RESIDUAL COMPONENT MODES IN THE ANALYSIS OF COMPOSITE PRIMARY-SECONDARY SYSTEMS

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ABSTRACT

A modal correction method is here extended for the analysis of composite primary-secondary structures, in both the cases of deterministic and stochastic input. It allows to take into account the influence on the response of the residual components modes, neglected in the component-mode synthesis, by means of the presence of particular pseudo-static forces. The method appears to be very amenable from a computational point of view taking into account the residual modes of the whole composite structure without an evident increment of computational effort. The applications of the method to some structural systems has always revealed a good improvement respect to the results obtained by the classical modal analysis.

NOMENCLATURE

- referred to the composite system
- referred to the primary subsystem
- referred to the secondary subsystem
- referred to the coupling between the subsystems
- referred to the increment in the primary system due to the presence of the secondary one
- referred to the condensed coordinates
- referred to approximated quantities
- referred to quantities evaluated with a reduced number of modes
- referred to the particular integral
- degrees of freedom
- condensed degrees of freedom
- mass matrix
- damping matrix
- stiffness matrix
- displacements vector
- forcing vector
- time
- generic time instant
- condensed coordinates vector
- transformation matrix
- mass-normalized eigenvector matrix
- pseudo-static influence matrix
- diagonal matrix of natural frequencies
- mass matrix in condensed coordinates
- damping matrix in condensed coordinates
- identity matrix
- matrix in Eq.(10)
- matrix in Eq.(10)
- matrix in Eq.(10)
- matrix in Eq.(11)
- state variable vector
- dynamical matrix
- participation factors matrix
- transition matrix
- eigenvector matrix
- eigenvalue matrix
- load matrix
- matrix in Eq.(24)
- j-th correlation of f
- stochastic average
- Kronecker product
- intensity vector of white noise
- Wiener processes vector
- temporal differential
- j-th Kronecker power
- dynamical matrix in Eq.(35)
- transformation matrix in Eq.(37)
- transition matrix in Eq.(39)
- load matrix in Eq.(41)

1. INTRODUCTION

The response behaviour of secondary systems attached to primary structural systems has been an area of considerable research interest in recent years [1]. In particular, in structural engineering, the seismic performance of secondary systems, such as equipment, panels, piping systems and structural partitions attached to building structures, is of great importance since they serve a vital function and their failure may have for-reaching ramifications.

Since both primary and secondary systems are often multi-degrees-of-freedom systems, and the number of degrees of freedom of the combined system can be prohibitively large, the component-mode synthesis methods [2] are usually adopted for reducing the number of generalized coordinates required.
in the dynamic analysis. These methods reduce the number of differential equations by using a coordinate transformation involving the first natural modes of the two substructures separately taken. Since a really efficient component-mode synthesis method should contain the smallest number of normal modes, it follows that it gives an approximate solution. Methods to improve this approximate solution have been proposed, introducing the residual stiffness and the residual mass matrices [3,4,5]. These methods include a pseudo-static representation of the forces due to higher modes of the single substructure (the so-called residual component modes). The main drawback in the application of these methods lies on the correct representation of the constraint and equilibrium conditions at the boundary between the substructures.

Here an alternative method is presented that overcomes the drawback of the traditional methods and, for deterministic input, it only requires the inversion of a symmetric and well conditioned matrix, allowing to obtain sufficiently good results with a limited computational effort. This method is an extension to the case of composite structures of a mode-superposition method, used in literature for non-composite multi-degrees-of-freedom systems [6].

At last, it is shown that the proposed approach can be advantageously applied to the stochastic analysis of composite primary-secondary structures without a real increasing of computational effort.

2. DETERMINISTIC RESPONSE

The motion equation of a secondary substructure having \( n_s \) degrees of freedom, connected with a primary substructure having \( n_p \) degrees of freedom, and subject to seismic excitations can be written, in terms of total coordinates, as follows:

\[
M_s \ddot{u}_s + C_s \dot{u}_s + K_s u_s = f(t) \quad (1)
\]

where the dot upon a function indicates the temporal derivative of the function, \( f(t) \) represents the forcing function vector of order \( n \) and \( u_s \) is the vector of the total displacements, of order \( n = n_s + n_p \), given by:

\[
u^T_s = (u^T_s \quad u^T_p) \quad (2)
\]

\( u_s \) and \( u_p \) being the vectors of the displacements of the secondary and primary substructures respectively, evaluated with respect to the ground. In the equation (1), \( M_s \), \( C_s \) and \( K_s \) represent the mass, damping and stiffness matrices of the composite system and they have the following expressions:

\[
M_s = \begin{pmatrix} M_{ss} & M_{sp} \\ M_{ps}^T & M_{pp} + M_0 \end{pmatrix} ; \quad C_s = \begin{pmatrix} C_s & C_{sp} \\ C_{ps}^T & C_p + C_0 \end{pmatrix} ; \quad K_s = \begin{pmatrix} K_s & K_{sp} \\ K_{sp}^T & K_p + K_0 \end{pmatrix} \quad (3)
\]

where \( M_s \), \( C_s \), \( K_s \), and \( M_p \), \( C_p \), \( K_p \) are the mass, damping and stiffness matrices of the secondary and primary substructures respectively, built considering the two systems fixed at their bases; that is, the primary subsystem is considered fixed at the ground and the secondary subsystem is considered fixed at the attached point with the primary one and, eventually, also at the ground; \( M_{sp}, C_{sp} \) and \( K_{sp} \) are the matrices defining, from a physical point of view, the coupling between the two substructures; at last, the matrices \( M_0, C_0 \) and \( K_0 \) represent the increments to the mass, damping and stiffness matrices of the primary substructure due to the presence of the secondary one.

As clarified in [2], the dynamical response of a composite structure can be advantageously represented if the component-mode synthesis, defined by the following coordinates transformation, is adopted:

\[
u_i = \Gamma_i q \quad (4)
\]

where \( q \) is the vector of condensed coordinates, while \( \Gamma_i \) is the transformation matrix; these two quantities are given by:

\[
q = \begin{pmatrix} q_s \\ q_p \end{pmatrix} ; \quad \Gamma_i = \begin{pmatrix} \Phi_s & N\Phi_p \\ 0 & \Phi_p \end{pmatrix} \quad (5)
\]

where \( N \) is called pseudostatic influence matrix [7], defined as:

\[
N = -K_i^{-1}K_{sp} \quad (6)
\]

In equation (5), \( \Phi_s \) and \( \Phi_p \) are the modal matrices of the two subsystems (of order \( n_s \times n_s \) \( \quad n_p \times n_p \)), normalized with respect to the mass matrices \( M_s \) and \( M_p \) respectively and obtained solving the following eigenvalues problems:

\[
M_s \Phi_s, \Omega^2_s = K_s \Phi_s ; \quad M_p \Phi_p, \Omega^2_p = K_p \Phi_p \quad (7)
\]

\( \Omega_s \) and \( \Omega_p \) being two diagonal matrices whose diagonal terms are the natural frequencies \( \omega_i \) of the two substructures separately taken.

Using the transformation defined into equation (4), the equation (1) becomes a set of \( m \leq n \) differential equation (with \( m = m_s + m_p \) \( \quad n = n_s + n_p \)), which can be written in the following form:

\[
u_i + \Xi \ddot{q} + \Omega^2 \dot{q} = \Gamma_i^T f(t) \quad (8)
\]

where \( \Xi \) and \( \Omega^2 \) are symmetric and positive definite matrices given, respectively, by the following expressions:

\[
\Xi = \begin{pmatrix} \Xi_s & \Xi_{sp} \\ \Xi_{sp}^T & \Xi_p \end{pmatrix} ; \quad \Omega^2 = \begin{pmatrix} \Omega^2_s & \Omega^2_{sp} \\ \Omega^2_{sp} & \Omega^2_p + \Omega^2 \end{pmatrix} \quad (9)
\]

In these equations \( I_{m_s} \) \( \quad (r = s, p) \) is the identity matrix of order \( m_s \times m_s \) and \( 0 \) is the zero matrix and:

\[
\Delta M = M_0 + N^T M_s N + N^T M_{sp} \quad (10)
\]

\[
\Delta C = C_0 + N^T C_s N + N^T C_{sp} \quad (10)
\]

\[
\Delta K = K_0 + K_{sp}^T K_i^{-1} K_{sp} \quad (10)
\]
In equations (9), the matrices $\mathbf{Z}$ and $\mathbf{N}$ are diagonal only if the two subsystems, separately taken, are classically damped. It is worth noting that, usually, the matrix $\mathbf{Z}$ is not diagonal; but often it is assumed that the two subsystems are classically damped and the out-of-diagonal terms of the matrix $\mathbf{Z}$ are sufficiently small to be neglected. In this case it is supposed that $\mathbf{C}_p + \mathbf{C}_p \mathbf{N} = 0$ and $\Delta C = 0$; in this way the matrix $\mathbf{Z}$ becomes diagonal. At last, the matrix $\Delta \mathbf{D}^2$ is given by:

$$\Delta \mathbf{D}^2 = \mathbf{Z}_p \Delta \mathbf{K} \mathbf{Z}_p$$

(11)

this matrix is usually non diagonal and becomes a zero matrix if the secondary substructure is mono-connected only to the primary one.

Since, even in the particular case when the two matrices $\mathbf{D}_p^2$ and $\mathbf{Z}$ are diagonal, the matrix $\mathbf{m}$ is not diagonal and the matrices $\mathbf{D}_p^2 \mathbf{m}^{-1} \mathbf{Z}$ and $\mathbf{Z} \mathbf{m}^{-1} \mathbf{D}_p^2$ do not commute [8], the solution of equation (8) can be obtained by means of the introduction of the state variables vector. In this way, equation (8) can be rewritten as follows:

$$\mathbf{z}_e = \mathbf{D}_e \mathbf{z} + \mathbf{V}_e \mathbf{f}(t)$$

(12)

where:

$$\mathbf{z}_e = \begin{pmatrix} \mathbf{q}_e \\ \mathbf{q}_e \end{pmatrix}; \quad \mathbf{D}_e = \begin{pmatrix} 0 & \mathbf{I}_m \\ -\mathbf{m}^{-1} \mathbf{D}_p^2 & -\mathbf{m}^{-1} \mathbf{Z} \end{pmatrix};$$

$$\mathbf{V}_e = \begin{pmatrix} 0 \\ \mathbf{I}_m \end{pmatrix} \mathbf{I}$$

(13)

Equation (12) represents a set of $2m = 2m_p + 2m_p$ differential equations of the first order which can be simply solved, once that the eigenvalue and the eigenvectors of $\mathbf{D}_e$ are known. In fact the solution of equation (12) can be written as follows:

$$\mathbf{z}_e(t) = \Theta_e(t - t_0) \mathbf{z}_e(t_0) + \int_{t_0}^{t} \Theta_e(t - \tau) \mathbf{V}_e \mathbf{f}(\tau) d\tau$$

(14)

where $\mathbf{z}(t_0)$ is the vector of initial conditions and $\Theta(t)$ is the so-called transition matrix or fundamental matrix [9,10] of the composite system that, if all the eigenvalue of $\mathbf{D}_e$ are distinct, can be obtained as follows:

$$\Theta_e(t) = \mathbf{F}_e \exp(\mathbf{A}_e t) \mathbf{F}_e^{-1}$$

(15)

where $\mathbf{F}_e$ and $\mathbf{A}_e$ are complex matrices that can be obtained solving the following eigenvalue problem:

$$\mathbf{D}_e \mathbf{F}_e = \mathbf{F}_e \mathbf{A}_e$$

(16)

$\mathbf{D}_e$ being the dynamical matrix defined in equation (13).

In many cases of practical interest, the closed form solution of the equation (14) can not be evaluated and a method of numerical solution has to be adopted. For this purpose, once that the temporal axis is divided in intervals $\Delta t$ having the same length by means of the subdivision times $t_0 = 0, t_1, \ldots, t_n$, a forcing function $\mathbf{f}(t)$, constant in each interval is considered; hence, equation (14) can be rewritten in the following step-by-step form:

$$\mathbf{z}_e(t_{k+1}) = \Theta_e(\Delta t) \mathbf{z}_e(t_k) + \mathbf{L}_e(\Delta t) \mathbf{V}_e \mathbf{f}(t_k)$$

(17)

where:

$$\mathbf{L}_e(\Delta t) = \int_{t_k}^{t_{k+1}} \Theta_e(t_{k+1} - \tau) d\tau = \int_{0}^{\Delta t} \Theta(t_{k+1} - \tau) d\tau$$

$$= [\Theta_e(\Delta t) - I_{zm}] \mathbf{D}_e^{-1}$$

(18)

and it is defined load matrix. Equation (17) gives an unconditional stable step-by-step procedure in which the error is connected only with the modelling of the forcing function as a piece-wise constant function; at this purpose, it is important to note that it is possible considering other approximating shapes for the forcing function in each interval, in order to improve the accuracy of the approach [2].

3 RESIDUAL MODAL COMPONENTS FOR DETERMINISTIC INPUTS

The procedure described in the previous section needs the solution of two real eigenvalue problems, defined in equation (7), and of a complex eigenvalue problem, defined in equation (16). Hence, this procedure is really effective only if the number of real eigenvalues considered for the analysis is such that $(m_p + m_p) \ll (n_1 + n_2)$. This implies that the nodal response $u(t)$ is approximated; then, applying the apex to all the quantities considered with a reduced numbers of modes, and the apex $\tilde{}$ to all the quantities approximated, equation (4) assumes the following form:

$$\mathbf{u}_t^e(t) = \mathbf{F}_1 \mathbf{q}(t)$$

(19)

Using this coordinate transformation, the equations system given in equation (8) becomes a set of $m$ differential equations (with $m = m_p + m_p \ll n = n_1 + n_2$).

As happens in the dynamical analysis of traditional non composite structures, an improvement of the accuracy of the nodal response is obtained, as showed by various authors [3,4,5], taking into account the modes non considered in the modal analysis in a pseudostatic way. Here an alternative approach, that takes into account these residual modes, is presented. This method is conceptually similar to the dynamical correction method proposed into [6]. The extension of this technique to the case of composite systems gives:

$$\mathbf{u}_t^e(t) = \mathbf{F}_1 \mathbf{q}(t) + \left[ \mathbf{u}_t^e(t) - \mathbf{F}_1 \mathbf{q}(t) \right]$$

(20)

where the apex $\tilde{}$ indicates that the nodal response has been evaluated taking into account the modal correction; the quantities $\mathbf{u}_t^e$ and $\mathbf{q}(t)$ are the particular solutions of the equations (1) and (8) respectively. Hence, the term in square brackets in the second member of equation (20) takes into account, in pseudostatic form, the residual components modes. It is important to note that the neglected modes are always those corresponding to the largest natural frequencies, without considering if they belong to a subsystems or to the other one.
Under the assumption of piece-wise constant forcing function inside each time interval $\Delta t$, the particular solutions $u_i(t)$ and $q_i(t)$, considered before, assume, for $t = t_{k+1}$, the following expressions:

$$u_i(t_k) = K_i^{-1}f(t_k); \quad q_i(t_k) = \hat{\Omega}^{-\frac{3}{2}}\hat{I}_i f(t_k) \quad (21)$$

Replacing these expressions into equation (21), it writes:

$$u_i(t_{k+1}) = \hat{I}_i q_i(t_k) + [K_i^{-1} - \hat{I}_i \hat{\Omega}^{-\frac{3}{2}} \hat{I}_i]^T f(t_k) \quad (22)$$

It is important to note that the modal correction needs a higher computational effort (with respect to the classical modal analysis) which is only connected to the inversion of $K_i$, which is a matrix of order $(n_i + n_p) \times (n_i + n_p)$. However it can be easily shown that the following relationship holds [2]:

$$P^T K_r P = K_r \quad (23)$$

where:

$$P = \begin{pmatrix} I & N \\ 0 & I \end{pmatrix} \quad K_r = \begin{pmatrix} K_r & 0 \\ 0 & K_r + \Delta K \end{pmatrix} \quad (24)$$

hence, we have:

$$K_i^{-1} = \hat{P} K_i^{-1} \hat{P}^T \quad (25)$$

with:

$$K_i^{-1} = \begin{pmatrix} K_i^{-1} & 0 \\ 0 & (K_r + \Delta K)^{-1} \end{pmatrix} \quad (26)$$

In this way, by means of a coordinate transformation, the inversion of a matrix of order $(n_i + n_p) \times (n_i + n_p)$ is replaced by the inversion of two matrices, one of order $n_i \times n_i$ and the other one of order $n_p \times n_p$. Moreover, it is important to note that the inversion of the matrix $K_i$ has to be carried out in any case for the evaluation of the matrix $N$; so, the only incremented computational effort, when the dynamical correction is applied, lies on the inversion of a matrix of order $n_p \times n_p$.

Replacing equation (6) into equation (27) and observing that:

$$\hat{I}_i = \hat{P} K_i^{r} \hat{P}; \quad \hat{I}_r = \begin{pmatrix} \hat{K}_{r} & 0 \\ 0 & \hat{P} \end{pmatrix} \quad (27)$$

we obtain:

$$u_i(t_k) = \hat{P} K_i^{r} q_i(t_k) + \hat{P} [K_i^{-1} - \hat{I}_i \hat{\Omega}^{-\frac{3}{2}} \hat{I}_i]^T f(t_k) \quad (28)$$

In this equation the first term on the right hand side is the dynamic contribution due to the first $m$ modes, while the second term is the pseudo-static contribution due to the higher modes neglected.

4. RESIDUAL MODAL COMPONENTS FOR STOCHASTIC WHITE NOISE INPUT

If $f(t)$ is a vector of stochastic processes, then $u(t)$ is a vector of stochastic processes too and it has to be characterized in a probabilistic sense. If $f(t)$ is a vector of zero-mean normal processes, then, as the structural system under examination is linear, $u(t)$ and $\dot{u}(t)$ are also zero-mean normal processes and the complete characterization of both input and output processes can be obtained by the knowledge of the first two correlation functions, that for zero-mean processes coincide with a zero-vector (the first order correlation function) and with a second order moments vector.

In the present work, the analysis is limited to the case in which $f(t)$ is a normal zero-mean white noise process, characterized by having the first two correlation functions in the form:

$$R_f^{(1)}(t) = E[f(t)] = 0 \quad (29)$$
$$R_f^{(2)}(t_1, t_2) = E[f(t_1) \circ f(t_2)] = S(t_1)6(t_2 - t_1)$$

where $E[\cdot]$ means stochastic average, the symbol $\circ$ indicates Kronecker product [8], $S$ is the vector collecting the intensities of the white noise vector and, at last, $6(\cdot)$ is the Dirac delta function.

Since, for linear systems, the second order correlation function of the response $R^{(2)}(t_1, t_2) = E[u(t_1) \circ u(t_2)]$ can be simply evaluated, once that the second order moments at a single time $t_1$ are known [9,10], in the following, the differential equations governing these moments will be written. In order to do this, we remember that, if $f(t)$ is a vector of white noise processes, its components are not Riemann integrable in mean square value [11] and, consequently, equation (1) has not mathematical sense and it is necessary to consider the following stochastic differential equation, written in terms of state variables:

$$dz_i = D_i z_i dt + V_i dB(t) \quad (30)$$

where:

$$D_i = \begin{pmatrix} 0 & \left( -M_i^{-1} K_i -M_i^{-1} C_i \right) \\ \left( -M_i^{-1} K_i -M_i^{-1} C_i \right)^T & 0 \end{pmatrix} \quad (31)$$

and:

$$dB(t) = f(t) dt \quad (32)$$

is the vector of order $n$ collecting the Wiener processes related to the white noise processes considered before; that is, it is such that the following relationships hold:

$$E[\dot{w}(t)] = 0; \quad E[\dot{w}(t) \circ \dot{w}(t)] = S(t) dt \quad (33)$$

The differential equations governing the second order moments can be obtained, starting from the stochastic differential equation (30), applying the Itô differential rule [11] in a suitable way; they have the following expression:

$$\dot{E}[x_{i+1}^{[2]}] = D_{i+1} E[x_{i+1}^{[2]}] + V_{i+1}^{[2]} S(t) \quad (34)$$

where the exponent into square brackets indicates the Kronecker power, that is $x_{i+1}^{[2]} = x_i \otimes x_i$, and $D_{i+1}$ has the following Lyapunov form:

$$D_{i+1} = D_i \otimes I_{2n} + I_{2n} \otimes D_i \quad (35)$$

$I_{2n}$ being the identity matrix of order $2n$. 

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As happens in the deterministic case, even in the stochastic case it is opportune to work in terms of condensed coordinates; at this purpose, it is easy to show that the following relationship holds between the second order moments expressed in terms of total and condensed coordinates:

\[ E[z^2] = R_i^2 E[y^2] \]  
(36)

where:

\[ R = \begin{pmatrix} 1 & 0 \\ 0 & R_i \end{pmatrix} \]  
(37)

In this way the equations governing the second order moments can be rewritten in terms of condensed coordinates as follows:

\[ \dot{E}[y^2] = D_{c,2} E[y^2] + V_{c}^2 S(t) \]  
(38)

where \( D_{c,2} \) is connected to \( D_c \) by means of a relationship equal to that given in equation (35). Equation (38) represents a set of \((2m)^2\) differential equations of first order, which can be solved once the eigenvalues and eigenvectors of \( D_{c,2} \) are known. But it can be shown that the eigenproblem of the matrix \( D_{c,2} \) is connected to that of the matrix \( D_c \) in such a way that the following relationship holds between the two corresponding transition matrices [12]:

\[ \Theta_{c,2}(t) = \Theta_{c}(t) \]  
(39)

This implies that the stochastic analysis does not require the solution of an additional eigenproblem with respect to the deterministic case.

Even in this case it is possible to apply a step-by-step procedure considering the forcing function as piece-wise constant in each step; this procedure gives the following solution for the second order moments of the condensed coordinates:

\[ E[y^2(t_{k+1})] = \Theta_{c,2}(\Delta t) E[y^2(t_k)] + L_{c,2} V_{c}^2 S(t_k) \]  
(40)

where:

\[ L_{c,2} = [\Theta_{c,2} - I_{4m^2}] D_{c,2}^{-1} \]  
(41)

The requirement to reduce the number of modal coordinates evidenced in the deterministic case, becomes more and more preminent in the stochastic analysis, because the number of unknown quantities, that is the components of \( E[y^2] \), becomes equal to \( m^2 \); hence, setting \( m_s + m_p = m \ll n = n_s + n_p \) and considering the residual modes as in the deterministic case, the response, in terms of second moment of the corrected total coordinates, is given by:

\[ E[z^2] = R_i^2 E[y^2] + \{ E[z^2] - R_i^2 E[y^2] \} \]  
(42)

where \( E[y^2] \) and \( E[z^2] \) are the particular integrals of the differential equations (34) and (36) respectively. Using a step-by-step solution, once that \( E[z^2](t_{k+1}) \) is evaluated by means of equation (40), the response, in terms of second moments of corrected condensed coordinates, is:

\[ E[z^2](t_{k+1}) = R_i^2 E[y^2](t_{k+1}) - [D_{c,2}^{-1}] V_{c}^2 S(t_{k+1}) \]  
(43)

It is important to note that, in the case of stationary input, that is when the intensities of the white noise processes are independent of \( t \), from equation (38) in which \( E[y^2] = 0 \), we obtain:

\[ E[z^2] = E[y^2] - D_{c,2}^{-1} V_{c}^2 S = E[z^2] \]  
(44)

and substituting this expression into equation (44), it follows:

\[ E[z^2] = -D_{c,2}^{-1} V_{c}^2 S = E[z^2] \]  
(45)

This means that, for stationary inputs, the corrected second order moments coincide with the exact ones, as can be easily seen setting \( E[z^2] = 0 \) into equation (34).

5. NUMERICAL EXAMPLE

In this section the structural system depicted in Fig.1 has been considered; this system is composed by a primary subsystem having four degrees of freedom and by a secondary subsystem having three degrees of freedom, connected both at the primary one and at the ground. The motion equation governing the dynamical behaviour of this structure is that defined into equation (1), where, remembering equations (3), the following matrices must be considered:

\[ M_s = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad M_p = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_4 & 0 \\ 0 & 0 & m_5 \end{pmatrix} \]

\[ K_s = \begin{pmatrix} k_s + k_0 & -k_s & 0 \\ -k_s & 2k_s - k_s & -k_s \\ 0 & -k_s & 2k_s + k_0 \end{pmatrix} \quad K_p = \begin{pmatrix} 4k_p & 0 & -2k_p \\ 0 & 4k_p & 0 \\ -2k_p & 0 & 4k_p \end{pmatrix} \]

\[ K_s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad K_p = \begin{pmatrix} k_0 & 0 & 0 \\ 0 & k_0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

In which \( m_1 = m_2 = m_3 = 5k\); \( m_4 = m_5 = m_6 = m_7 = 10k\); \( k_s = k_p = k_0 = 5000N/m\); moreover it has been assumed that the two subsystems are both classically damped with, damping coefficients \( \zeta_p = 0.05 \) and \( \zeta_s = 0.01 \). In Tab.1 the natural frequencies of the two subsystems separately taken are reported. In the deterministic case it has been considered, as forcing, a sinusoidal function acting on all the degrees of freedom of the structure, that is:

\[ f(t) = -M_s \tau \sin(\omega_0 t) \]  
(47)

in which \( \tau \) is a vector with all the components equal to the unit and \( \omega_0 \) is the forcing frequency. Only two modes have been considered in the analysis, and we have reported the results obtained by the modal correction method proposed in this work and by the classical component-mode synthesis, compared with the exact response, evaluated considering all the modes. In Fig.2 the response of the mass 5, for \( \omega_0 = 1rad/sec \), is
depicted. From this figure, the correction operated by the proposed method is clear; but it is more and more evident if we report the results in terms of percentage error, as made in Fig.3. In Fig.4 the percentage error related to the response of mass 1 is reported. From these figures we observe that the modal correction method gives good results, above all for the mass 3.

In the stochastic case it has been considered the following vector forcing:

\[ f(t) = -M_1 \sigma f_0(t) \]  \hspace{1cm} (48)

\( f_0(t) \) being a white noise with unitary intensity, modulated by a shaping function having the expression \( \sqrt{t/2} \exp(1-t/2) \). The same modes of the deterministic case have been dropped in the analysis. In Fig.5 the response, in terms of second order moments of the velocity of the mass 1, evaluated with the classical component-mode synthesis, with the modal correction method and with all the modes, are depicted. In Figs.6-9 the percentage errors related to the second order moments of displacements and velocities of the mass 5 and 1 are reported. From these figures we can observe that, even in the stochastic case, the modal correction method proposed in this work gives good results.

6. CONCLUSIONS

In the present work, a modal correction method, used in literature for non-composite structural system, is extended for the deterministic and stochastic analysis of composite primary-secondary structures. This method allows to substantially improve the accuracy reached with the application of the classical component-mode synthesis, without a relevant increment of computational effort; in particular, for the deterministic analysis, it only needs the inversion of a matrix of order equal to the degrees of freedom of the primary structure. The application of this method to the deterministic and stochastic analysis of a simple composite structure has evidenced its goodness.

7. REFERENCES


![Fig. 1 Structural system](image)

<table>
<thead>
<tr>
<th>( \omega_1 ) [rad/s]</th>
<th>( \omega_2 ) [rad/s]</th>
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<td>8.68</td>
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<tr>
<td>13.32</td>
<td>19.50</td>
</tr>
<tr>
<td>18.79</td>
<td></td>
</tr>
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</table>

Tab. I Natural radians frequency of the two subsystems
Fig. 2 Mass 5 displacement; $\omega_0 = 1$ [rad/sec]

Fig. 3 Percentage error of mass 5 displacement

Fig. 4 Percentage error of mass 1 displacement

Fig. 5 Second order moment of mass 5 velocity

Fig. 6 Percentage error of second order moment of mass 5 displacement

Fig. 7 Percentage error of second order moment of mass 5 velocity

Fig. 8 Percentage error of second order moment of mass 1 displacement

Fig. 9 Percentage error of second order moment of mass 1 velocity