Digital Image Correlation Analyses Using Moving Least Squares Approximation

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Introduction
Moving least squares approximation is one kind of meshless methods in which the approximation is built from nodes only. Comparing to mesh-based methods, meshless methods have advantages on (1) refining is simpler to incorporate, (2) moving discontinuities can be treated easily, (3) large deformation can be handled robustly, (4) higher-order continuous shape functions, (5) non-local interpolation character and (6) no mesh alignment sensitivity[1]. Moving least squares approximation has been used for constructing smooth approximations to fit a specific cloud of points in many surface generation applications. One of the most famous applications is probably the meshless or element-free Galerkin methods[2].

In this talk, a recent developed method of one-dimensional digital image correlation (DIC) analyses using moving least squares (MLS) approximation will be discussed. Specifically, MLS approximation has been applied to (1) subpixel interpolation of image grayscales; (2) deformation of a subset in a digital image correlation analysis. The new analyses algorithms are implemented in a MATLAB program. The results of MLS approximation are compared against other conventional DIC methods, such as linearized DIC and finite element DIC, in image subpixel interpolation and subset deformation mapping.

Subpixel Interpolation by Moving Least Squares
In this subpixel interpolation method, the moving least squares approximant of subpixel interpolation grayscale \( g^h(X) \) on a specific point \( X \) can be defined as

\[
g^h(X) = \Phi^T(X)\hat{g}
\]

(1)

where \( \Phi^T(X) = [\phi_1(X),\phi_2(X),\ldots,\phi_N(X)] \) are the shape functions, and \( N \) is the number of randomly located nodes, \( \hat{g}^T = [\hat{g}_1,\hat{g}_2,\ldots,\hat{g}_N] \) are the grayscale values on nodes. Here it should be noted that \( \hat{g} \) in Eq. (1) are treated as the fictitious nodal values, and are not the nodal values of the unknown trial function \( g^h(X) \) in general.

MLS subpixel interpolation is a useful additional choice for the conventional DIC analyses. It is a flexible and adaptable solution of subpixel interpolation which can be widely applied without losing accuracy.

Subset Deformation Mapping Approximation by Moving Least Squares
In the subset deformation mapping approximation by moving least squares, the moving least squares approximant of subset displacement \( u^h(X) \) on a specific point \( X \) can be defined as

\[
u^h(X) = \Phi^T(X)\tilde{u}
\]

(2)

where \( \tilde{u}^T = [\tilde{u}_1,\tilde{u}_2,\ldots,\tilde{u}_N] \) are the displacement values on the fictitious nodes. It should be noted that here \( \tilde{u}_I \), \( I = 1,2,\ldots,N \) are unknowns. Consequently \( \tilde{u} \) have to be solved by linearized digital image correlation algorithm before Eq. (2) is applied.

Considering errors from various sources may be generated during approximation process, Eq. (2) can be written in the following form

\[
u^h(X) + \Delta u^h(X) = \Phi^T(X)(\tilde{u} + \Delta \tilde{u})
\]

(3)

The error of the approximation \( \Delta u^h(X) \) can be evaluated by comparing to the exact solution.
An Example

In this example, numerical generated image pairs are used to evaluate error levels of moving least squares DIC analyses and conventional nonlinear, linearized, and finite element DIC analyses.

For MLS DIC analysis, fictitious nodes are generated uniformly with an interval between each other along x-direction. The monomial basis is linear ($m = 2$) and three nodes are picked for local processing in each domain of influence ($N = 3$). Therefore the domain of influence is defined by the nodal interval $\Delta N$. The nodal interval can be treated as a critical parameter to compare to the subset size $n$ in nonlinear and linearized DIC analyses and the element size $n$ in finite element DIC analysis. The root mean square (RMS) of the differences between the approximations and the exact solutions is used as an indicator of the error level. The comparison results are plotted in Figure 1.

Because MLS analysis is flexible and adaptable on setting up parameters, it is a critical problem for research that how each parameter influences results. Figure 2 shows the RMS of the differences between the approximations and the exact solutions versus the inverse of the square root of the nodal intervals by changing the following parameters, the order of monomial basis ($m – 1$), the number of nodes $N$, and the radius of supports of nodes $r_j$.

Conclusions

By using the element-free MLS approximation, digital image correlation analyses can be more flexible and more widely applied without losing accuracy and reliability compared to other conventional DIC algorithms. These results can be generalized to both two-dimensional and three-dimensional image correlation analyses.

More comparison results between the MLS DIC approximation and other conventional DIC approximation will be researched in order to utilize the advantages of the MLS approximation and to optimize MLS approximation processings.

References


![Figure 1. RMS of approximation errors versus subset sizes in nonlinear and linearized DIC, element sizes in finite element DIC, and nodal intervals in MLS DIC analyses](image1)

![Figure 2. RMS of approximation errors versus nodal intervals in MLS DIC analyses with various parameters](image2)