THE TIME-FREQUENCY SPECTRAL ESTIMATION OF NONLINEAR VIBRATION SYSTEMS

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ABSTRACT

In this paper, we use the Wigner distribution (WD) to estimate the time-frequency spectral of the nonlinear vibration of a cantilever coupled-beam with mini-gap. The amplitude distortion and the frequency varying phenomenon is presented. The comparison of the WD method with fast Fourier transform (FFT) method is given. The results of the WD method is excellent.

NOMENCLATURE

- $e$: the base of natural logarithm
- $f$: signal frequency
- $f_s$: sampling frequency
- $i$: imaginary number
- $K_0(t)$: the WD kernel sequence
- $K_{m}(t)$: the modified kernel sequence
- $L$: Positive integers
- $t$: time
- $T$: sampling period
- $T$: FM impulse length
- $w(t)$: the window function
- $W_x(t, \omega)$: the cross-WD
- $W_x(t, \omega)$: the auto-WD
- $x(t)$: time signal
- $X(\omega)$: Fourier transform of $x(t)$
- $y(t)$: time signal
- $\omega$: frequency
- $*$: complex conjugate

1. INTRODUCTION

Nonstationary signals arise in many scientific testing and engineering measurement. The time-frequency spectral estimation is a important way of studying the test object. It was Wigner Distribution(WD) that recently has arised and obtained considerable attention in nonstationary signal processing.

The traditional methods of nonstationary signal spectral estimation are based on the assumption that on a short-time basis the signal is stationary. This has the important drawback that the length of the assumed short-time stationarity determines the frequency resolution which can be obtained. To increase the frequency resolution one has to take a longer measurement interval (window), which means that nonstationarity occurring during this interval will be smeared out in time and frequency. However, the WD method has a high time-frequency resolution that the traditional methods have not. Since 1980, many papers have been presented, which adopted the WD to estimate time-frequency properties of the nonstationary signals, and gave many excellent results[1, 2, 3, 4, 5, 6].

In this paper, we adopt the WD method to estimate the time-frequency spectral of a nonlinear vibration testing signal. The fundamental principle is presented in section 2. in sec. 3 we give a numerical illustration. The experimental procedure and results is described in sec. 4. Sec. 5 is devoted to some conclusions.

2. THE THEORY

2.1 Definition of WD and its basic properties

The cross-Wigner distribution of two signals $x(t)$ and $y(t)$ is defined by:

$$W_{xy}(t, \omega) = \int_{-\infty}^{\infty} e^{-i\omega t} x(t+\tau/2) y^*(t-\tau/2) d\tau$$ (1)

The auto-Wigner distribution of a signal is then given by:

$$W_{x}(t, \omega) = \int_{-\infty}^{\infty} e^{-i\omega t} x(t+\tau/2) x^*(t-\tau/2) d\tau$$ (2)

When no confusion can arise we will call both functions a Wigner distribution.
There are a number of important properties of the WD. As a base of this paper, we give the two basic properties of them.

(I) The frequency-domain integral property
\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} W_x(t,\omega) d\omega = |x(t)|^2
\]
This means that the integral of the WD over the frequency variable at a certain time \(t\) yields the instantaneous signal power at that time.

(2) The time-domain integral property
\[
\int_{-\infty}^{\infty} W_x(t,\omega) dt = |X(\omega)|^2
\]
This means that the integral of the WD over the time variable at a certain frequency \(\omega\) yields the energy density spectrum of \(x(t)\) at this frequency.

2.2. Discrete WD and its efficient computation

For a discrete-time signal \(\{x(nT_s)\}\), the WD is defined as:
\[
W_x(nT_s,f) = 2\sum_{l=-N/2}^{N/2-1} K_n(l) x(nT_s+lT_s) \cdot w(l) \cdot (-i) \exp(-i\pi flT_s)
\]
where \(T_s\) is the sampling period, \(T_s \leq 1/4B, B\) is the maximum frequency bandwidth of the signal, and \(\omega(l) = 0, \text{ for } |l| > N/2\), positive integers. Let \(T_s = 1, f = m/N\), from equation (3) we have
\[
W_x(n,m) = 2\sum_{l=0}^{N-1} K_n(l) W^{l l'}
\]
where
\[
N = 2L + 2, W_l = \exp(-i\pi l/N)
\]
\[
K_n(l) = \begin{cases} 0 \quad & \text{if } 0 \leq l \leq N/2 - 1 \\ K_n(l-n) \quad & \text{if } N/2 + l \leq N - 1 \\ K_n(l) & \text{if } l = N/2 \end{cases}
\]
Clearly, equation (4) matches the standard form of a DFT and can be evaluated efficiently using standard fast Fourier transform (FFT) algorithm. The only difference is that the discrete-frequency at point \(m\) is \(f = m/(2NTs)\).

3. THE NUMERICAL EXAMPLES

In this section, the comparison of time-frequency properties of WD with DFT is made, by using of a numerical example. We consider a chirp signal whose two frequencies vary linearly with time. Let
\[
\begin{align*}
q_1(t) &= nft/T, \quad q_2(t) = -nf(t-T)/T \\
\end{align*}
\]
then
\[
\begin{align*}
\omega_1(t) &= 2nf(t/T, \quad \omega_2(t) = -2nf(t-T)/T \\
x(t) &= A_1 \sin[q_1(t)] + A_2 \sin[q_2(t)]
\end{align*}
\]
\(t \in [0,T]\)
Select \(f = 25\text{Hz}, T = 10\text{s}, A_1 = A_2 = 2\) and sampling frequency \(f_s = 200\text{Hz}\), we can get the discrete-time signal \(\{x(n)\}\). Fig. 1 indicates the regular pattern of frequencies varying with time. The number of data points used in each WD was \(N = 512\), and the corresponding frequency varying region was \(\Delta f = 6.2875\text{Hz in every data interval.}\) Fig. 2 depicts the time-frequency estimation of the signal by WD method. Fig. 3 depicts the time-frequency estimation of the same signal by DFT method. The peak frequency estimation results at each time are listed at table 1. From these results we can observe that the WD method can best estimate the instantaneous frequency of the nonstationary signal, but the DFT method can only give a bandwidth estimation of the signal, not the instantaneous frequency.

4. THE EXPERIMENTAL PROCEDURE AND RESULTS

4.1 The experimental principle

The testing structural model is a wave guide with "U" shape, as shown in Fig. 4. Since its special consturcture, its natural frequencies has tuning fork like beat frequency, \(f_1 = 19.5\text{Hz}, f_2 = 20.25\text{Hz}\). After a long time using in teaching experiment, a minigap appeared at the weld line and the nonlinear vibration phenomenon arose from the experiment. The experimental principle is shown in Fig. 5.

4.2 The experimental results

A sinusoidal forcing function is used to excite the structure. The frequency of the sinusoidal signal is settled at \(f = 21.25\text{Hz}\), a bit higher than the second mode natural frequency. When the amplitude of the input signal is small, the vibration of the structure displays a linear property. Its power spectrum of response, say \(\{x_1(n)\}\), is shown in Fig. 6. Increasing the strength of the input, the nonlinear vibration phenomenon appeared. Fig. 7 shows the spectral of response \(\{x_2(n)\}\), and Fig. 8 shows the spectral of re-
4. The comparison of the WD and the FFT

In this section, we use the WD method and the FFT method to estimate the time-frequency spectral of the experimental response signals respectively. The sampling frequency is \( f_s = 1.28 \text{KHz} \), the time resolution is \( \Delta t = 128 \) points, that is \( \Delta t = 0.1 \) second. The points of data number used in each WD and FFT is \( N = 512 \). Fig. 9, Fig. 11 and Fig. 13 depict the time-frequency spectral of \( \{x_1(n)\}, \{x_2(n)\} \) by FFT method respectively. Fig. 10, Fig. 12 and Fig. 14 depict the corresponding results by WD method.

From these results, we can see that for linear vibration, the WD and FFT give a similar results, shown in Fig. 9 and Fig. 10. We can also see that for the nonlinear vibration case, the FFT method gives the varying spectral in time direction, see Fig. 11 and Fig. 13, but the WD method gives a different time-frequency property, and the amplitude-jump phenomenon arose, see Fig. 12 and Fig. 14.

5. CONCLUSION

The WD method was adopted to estimate the time-frequency spectral of the nonlinear vibration system. Comparisons of the WD method with FFT method by numerical illustration and experimental signal processing showed the effectiveness of the WD in time-frequency spectral estimation.

6. REFERENCES


Fig. 4. The testing structural model


Fig. 5. The experimental principle diagram

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<tr>
<th>Time</th>
<th>t/s</th>
<th>1.275</th>
<th>2.275</th>
<th>4.275</th>
<th>4.775</th>
<th>6.275</th>
<th>7.275</th>
<th>8.275</th>
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<td>ACTURE</td>
<td>$f_1(t)$</td>
<td>3.188</td>
<td>5.688</td>
<td>10.688</td>
<td>11.938</td>
<td>15.688</td>
<td>18.188</td>
<td>20.688</td>
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<tr>
<td>WD</td>
<td>$f_1(t)$</td>
<td>3.125</td>
<td>5.664</td>
<td>10.938</td>
<td>11.914</td>
<td>16.016</td>
<td>18.555</td>
<td>21.484</td>
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<tr>
<td>FFT</td>
<td>$f_1(t)$</td>
<td>5.087</td>
<td>7.422</td>
<td>12.891</td>
<td>10.938</td>
<td>17.578</td>
<td>17.578</td>
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Fig. 6 Spectral of $x_1(t)$

$x_1(\omega)$

Fig. 7 Spectral of $x_2(t)$

$x_2(\omega)$

Fig. 8 Spectral of $x_3(t)$

$x_3(\omega)$

Fig. 9 Time-frequency spectral of $x_1(t)$ by FFT

Fig. 10 Time-frequency spectral of $x_1(t)$ by WD

Fig. 11 Time-frequency spectral of $x_2(t)$ by FFT
Fig. 12 Time-frequency spectral of $x_1(t)$ by WD

Fig. 13 Time-frequency spectral of $x_1(t)$ by FFT

Fig. 14 Time-frequency spectral of $x_1(t)$ by WD