). Rather than computing the estimate (6) for this \( T \) and a large number of \( \tau \) values, and then applying the discrete version of (7), which amounts to a DFT-type periodic spectral estimate, it was decided to compute an AR-type periodic spectral estimate. The reasons for this decision are similar to those given for the case of spectral analysis of wss data; namely a more parsimonious and computationally efficient estimate with potentially higher resolution. A discrete-time random process \( X_t \) is said to be a w.s.s AR\((p)\) process with period \( T \), if for each \( t = 1, 2, \ldots , T \), \( X_t \) has an AR\((p)\) representation with predictor vector \( a_t = [a_{t1}, a_{t2}, \ldots , a_{tp}] \) and white noise variance \( \sigma_t^2 \), where both the predictor vector and noise variance are periodic with period \( T \), and \( p = [p_1, \ldots , p_T] \). The least squares estimate of \( a_t \), as well as the estimate (6), involves partitioning the data into blocks of length \( T \). If the difference between the true and computed periods is ignored, then for a diesel data length of 32,000 and a block size of \( T = 154 \) the beginning of the last block will actually be located at \( 0.2(32,000/154) \approx 42 \) points, or approximately one third of a period preceding the start of a period. This type of precession of the computed period will have the effect of smearing the time-varying spectral structure of the data across the period. To demonstrate this consider the periodic AR\((1)\) process

\[ X_t = a_t X_{t-1} + e_t; a_t = 0.5 + 0.4 \sin(2\pi t/T_{\text{true}}) \]  

where \( e_t \) is a wss white noise process. Using 10,000 samples of (8) the AR\((1)\) spectrum was estimated for a true period \( T_{\text{true}} = 20 \) and for \( T_{\text{true}} = 20.2 \), while in both cases \( T = 20 \). These spectra are shown in Figure 4(a-b). Assuming a true and fixed period \( T_{\text{true}} \) is known, and the difference between it and the closest integer-valued computation period \( T \) is \( T_{\text{true}} - T = \Delta \), then it is a simple matter to accommodate this difference. Specifically, let \( X_n = [x_{nT}, x_{nT+1}, \ldots , x_{nT+1}] \) be all of the remaining data after the \( n \)th block. Beginning with \( n=1 \), partition the original data until the condition \( n\Delta > 0.5 \) is noted. This implies that the beginning of the computed \((n+1)\)st period is at least 0.5 samples behind the true \((n+1)\)st period. Correct this condition by discarding the first point in \( X_n \), and proceed to partition this remaining modified data. If the condition \( n\Delta < -0.5 \) occurs, then a sample point needs to be added. This can be done in a number of ways. For simplicity we chose to add the average of \( x_{nT} \) and \( x_{nT+1} \) to the beginning of \( X_n \). The result of applying this fractional period correction scheme is demonstrated in Figure 4(c). Comparison with Figure 4(a) indicates that it is able to accommodate a fractional period with minimal distortion to the periodic spectral estimate.

The result of applying this period correction method to the diesel data is shown in Figure 5, which includes a periodic spectrum estimate using the first 12,500 points of the diesel data. (The reason for not using all the data will be explained in the next section.) All of the AR spectra in this figure utilized the same order \( p = 10 \), due to the fact that, as will be seen, tones dominate the process. From this figure it is not clear whether the spectral data is time-varying. This could be due in part to the fact that the true period is not constant, so that the period correction scheme has limited effect. In fact, while not plotted, the corresponding spectral plots without period correction appeared very similar to those in Figure 5. Another obvious reason for the time-invariant nature of the plots in Figure 5 is that they are dominated by the tone components in the 400 Hz and 600 Hz regions. This dominance is not surprising in view of the MV analysis in Section 4.
6. ADAPTIVE TONE SUPPRESSION

The diesel data analysis up to this point has highlighted two issues that must still be addressed in order to have any chance of extracting periodic spectral information with any real temporal structure from the diesel data. One is that inaccurate period estimates, when used with long enough data lengths, will smear any time-varying spectral structure into a time-invariant structure. The period correction method proposed in Section 5 will improve matters if the interval over which the shaft period can assume to be constant is long enough to obtain statistically reliable spectral estimates, and if this period can be estimated with sufficient accuracy. The second is that tones with any strength must be removed. Since there can be little doubt that the shaft period is slowly time-varying to some extent, it is not likely that tone cancellation assuming a fixed period will work. A more natural approach is to track the tone information (magnitude, frequency and phase) and adaptively remove it. This period tracking and tone rejection is achieved by using an extended Kalman filter (EKF). The state model is assumed to be first order, and the state variables include the magnitudes, frequencies and phases of the tones identified by the MV analysis that we desire to reject. The data is assumed to be the sum of these tones plus white noise. For details concerning the formulation of an EKF see [7]. Figure 6 shows the MV spectra subsequent to tracking four specified sinusoids identified in Figure 3. Comparing these figures, we see that the 024 Hz peak, for example, is attenuated by approximately 15 dB. A periodic spectral estimate corresponding to this tone-adjusted data is given in Figure 7. It is based on the first 12,500 data points, since beyond this number the EKF frequency tracking could no longer be considered constant, as shown in Figure 8.

7. SUMMARY AND CONCLUSIONS

The goal of this work was to investigate the potential for extracting periodically time-varying spectral information from vibration data measured from a diesel engine operating at nominally constant speed. Two theorems were presented which predict the time-invariant spectral nature of any periodic spectral estimates in the event that period bias and randomness are not accounted for. It was also shown that any dominant tones in the data (which is often the case with systems such as rotating machinery) will camouflage time-varying spectral structure, and so must be identified and removed. To determine the data lengths over which the period can be assumed constant and perform the tone removal an extended Kalman filter was incorporated. To identify the tones and obtain initial magnitude and frequency estimates for the EKF the convergence properties of the MV(n) spectra were used. Finally, to accommodate non-integer period estimates a simple interpolation/extraction correction scheme was used.

These signal processing strategies, when applied to the diesel data resulted in periodic spectral estimates with notably improved temporal structure, but the estimates retained a fair amount of time-invariant structure. It is not known whether this structure is real, or whether it is the result of signal processing limitations. Even though the signal processing tools developed for this purpose did not extract obvious intra-cycle spectral information related to the diesel vibration signal, it is believed that enough time-varying information was sufficiently enhanced to motivate further efforts along these lines. Examples of such efforts might be inclusion of a wcs structure in the EKF. The EKF used in this work assumes the non-tone process to be white. Even so, it performed reasonably well in a very non-white and in fact nonstationary noise environment.

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REFERENCES


**Figure 1.** Diesel data periodogram obtained by averaging 32 1024-point spectra.

**Figure 2.** Diesel data AR(n) spectra for n=20, 40, 80, 160.

**Figure 3.** Diesel Data MV(n) spectral estimates for n=20, 40, 80, 160.
Figure 4. AR(1) spectral estimates of the process given by Eqn. (8), including (a) $T_{true}=20$, where $T_{true}$ is the true period and $T$ is the computation period; (b) $T_{true}=20.2$ and $T=20$, and (c) same as (b) but with the period correction scheme described in Section 5 applied.

Figure 5. AR(10) periodic spectrum estimate based on the first 12,500 points of the diesel data, and using period correction under the assumption $T_{tr}=154.2$ points and $T=154$ points.

Figure 6. MV(n) spectra after EKF-based tone rejection, for $n=20, 40, 60$, and 80.

Figure 7. AR(10) periodic spectrum estimate after tone removal.