IDENTIFICATION OF FLEXIBLE SPACE STRUCTURE MODELS USING OPERATING DATA

Albert Bosse, Doctoral Research Assistant
Gary Slater, Professor
Department of Aerospace Engineering and Engineering Mechanics
University of Cincinnati
Cincinnati, OH 45221-0070 USA

Abstract

A procedure called Reference Model Adaptation (RMA) is presented whose primary objective is the estimation of impulse response functions of a multivariable flexible structure under closed loop control. The RMA scheme, which operates on input and output time histories, is envisioned to track slowly time-varying system characteristics which may be present in flexible spacecraft systems due to anomalies, maintenance, or evolving substructures. Primary considerations are the minimization of disturbances required by the identification process in extracting system models from the operating data, and the provision of updated models for the redesign or reconfiguration of active vibration controllers. Early development efforts have focused on the problem of closed loop identification of system impulse response functions, results from which are given in this paper. With impulse response functions in hand, active controls can then be designed directly, or a state-space model can be generated from which active controls may be designed. Experimental results are presented for a five meter space truss instrumented with proof mass actuators and accelerometers located in the University of Cincinnati Structural Dynamics Research Laboratory.

Nomenclature

- $\hat{u}$: total system input
- $w$: noise process
- $x$: state vector
- $y$: output vector
- $u$: input vector, control signal
- $A$: discrete system matrix (state-space model)
- $B$: input influence matrix
- $C$: output influence matrix
- $D$: direct transmission matrix
- $n$: number of states, observer order
- $p$: number of IRF's to estimate
- $m$: number of inputs
- $q$: number of outputs
- $i$: time step index
- $h(i)$: IRF matrix at time step $i$
- $[A \mid B]$: matrix partitions
- $\hat{x}$: observer state vector
- $\hat{y}$: observer output estimate
- $\hat{A}$: observer system matrix
- $\hat{B}$: observer input influence matrix
- $\nu(i)$: observer system input vector
- $[0]$: zero matrix
- $\lambda$: vector of assigned observer eigenvalues
- $\lambda_k$: $k$th element of $\lambda$
- $\Gamma^T$: observer identification data matrix

1 Introduction

In the last 15 years, much attention in the literature has been given to identification and control of large space
structures (LSS). A variety of methods have been developed and tested using analytical data, while very few have been tested on real systems. Based on previous experience during the NASA Controls-Structures Interaction Guest Investigator program, an approach to integrated identification and control of flexible structures, called Reference Model Adaptation (RMA), has been developed. RMA is a common-sense approach to the problem, for which the following scenario is quite likely:

1. Using analytical or finite-element techniques, an initial system model is constructed.

2. Based on ground testing of the full spacecraft, of scale models, or of sub-structures, the initial system model is modified to closely match the experimental data. This is called the initial verified model.

3. The spacecraft is launched and deployed, and the operating data show that the initial verified model is inaccurate.

The deviations of the initial verified model from the orbiting spacecraft can be expected due to the inability of ground tests to replicate the free-free boundary conditions and zero-g environment of space. Therefore, the most important consideration in the active control of flexible spacecraft, which can lead to performance degradation or even instabilities, is model uncertainty. Likewise, whether the objective is to actively control a test structure on the ground or an orbiting spacecraft, the capability to identify a system model or to modify an existing model so that it more closely matches experimental data should be of paramount interest. Modification of the reference model using the spacecraft operating data is the focus of Reference Model Adaptation.

Fig. 1 depicts the Reference Model Adaptation scheme, which is an indirect adaptive control approach consisting of a reference model adaptation mechanism and a controller redesign mechanism. Such a system architecture is common in the area of adaptive control, and variations have been proposed for on-line robust control design. In the RMA approach, it is assumed that a spacecraft attitude controller and possibly a conservative vibration controller based on the initial verified model are operating after deployment of the spacecraft. Spacecraft vibrations are observed due to maneuvering, payload disturbances, or environmental disturbances. If the reference model does not match the operating data accurately enough, the reference model adaptation mechanism processes the input-output time histories in an attempt to identify a more accurate system model. Once the updated reference model has been found, it is passed on to the controller redesign mechanism which synthesizes a new vibration controller. The symbol $y$ refers to the system output vector (e.g., accelerometer signals), $u$ refers to the controller output vector (e.g., voltage commands to force actuators), and $\hat{u}$ refers to the system input vector, which may differ from $u$ due to the addition of low-level test signals needed to satisfy identifiability conditions. A variety of algorithms and procedures are required to implement an RMA system. This paper focuses on closed loop system identification which is an integral part of the reference model adaptation mechanism.

Researchers have long recognized the need for on-orbit identification of large space structures because of the inability of analytical and ground-based testing techniques to accurately model the flexible spacecraft in its operating environment. For any on-orbit identification, the testing technique should not render the structure inoperable over considerable periods of time; therefore, an approach which allows for the identification to take place while the spacecraft is under closed loop control offers added flexibility. In this effort, the closed loop system identification is applied to both analytical and experimental data by adapting existing time and frequency domain techniques which estimate impulse response functions (IRF's). For traditional structural system identification algorithms, impulse response functions are a common input data type. Once impulse response functions have been obtained, active controls can be designed directly, or a state space model can be generated from which active controls may be designed. A state-space model, rather than a modal model, is used because experimental vibration data for a flexible spacecraft reflect not only structural dynamics, but also the dynamic effects due to actuators, sensors, signal conditioning filters, and control computer with its inherent time delays. The ramifications of doing identification under closed loop control are discussed below.

### 2 IRF Estimation

For open loop identification, test signals (e.g., pure or burst random) are used as commands to the system inputs, the outputs are measured, and then some type of estimation algorithm is applied either directly or to filtered versions of the input and output time histo-
2.1 Time Domain Approach

A general linear, time-invariant, discrete state-space model is given by

\[ x(i + 1) = Ax(i) + Bu(i) \]  \hspace{1cm} (1)

\[ y(i) = Cx(i) + Du(i) \]  \hspace{1cm} (2)

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \), and \( y \in \mathbb{R}^p \) refer to the system state, input and output vectors, respectively, \((A,B,C,D)\) are the model matrices, and \( i \) is a time step index. From the state-space model, a sequence of input vectors \( u(i) \), along with the initial state vector \( x(0) \), give rise to a sequence of output vectors \( y(i) \). A sequence of \((q \times m)\) matrices, \( h(i) \), referred to here as the impulse response functions, or Markov parameters, are defined such that

\[ h(i) = CA^iB \hspace{1cm} i = 1,2,\ldots \]  \hspace{1cm} (3)

For \( i = 1 \) the \((j,k)\) element of \( h(1) + D \), and for \( i = 2,3,\ldots \) the \((j,k)\) element of \( h(i) \) represents exactly the discrete time response for output \( j \) due to a unit impulse for input \( k \) with all other inputs and the initial state vector equal to zeros, i.e., \( u_0(0) = 1 \), \( u_0(i) = 0 \) \( \forall i \neq 0 \), \( u_k(0) = 0 \forall k \neq l \), and \( x(0) = [0] \), where \( u_k(0) \) denotes the \( k \)th element of the input vector at time step 0.

The time-domain algorithm for computing impulse response functions, i.e., Markov parameters, was developed by Phan et al.\cite{4} Because it identifies an autoregressive-moving-average (ARMA) model based on an observer of the system with assigned stable eigenvalues from which the system Markov parameters are recovered, it is appropriate for both stable and unstable systems. Also, the formulation allows for the estimation of IRF's using short blocks of general input-output data, making it easily adaptable for the closed loop identification problem. In this paper, a slightly different presentation of the observer identification algorithm is given, which was extended to handle the case of acceleration measurements (i.e., a \( D \) term was added to the state space model output equation). The main elements of the algorithm are as follows:

1. A linear system is constructed from filtered inputs and outputs based on an observer model of the system having assigned eigenvalues.
2. Observer parameters are solved by a batch singular value decomposition (SVD) pseudo-inverse approach or a recursive estimation approach.
3. The observer system Markov parameters are computed from the observer parameters and assigned eigenvalues.
4. The system Markov parameters are computed from the observer system Markov parameters.

2.1.1 Finite Impulse Response Description

The final system of linear equations describes an ARMA model that is derived using an observer description of the system in (1) and (2). Before describing the observer identification, the finite impulse response model will be developed which connects the impulse response functions to the input-output data. The solution to the state Eq. (1) is given by

\[ x(i) = A^i x(0) + \sum_{\tau=0}^{i-1} A^{i-\tau-1}Bu(\tau) \]  \hspace{1cm} (4)

The output response is obtained by premultiplying the state vector by \( C \) and adding \( Du(i) \)

\[ y(i) = CA^i x(0) + \sum_{\tau=0}^{i} CA^{i-\tau-1}Bu(\tau) + Du(i) \]  \hspace{1cm} (5)

The first term in (5) reflects the contribution of the initial state vector, \( x(0) \), to the outputs, while the second term, which is a sum of outputs weighted by the Markov parameters, and third term represent forcing terms. In general, the initial state vector, the Markov parameters, and the matrix \( D \) are unknowns, whereas the input and output time histories are known. In order to limit the number of Markov parameters appearing in (5) to a finite number, \( p \), and to neglect the contribution of the initial state vector, the number \( p \) is chosen such that \( A^p \approx [0] \). In the sequel, \( p \) represents the number of Markov parameters desired for estimation.

By introducing the approximation \( A^p \approx [0] \) into (5), an approximate equation for the outputs is obtained that
is valid $\forall i > p$

$$\begin{align*}
y(i) & \simeq \sum_{r=0}^{p} C A^{i-r-1} B u(r) + D u(i) \\
& = \begin{bmatrix} h(p) & h(p-1) & \cdots & h(1) & D \end{bmatrix} \times \begin{bmatrix} u(i-p) \\
u(i-p+1) \\
\vdots \\
u(i-2) \\
u(i-1) \\
u(i) \end{bmatrix}
\end{align*}$$

Eq. (6) represents a moving-average (MA) model of the system, i.e., the outputs are a linear combination of the prior inputs weighted by the first $p$ Markov parameters and $D$. A solution procedure may be developed based on (6) which estimates the Markov parameters and the matrix $D$. This involves solving an over-determined system of equations obtained by writing (6) for a large number, $k$, of output vectors (where $k \geq m \times (p+1)$), and solving the matrix equation, $Y^T = X^T U^T$.

$$\begin{bmatrix} y(p+1) & y(p+2) & \cdots & y(p+k) \\
h(p) & h(p-1) & \cdots & h(1) & D \end{bmatrix} \times \begin{bmatrix} u(1) & u(2) & \cdots & u(k) \\
u(2) & u(3) & \cdots & u(k+1) \\
\vdots & \vdots & \ddots & \vdots \\
u(p) & u(p+1) & \cdots & u(k+p-1) \\
u(p+1) & u(p+2) & \cdots & u(p+k) \end{bmatrix}$$

where $Y^T$ is a matrix of shifted output vectors, $X^T$ is the unknown matrix containing the first $p$ Markov parameters and the matrix $D$, and $U^T$ is a generalized Hankel matrix made up of the input vectors. Eq. (7) can be solved either by a batch method (e.g., SVD pseudo-inverse) or by recursive estimation methods (e.g., recursive least-squares). The formulation outlined above has several drawbacks, the major one being the requirement not only that the system to be identified be stable, but further that $A^p \simeq 0$. In general, this is a difficult assumption to make since the system matrix $A$ is unknown and, even if stable, may have eigenvalues with magnitudes nearly equal to $1$. In order to have complete freedom to assign the eigenvalues of the system matrix to be identified so that the assumption $A^p \simeq 0$ can be valid for both stable and unstable systems, an ARMA model of an observer of the system is identified, from which observer and system Markov parameters may be computed.

### 2.1.2 Observer Identification

A Luenberger observer of the system in (1) and (2) has the form

$$\dot{\hat{x}}(i+1) = (A + MC)\hat{x}(i) + [(B + MD) \mid - M] \begin{bmatrix} u(i) \\
y(i) \end{bmatrix}$$

where $M$ is a filter gain matrix. If the system in (1) and (2) is completely observable, then $M$ may be chosen so as to place the eigenvalues of the observer system matrix

$$\tilde{A} = (A + MC)$$

in any desired configuration of complex-conjugate pairs. Thus, the eigenvalues of $\tilde{A}$ may be assigned so that $A^p \simeq 0$. A solution procedure analogous to the one developed for the asymptotically stable system may be developed for the asymptotically stable observer system, where the $A$, $B$, and $u(i)$ are replaced by $\tilde{A}$, $\tilde{B}$, and $v(i)$. The output response for the observer system corresponding to (5) is

$$\hat{y}(i) \simeq \sum_{r=0}^{p} C \tilde{A}^{i-r-1} \tilde{B} v(r) + D u(i)$$

Eq. (9) represents an ARMA model of the original system, in that the outputs are a linear combination of the prior inputs and outputs weighted by the first $p$ observer Markov parameters and $D$. In (9), the true output $y(i)$ is used in place of the estimated output $\hat{y}(i)$, which essentially assumes that the asymptotically stable observer has reached steady-state. Analogous to the system Markov parameters, $h(i)$, the observer Markov parameters are a sequence of $(q \times \lfloor m + q \rfloor)$ matrices, $\tilde{h}(i)$, such that

$$\tilde{h}(i) = C \tilde{A}^{i-1} \tilde{B}$$

The $\tilde{h}(i)$ are only partially unknown, since the eigenvalues of $\tilde{A}$ are assigned to be stable. Development of the eigenvalue assignment procedure involves incorporating the knowledge of the stable eigenvalues of $\tilde{A}$ into the ARMA model described by (9). As before, the true impulse response for the observer system for $i = 1$ is obtained by adding $[D \mid 0]$ to $h(1)$.

For a set of desired eigenvalues of $\tilde{A}$, there exists a similarity transformation such that

$$\tilde{A} = T^{-1} \Lambda T$$

where $\Lambda$ is a diagonal matrix made up of the $n$ assigned eigenvalues, $\lambda_1, \lambda_2, \ldots, \lambda_n$, which are selected in advance. Then the observer Markov parameters become

$$C \tilde{A}^k \tilde{B} - CT^{-1} \Lambda^k T \tilde{B} - C^* \Lambda^k B^*$$

where

$$C^* = CT^{-1} \text{ and } B^* = T \tilde{B}$$

If the elements of $C^*$ and $B^*$ are written explicitly as

$$C^* = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\
c_{21} & c_{22} & \cdots & c_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

$$B^* = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\
b_{21} & b_{22} & \cdots & b_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

1440
then the $l$th row of the matrix product in (11), denoted by $\hat{b}_l^T$, is equal to
\[
\hat{b}_l^T = \left[ \begin{array}{c}
\sum_{i=1}^{n} \lambda_i^l c_i^T v_i, \sum_{i=1}^{n} \lambda_i^l c_i^T b_i, \ldots, \sum_{i=1}^{n} \lambda_i^l c_i^T m_i \\vdots \\vdots \\vdots \\
\sum_{i=1}^{n} \lambda_i^l c_i^T m_i, \sum_{i=1}^{n} \lambda_i^l c_i^T m_i, \ldots, \sum_{i=1}^{n} \lambda_i^l c_i^T m_i
\end{array} \right]
\]
(12)

In order to separate the known assigned eigenvalues from the unknown parameters, the parameter vectors $\alpha_i$ and $\beta_i$ are defined as
\[
\alpha_i = \left[ \begin{array}{c}
c_i^T b_{1r}^*, c_i^T b_{2r}^*, \ldots, c_i^T b_{nr}^*
\end{array} \right]^{T}
\]
\[
\beta_i = \left[ \begin{array}{c}
-c_i^T m_{1s}^*, -c_i^T m_{2s}^*, \ldots, -c_i^T m_{ns}^*
\end{array} \right]^{T}
\]
for $i = 1, 2, \ldots, q$ and $r = 1, 2, \ldots, m$.

Then (12) can be written as
\[
\hat{b}_l^T = \left[ \begin{array}{cccc}
\alpha_{i1}^{T} \lambda_1^l & \alpha_{i2}^{T} \lambda_2^l & \cdots & \alpha_{iM}^{T} \lambda_M^l \\
\beta_{i1}^{T} \lambda_1^l & \beta_{i2}^{T} \lambda_2^l & \cdots & \beta_{iN}^{T} \lambda_N^l
\end{array} \right]
\]  
(13)

and (9) can be expressed as
\[
y(i) = \left[ \begin{array}{c}
\phi_1(i) \\
\phi_2(i) \\
\vdots \\
\psi_1(i) \\
\psi_2(i) \\
\vdots \\
u_1(i) \\
u_2(i) \\
\vdots \\
\psi_q(i)
\end{array} \right]
\times \left[ \begin{array}{c}
\alpha_{i1}^{T} \lambda_1^l \\
\alpha_{i2}^{T} \lambda_2^l \\
\alpha_{iM}^{T} \lambda_M^l \\
\beta_{i1}^{T} \lambda_1^l \\
\beta_{i2}^{T} \lambda_2^l \\
\beta_{iN}^{T} \lambda_N^l \\
\vdots \\
\vdots \\
\psi_1(i) \\
\psi_2(i) \\
\psi_q(i) \\
u_1(i) \\
u_2(i) \\
\psi_1(i) \\
\psi_2(i) \\
\psi_q(i)
\end{array} \right] 
\]  
(14)

where
\[
\phi_s(i) = \sum_{r=1}^{i-1} \lambda_i^{r-1} u_r(i)
\]  
(15)
\[
\psi_s(i) = \sum_{r=1}^{i-1} \lambda_i^{r-1} y_r(i)
\]  
(16)

Eq. (14), along with (15) and (16), form the basis for the observer identification. Given a set of input-output data, a value for the order is chosen and the eigenvalues for the observer system are assigned to satisfy $|\lambda|_{\text{max}} \approx 0$, where $p$ is the number Markov parameters desired for estimation. In this work, a batch solution procedure is carried out using an SVD pseudo-inverse solution to a set of over-determined equations obtained by writing (14) for a large number, $k$, of output vectors (where $k \geq n + m + q + m$), and solving the matrix equation,
\[
Y^T = X^T \Gamma^T, \quad X
\]

\[
\begin{bmatrix}
y(p+1) \\
y(p+2) \\
\vdots \\
y(p+k)
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_{i1}^{T} \\
\alpha_{iM}^{T} \\
\beta_{i1}^{T} \\
\beta_{iN}^{T} \\
\vdots \\
\psi_1(i) \\
\psi_2(i) \\
\psi_q(i) \\
u_1(i) \\
u_2(i) \\
\psi_1(i) \\
\psi_2(i) \\
\psi_q(i)
\end{bmatrix}
\]  
(17)

2.1.3 Markov Parameters

Once the $\alpha_i$ and $\beta_i$ observer parameter vectors have been determined, the observer Markov parameters are found using (13). The system Markov parameters may be expressed as a convolution product of past values of system and observer Markov parameters,
\[
h(k) = \hat{h}^{(1)}(k) + \tilde{h}^{(2)}(k)D + \sum_{i=1}^{k-1} \tilde{h}^{(2)}(i) h(k-i)
\]
(18)

where $\hat{h}^{(1)}$ and $\tilde{h}^{(2)}$ are $(q \times m)$ and $(q \times q)$ partitions of $\hat{h}$, respectively
\[
\tilde{h}(k) = \left[ \begin{array}{c}
\hat{h}^{(1)}(k) \\
\hat{h}^{(2)}(k)
\end{array} \right]
\]

As shown in the sequel, the identifiability of the system is intimately tied to the rank of the $\Gamma$ matrix containing the $\phi$'s and $\psi$'s, and the effect of doing identification under closed loop control is that rows of $\Gamma$ become linearly dependent, as the controls are dependent on the measurements. By adding low-level pure random signals to the control signals, or increasing the order of the observer, it is possible to increase the rank of this matrix and to confidently estimate impulse response functions of the open loop system while the controller is in operation.
3 Examples

3.1 Analytical Model Data

A 2-mode, 2-input-2-output model of the AGS/Rate Gyro system was constructed based on an identified model of the 13 meter ACES ground test structure located at the NASA Marshall Space Flight Center in Huntsville, AL. The system consists of 2 torque motors colocated with 2 rate gyros which measure angular velocities about the global x and y axes. Data were generated using the model matrices for a total of 3 identification experiments, 1 open loop and 2 closed loop. The model matrices corresponding to a sample rate of 10 Hz and having modes at 0.162 and 0.140 Hz are

\[
A = \begin{bmatrix}
0.9406e+01 & 0.9904e+02 & 0.0 & 0.0 \\
0.0 & 0.9614e+01 & 0.0 & 0.9453e+01 \\
0.0 & 0.7857e+01 & 0.0 & 0.9742e+01 \\
-1.0541e+01 & 0.0 & 0.0 & 0.0
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.3407e+02 & 0.9173e+01 \\
0.5922e+01 & 0.2436e+03 \\
0.7603e+01 & 0.1768e+02 \\
0.1839e+02 & 0.3026e+02
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0.0 & 0.0 & 7.0260e+00 & 1.1900e+01 \\
0.0 & 0.0 & 1.1900e+01 & 4.9900e+00
\end{bmatrix}
\]

For each experiment, 400 data points were used, the correct system order was chosen for the order of the observer (i.e., \(n = 4\)), the assigned observer eigenvalues were the same in each case, \(\lambda' = [0.65 \ -0.65 \ 0.75 \ -0.75]^T\), and 100 Markov parameters were estimated (i.e., \(p = 100\)).

Open Loop Identification

The system was driven by pure random inputs having a normal distribution with zero mean and variance of 1.0. The input and output time histories corresponding to the x-axis are shown in Fig. 2, and the singular values of the \(\Gamma\) matrix are shown as the o's in Fig. 3. Using subset selection analysis,[5] it was found that all 8 (= \(n \times m\)) of the \(\alpha\) parameters were significant, whereas only 4 (= \(n\)) of the 8 (= \(n \times q\)) \(\beta\) parameters were determined to be significant. The rank deficiency of the observer identification data matrix implies a redundancy among the \(\alpha\) and \(\beta\) parameters that comprise the ARMA model. The subset selection results for analytical data indicate that the insignificant \(\beta\) unknowns may be ignored in the model, i.e., set equal to zero, without adding significantly to the size of the residual from estimation of the ARMA model parameters. Since the \(\beta\) parameters involve products of \(a_{-c}\) and \(m_{-p}\) (elements of transformed output and filter matrices, respectively) and only \(n\) observer eigenvalues are assigned, this leaves \([n - 1] \times q\) \(\beta\) parameters unconstrained. Finally, the 100 Markov parameters estimated using the observer identification are plotted as the solid curve versus the actual Markov parameters which are plotted as the dashed curve, corresponding to output 1 and input 1. As shown by this representative result, the Markov parameter estimates are nearly perfect.
Closed Loop Identification

An ideal digital output feedback controller (with no time delays) was implemented for vibration suppression:

\[ u(k) = \begin{bmatrix} -1.56 & 0 \\ 0 & -4.11 \end{bmatrix} y(k) \]

The open versus closed loop response for output 1 is shown for a free decay, \( x(0) = [1 \ -1 \ 1 -1]^T \), in Fig. 5, where no noise was added to the control signals for the closed loop case and significant damping was achieved. The observer identification was carried out using the closed loop control torque commands and well-damped rate gyro responses. The singular value plot for this case is shown as ±4's in Fig. 3, where there are only 4 significant singular values. The effect of using closed loop data for the identification is that the \( \Gamma \) matrix becomes even further rank deficient than for the open loop case. Although the data do contain enough information that the identified ARMA model has satisfactory predictive accuracy, there is not enough information to accurately identify the Markov parameters. By adding low level pure random signals to the control signals, i.e., \( \epsilon = u + 10^{-6} \epsilon u \), where \( u \) is the pure random input used in the open loop identification experiment, it is possible to increase the rank of \( \Gamma \) to the same value as for the open loop case. The singular values for this case are shown as ±4's in Fig. 3, and the corresponding response for output 1 is identical to that shown as the dashed line in Fig. 5. Thus, the addition of a small amount of noise to the control signals has little effect on vibration control performance, yet the identified Markov parameters shown in Fig. 6 are identified nearly perfectly.

Because the example considered above is noise-free, it can be expected that larger amounts of noise must be added to make a real system containing noise identifiable under closed loop control. The next example considers closed loop identification using experimental data for a 5 meter truss located in the University of Cincinnati Structural Dynamics Research Laboratory.

3.2 Experimental Truss Data

Fig. 7 shows the experimental truss article which is suspended vertically in the UC-SDRL high-bay laboratory, where the \( z \)-axis points vertically from the ground. Each bay of the truss is a 0.5 m cube, and the total length is 4.6 m. The truss is instrumented with accelerometer sensors and proof mass actuators which accept voltage commands proportional to force and have a bandwidth extending from 2 Hz to well above 50 Hz. The system under consideration consists of the voltage commands to the 2 proof mass actuators denoted as e204yn and e205yn in Fig. 7, and 2 accelerometers that are attached to the base of each proof-mass actuator, which are called r104yn and r105yn. A typical driving point FRF measurement is shown in Fig. 8 for the sensor/actuator pair r104yn/e204yn. A series of 4 identification experiments were carried out, 1 open loop and 3 closed loop, each using 500 data points sampled at 128 Hz.

Open Loop Identification

The system was driven by a set of two continuous random noise sources having peak voltages of 1.94 and zero means. Input and output time histories for e204yn and
The value for the observer order was chosen as \( n = 20 \), and the observer eigenvalues were chosen as positive/negative pairs of numbers in the interval \([0.6, 0.7]\). The identified impulse response function corresponding to output \( r_{104y} \) and input \( e_{204yn} \) is shown as the solid curve in Fig. 10, and the impulse response function computed as the FFT of the frequency response function (using only 2 blocks of 1024 points each) is shown as the dashed curve. Because there was no identified modal model of the truss at the time of this writing, the similarity of impulse response functions—one set computed in the time domain and the other in the frequency domain—inspired confidence that the correct impulse response functions had been obtained. The singular values of the matrix \( \Gamma \) are shown as the \( o \)'s in Fig. 11, where there appear no clear breaks in the singular values. It should be noted that the observer order was chosen iteratively on the basis that increasing the order beyond 20 did not make great improvements in the predictive accuracy of the ARMA model (i.e., the norm of the error between the actual outputs and those predicted by the ARMA model leveled off at an order of 20). Also, a starting point for the observer order was obtained by counting different peak frequency values on the measured frequency response functions.
Closed Loop Identification

A single analog vibration suppression controller was designed and implemented for the sensor/actuator pair r104y/e204yn:

\[ e_{204yn} = \frac{-158}{(s + 2.51)^2} r_{104y} \]

where \( r_{104y} \) and \( e_{204yn} \) are in units of Volts. The controller consists of a high-pass filter with a break frequency of 0.4 Hz which attenuates components of the measured acceleration due to rigid body modes, an integrator that operates on frequencies above 0.4 Hz to give a local velocity estimate, and finally a negative gain factor which commands the proof-mass actuator to apply a force that opposes the estimated velocity. This local velocity feedback (LVFB) controller was effective in increasing the damping of the open loop system, as shown in Fig. 12. The first half of this plot shows the open loop response of \( r_{104y} \) (in g's) due to simultaneous pulses of 2 V amplitude and 46.9 ms duration, applied so that the proof-mass actuators \( e_{204yn} \) and \( e_{205y} \) yielded forces in the same direction, thereby exciting the longitudinal bending modes. The second half of the plot shows the acceleration response \( r_{104y} \) with the LVFB controller operating and the same disturbances acting.

For the closed loop identification, as for the open loop case, continuous random noise with 1.94 Volts peak and zero mean was used to drive the proof-mass actuator \( e_{205y} \); however, the total voltage command to proof-mass actuator \( e_{204yn} \) was the sum of both the control signal and a zero-mean noise with an adjustable peak voltage. Using no additive noise and an observer order of \( n = 20 \), the identified ARMA model was found to be reasonably accurate; however, the resulting open loop system Markov parameters were unstable. The singular values for the matrix \( \Phi \) for this case are shown as the x's in Fig. 11. The next two cases used an additive noise having 1.94 Volts peak (same level as for \( e_{205y} \)). Using an observer order of 20, the estimates of Markov parameters were closer to the true values, yet the plot of the singular values for this case which are shown as the +'s in Fig. 11 indicated the need for a higher value of the observer order. Finally, an observer order of 50 was selected, and the observer eigenvalues were chosen as positive and negative pairs of numbers in the interval \([0.5, 0.7] \). The first 100 singular values are shown as the *'s in Fig. 11. Fig. 13 shows the response reconstructed using the identified ARMA model as a solid curve overlaid on the actual response for accelerometer \( r_{105yn} \) which is the dashed curve. The estimated Markov parameters for output \( r_{104y} \) due to input \( e_{204yn} \) are shown in Fig. 14 as the solid curve, overlaid on those estimated in the open loop case (dashed curve in Fig. 14 same as solid curve in Fig. 10).

Both the open and the closed loop identified Markov parameters were used as input to the Eigensystem Realization Algorithm (ERA), which generated \((A,B,C,D)\) state-space models (40th order for the open loop IRF’s and 52nd order for the closed loop IRF’s) that synthesized the Markov parameters perfectly. An eigenvalue analysis of the discrete system matrices, the \( A \)'s, showed that approximately 13 modes were identified that appeared in the measured frequency response functions. As for the remaining system eigenvalues, many appeared to be poles that were assigned in the observer identification procedure. This effect of residual observer dynamics appearing in the identified open loop system Markov parameters was not observed in the analytical model case, probably because it did not include sensor noise. From the experimental data, it's apparent that an analysis should be made to determine how the measurement noise plays through the time-domain observer identification algorithm. For the case using experimental data, it was found that not only did the level of the additive noise have to be increased to make the system identifiable, but also, the observer order had to be increased. Nevertheless, accurate Markov parameters were found in both the analytical and experimental cases with vibration suppression controllers in operation.
4 Concluding Remarks

Analytical work will continue to determine the nature of the residual observer dynamics that appear in the identified Markov parameters. Also, more analysis will be done to prescribe identifiability conditions that the input signals must satisfy in order that accurate Markov parameters can be estimated using observer identification, without severely degrading controller performance. Finally, on the experimental side, work will continue to implement digital controllers based on the Markov parameters from the observer identification. Once the system identification and controller design have been integrated, a working version of the Reference Model Adaptation scheme may be implemented.

Acknowledgements

The authors wish to acknowledge Dr. Stuart Shelley of the University of Cincinnati for the arduous task of piecing together the UC-SDRL 5 meter truss facility, and for his cooperative efforts in obtaining the experimental data included in this paper. They also wish to thank the University of Cincinnati Structural Dynamics Research Laboratory for allowing generous use of the laboratory's facilities and equipment in carrying out the experimental component of this work.

References