Superimposed Fringe Projections for 3D Shape Acquisition by Image Analysis

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ABSTRACT
The aim of this work is the development of an image analysis technique for 3D shape acquisition, based on luminous fringes projections. More in detail the method is based on the simultaneous use of several projectors, which is desirable whenever the surface under inspection has a complex geometry, with undercuts or shadow areas. In these cases, the usual fringe projection technique needs to perform several acquisitions moving each time the projector or using several projectors alternately. Besides the procedure of fringes projection and phase calculation, an unwrap algorithm has been developed in order to obtain continuous phase maps. These are used in the subsequent stereoscopic calculation. With the technique of simultaneous projections, oriented in such a way to cover all the surface, it is possible to increase considerably the velocity of the acquisition process without significant loss of accuracy.

1. INTRODUCTION
In today industry there is an increasing need of shape acquisition instruments to be used mainly for quality control or reverse engineering applications. The adoption of optical full-field methods for 3D shape acquisition can significantly reduce the time of acquisition with good accuracy and high spatial resolution. A classic technique for full-field shape acquisition is based on the projection of luminous fringes on the surface under inspection [1-5]. The fringe pattern is shifted several times while one or more cameras grab images of the illuminated surface. In this way each point is invested by an oscillating luminous signal; extracting the phase of this signal it is possible to obtain phase maps of the surface. Points having the same phase values in the maps of different cameras correspond to the same real point of the surface; knowing their image coordinates (pixel), it is so possible to compute their 3D world coordinates by means of stereoscopic algorithms. Such a technique has been developed by authors [2] using a LCD projector and a number from 2 to 4 cameras. Results are good in terms of precision, size of acquisition area and elaboration time. On the other hand problems may arise when the complexity of the surface to be acquired is excessive, with presence of vertical faces and/or undercuts. In these cases, using only one source of light, it is necessary to move it so many times as one need to cover all the surface. Obviously, under these circumstances, acquisition time increases considerably. Other techniques with only one projector consist in the utilization of several cameras framing all the surface with multiple moving of the projector or in the utilization of only one camera and one projector with moving of the object. In all these cases anyway acquisition time increases and the several clouds of points obtained at each step need to be matched in a single one, increasing also elaboration time.

The present work suggests an extension of previous technique: three projectors are simultaneously activated in such a way to cover in one shot as much acquisition area as possible. In this way particular kind of luminous patterns are created with superimposition of fringes with different orientations and periods. Using the proposed system it is possible to cover with the projection all the surface in one shot, without any moving of the source of light or of the test object and reducing sensibly acquisition time.

Figure 1 shows the layout of the system with three projectors and three cameras for images acquisition. Details of projection technique are given in section 2. The drawback is that the signal grabbed by camera sensors is the result of the superimposition of three simple periodic signals and a further data elaboration is required to separate the contribute of each one of them. The separation algorithm is fully described in section 3; its output is a series of wrapped phase maps with values from 0 to $2\pi$. 
The following step is the phase unwrapping, required to obtain continuous phase maps without steps and ambiguities between phase values and pixels of the frames. Several technique are available in literature [5-8]. One of the most common is the gray code technique [2], based on multiple projections of pattern of rectangular fringes doubling each time their period. In this way, beside a phase value from 0 to $2\pi$, a binary sequence is associated to each pixel. Converting this sequence in a decimal number it is possible to obtain the absolute number of fringe n. Problems arise with this technique when different projection superimpose making the binary sequence decoding more complex. Moreover the intent of authors was also to avoid the phase of binary projection in order to reduce acquisition time. So the unwrap operation has been carried out by an algorithm inspired to the multi-spectral phase unwrapping [9-10]. This technique consists in the acquisition of phase maps with different periods. Values of periods are such as their least common multiple is greater than the maximum dimension of the projection (1024 pixel). The number of image acquisitions is lower than previous technique and obviously the time of the operation decreases. Section 4 explains in detail the unwrapping method. Finally in section 5 an example of application of the system is reported described.

2. MULTI-PROJECTION TECHNIQUE
Before starting to describe the multi-projection it can be useful to speak briefly about the original phase shift technique with one projection [1-2]. A PC generates a fringe pattern in a flexible way: fringes may be horizontal or vertical, with adjustable period, with sinusoidal distribution of grey level (sinusoidal fringes) or with a step distribution (rectangular fringes). A common LCD projector reproduces the pattern on the surface of the object. The result is shown in figure 2a: fringes are distorted because of the morphology of the surface and cameras (at least two as required in stereoscopic calculation, or only one if also the projector has been 3D calibrated [2]) grab images of the distorted pattern. This operation is repeated n times and each time fringes are shifted of an $n^{th}$ part.

Figure 2 – Phase shift technique; (a) Fringe pattern on the surface; (b) Phase map after phase shifting; (c) Phase map after unwrapping
of the period. In that way each pixel is invested during the acquisition by a signal of variable light intensity. By discrete Fourier transform of that signal it is possible calculate the phase for each pixel, obtaining a map of the phase values (figure 2b). This map has values from 0 to $2\pi$, with steps with the same period of the projected fringes. In order to obtain phase maps without discontinuities (figure 2c) the unwrap procedure is required. As mentioned, one of the classic techniques is the gray code: a series of images, made of black and white fringes with doubling period in each frame, are projected on the surface. In this way at the end each pixel can be associated with a univocal binary sequence. To calculate the number of fringe of the pixel it is sufficient to decode this sequence into a decimal number. When for each pixel of coordinates $(u,v)$ the number of fringe $n$ to which it belongs is known, the operation below is sufficient to obtain unwrapped phase:

$$\phi_{UNW}(u,v) = \phi(u,v) + n \cdot 2\pi$$

When two cameras are adopted, the respective phase maps are used to understand which points of the two frames corresponds to the same point of the physical surface. For example if phase maps obtained with both horizontal and vertical fringes are available for different cameras, a point of the test object surface will be associated with two particular values of horizontal and vertical phase and this values are the same in each frame. Using at least two cameras, 3D coordinates may be calculated by stereoscopic algorithm based on 3D calibration of cameras. For camera calibration several methods are available in literature. Among these the Zhang algorithm [11] has been used by authors. An interesting extension of the stereoscopic technique is the triangulation between a camera and the projector. This is possible if the projector is 3D calibrated such as camera, using the same algorithm. If this method requires the use of only one camera, on the other hand problems arise in the complex calibration procedure of LCD projector. Furthermore projectors often exhibit a quite large distortion that needs to be accurately corrected to avoid distortions even in the final acquired shape. At the end there is another direct technique to calculate 3D coordinates. In fact phase value of a point depends directly on the quote $Z$ of that point respect to the reference plane. So, if in a previous step two phase maps are acquired, one on the plane with $Z=0$, the other one on a plane parallel to the previous at a known $Z$, the expression below is valid:

$$Z = \frac{\phi - \phi_0 - Z}{\phi_Z - \phi_0}$$

Once the $Z$ coordinate of a point is known and after 3D calibration of camera, it is easy to calculate also $X$ and $Y$ coordinates. This technique has the great advantage to require the use of only one camera and does not need the calibration of the projector. On the other hand the acquisition of reference and $Z=Z$ planes is critical and requires a good precision.

A summary of several techniques has been done in this section, with some methods of unwrapping and 3D coordinates extraction. Problems with the phase shift with one projector become relevant when test surfaces are complex with shadows areas and undercuts. With only one projection, as is possible to see in figure 2, it is often impossible to cover all the surface by fringes. So the utilization of multiple simultaneous projections has been the natural idea of extension of the technique, trying to speed up the acquisition avoiding the alternately moving of the light source. The use of three projectors ($A, B, C$) in the layout shown in figure 1 creates a particular pattern resulting from superimposition of three simple patterns. Different areas are visible (figure 3), depending on the number of projectors that invest that zone and on the period and orientation of fringes. The acquisition procedure is divided in three steps. In the first step projector $A$ executes a sequence of $n$ projections; the period of the fringes is $P_0$ and they are shifted as described in the technique with one projector. At the same time projectors $B$ and $C$ execute the same sequence with fringes of period respectively $P_1$ and $P_2$. The values of $P_0$, $P_1$, $P_2$ are such as their least common multiple is greater than the maximum dimension of the projection. For example, if the resolution is $1024\times1024$, the condition becomes:

$$\text{l.c.m.}(P_0,P_1,P_2) > 1024$$

In the second step projectors rotate the period of their fringes; for examples projector $A$ generates fringes of period $P_1$, projector $B$ fringes of period $P_2$, projector $C$ fringes of period $P_0$. The sequence of
image projections is the same of step 1. In the last step projectors rotate again the periods and execute the same sequence. The number \( n \) of projections for each step is equal to the maximum period between \( P_0 \), \( P_1 \), \( P_2 \), that is conventionally \( P_2 \). So the total number of projections during the acquisition is \( 3 \cdot P_2 \). At the end each projector will have executed three phase shifts with three different period of fringes and this operation is simultaneous for \( A \), \( B \), \( C \). The only difference between the projectors is the order of rotation of fringe periods. In figure 4 three frames, one for each step, are shown; the values of periods are \( P_0=5 \), \( P_1=11 \), \( P_2=21 \) so that \( n=21 \) and the total number of projections is 63. The utilization of multiple source of light grants a greater coverage of the surface, reaching also vertical surfaces of different orientations and undercuts. Moreover the simultaneity of the process makes the acquisition process fast. Nevertheless a further elaboration is required to separate the contribute of each projector from the superimposition of the three signals. In fact the signal in each pixel is the composition of one, two or three single periodical signals, depending on how many projectors invest that pixel. Next section describes in details an algorithm, based on discrete Fourier transforms to execute the separation of signals.

3. PHASE EXTRACTION PROCEDURE

The starting point of this procedure is the sequence of frames grabbed during acquisition, that can be thought seen as a set of arrays, one for each pixel, containing the light intensity signal of that pixel. More in detail, each array is made of \( 3 \cdot P_2 \) samples of a signal resulting from superimposition of up to three single signals; considering only one of the three steps, samples will be obviously \( P_2 \). To extract phase values of single signals from the total signal a direct application of discrete Fourier transform (DFT) is not sufficient. In the areas of superimposition, it is necessary to investigate how signals with different frequency influence the DFT of total signal. In particular we need to calculate how a signal of period \( P_j \) (\( P_j < P_2 \)) and made of \( P_2 \) samples scatters its content in the spectrum of the DFT. We can consider the discrete total signal \( x_k \), received by a pixel, is the summation of three contribution due projector.

\[
x_k = \sum_{i=0}^{2} x_k^i \quad k = 0, 1, \ldots, P_2 - 1
\]  \hspace{1cm} (4)

where \( k \) is the index of sample and \( i \) is the index of the projector.
The DFT of the total signal will have the form (5), where \( f_q \) is a complex number relative to the contribute of the \( q^{th} \) harmonic of the signal and containing information about amplitude and phase of that harmonic.

\[
f_q = \frac{1}{P_2} \sum_{k=0}^{P_2-1} x_k \cdot e^{-\frac{j2\pi k q}{P_i}} \quad q = 0, 1, ..., P_2 - 1
\]  

(5)

Considering separately the signals \( x_k^i \), they can be expressed as a sinusoidal discrete signal where \( A_i \) is the amplitude and \( \phi_i \) is the phase value:

\[
x_k^i = A_i \cdot \sin \left( \frac{2\pi}{P_i} k + \phi_i \right) = A_i \left[ \sin \left( \frac{2\pi}{P_i} k \right) \cos \left( \frac{2\pi}{P_i} k \right) + \cos \left( \frac{2\pi}{P_i} k \right) \sin \phi_i \right]
\]  

(6)

From equation (6) and imposing

\[
\begin{align*}
a_i &= A_i \cdot \cos \phi_i \\
b_i &= A_i \cdot \sin \phi_i
\end{align*}
\]  

(7)

we obtain:

\[
x_k^i = a_i \cdot \sin \left( \frac{2\pi}{P_i} k \right) + b_i \cdot \cos \left( \frac{2\pi}{P_i} k \right)
\]  

(8)

\[
x_k = \sum_{i=0}^{2} a_i \cdot \sin \left( \frac{2\pi}{P_i} k \right) + b_i \cdot \cos \left( \frac{2\pi}{P_i} k \right)
\]  

(9)

Coefficients \( a_i \) and \( b_i \) can be seen as real and imaginary parts of the complex number \( a_i + j \cdot b_i \) that is the value relative to the first harmonic of the DFT of \( x_k^i \). Combining equations (9) and (5) we obtain

\[
f_q = \frac{1}{P_2} \sum_{i=0}^{2} \left[ a_i \sum_{k=0}^{P_2-1} \sin \left( \frac{2\pi}{P_i} k \right) \cdot e^{-\frac{j2\pi k q}{P_2}} + b_i \sum_{k=0}^{P_2-1} \cos \left( \frac{2\pi}{P_i} k \right) \cdot e^{-\frac{j2\pi k q}{P_2}} \right]
\]  

(10)

where the DFT of the cumulative signal is function of six unknown parameters \( a_i, b_i, i=1,2,3 \). It is possible to see that the two summations in equation (10) are the DFTs of unitary sinusoids or cosinusoids of period \( P_i \) and formed of \( P_2 \) samples. Defining them as in equations (11) it is possible to write equation (12).

\[
\begin{align*}
S_q^i &= \frac{1}{P_2} \sum_{k=0}^{P_2-1} \sin \left( \frac{2\pi}{P_i} k \right) \cdot e^{-\frac{j2\pi k q}{P_2}} \\
C_q^i &= \frac{1}{P_2} \sum_{k=0}^{P_2-1} \cos \left( \frac{2\pi}{P_i} k \right) \cdot e^{-\frac{j2\pi k q}{P_2}}
\end{align*}
\]  

(11)

\[
f_q = \sum_{i=0}^{2} \left( a_i \cdot S_q^i + b_i \cdot C_q^i \right)
\]  

(12)

While coefficients \( a_i, b_i \) are real, coefficients \( S_q^i, C_q^i \) are complex. Writing equation (12) in an expanded form we obtain a system of \( P_2 \) equations with six unknowns:
In (14) we called $f$ the array of the DFT of total signal, $[D]$ the matrix composed by the terms $S_{q,i}$ and $C_{q,i}$, $c$ the array of unknown parameters. Now it is easy to reverse the system and to calculate phase and amplitude of single signals using the equations below:

$$c = [D]^{-1} \cdot f$$

$$A_i = \sqrt{a_i^2 + b_i^2}$$

$$\varphi_i = \tan^{-1}\left(\frac{b_i}{a_i}\right)$$

At the end of this procedure three separate phase maps, one for each projector, are available for each step of acquisition. Considering all the steps of acquisition we will have nine maps, three for each projector with periods $P_0, P_1, P_2$ (figure 5).
4. PHASE UNWRAPPING ALGORITHM

As it is possible to see in figure 5 phase maps have now a step trend with values from 0 to \(2\pi\) inside each fringe. To create a univocal correspondence between pixels and their phase value it is necessary a procedure of unwrapping. The objective is to obtain a map without discontinuities. The technique to which authors inspired is the multi-spectral phase unwrapping [9-10]. It is based on the utilization of several phase maps with different periods as is possible to see in figure 6. In this way each pixel is associated with a sequence of phase values that univocally identifies its position inside the projection. For example, looking to figure 6, pixel A and B have the same value of \(\varphi_2\), but different values of \(\varphi_0\) and \(\varphi_1\).

If periods \(P_0\), \(P_1\), \(P_2\) are such as their least common multiple is greater than the maximum projection dimension, the sequence of phase values becomes univocal. Considering a pixel and its generic phase value \(\varphi_i\), the relative unwrapped value \(\varphi_i^u\) depends on the absolute number of fringe \(n_i\) to which the pixel belongs:

\[
\varphi_i^u = \varphi_i + 2\pi \cdot n_i \quad i = 0, 1, 2
\]  

(18)

At the same time another relation exists between unwrapped phase values of different periods:

\[
\varphi_0^u = \varphi_1^u \cdot \frac{P_1}{P_0} \quad \varphi_1^u = \varphi_2^u \cdot \frac{P_2}{P_1} \quad \varphi_2^u = \varphi_2^u \cdot \frac{P_2}{P_0}
\]  

(19)

Combining equations (18) and (19) it is possible to write the system (20) where \(n_0\), \(n_1\), and \(n_2\) are unknown.

\[
\begin{align*}
\varphi_0 + 2\pi \cdot n_0 &= (\varphi_1 + 2\pi \cdot n_1) \frac{P_1}{P_0} \\
\varphi_0 + 2\pi \cdot n_0 &= (\varphi_2 + 2\pi \cdot n_2) \frac{P_2}{P_0} \\
\varphi_1 + 2\pi \cdot n_1 &= (\varphi_2 + 2\pi \cdot n_2) \frac{P_1}{P_0}
\end{align*}
\]  

(20)

This system is not directly solvable because equations are linearly dependent. Further more values of \(n_i\) must be obviously integer. To this purpose, a recursive procedure has been developed where, by writing the first two equations of (20) as
\[
\begin{align*}
n_1 &= \left( \varphi_0 + 2\pi \cdot n_0 \right) \frac{P_0}{P_1} - \varphi_1 \right) \frac{1}{2\pi} \\
n_2 &= \left( \varphi_0 + 2\pi \cdot n_0 \right) \frac{P_0}{P_2} - \varphi_2 \right) \frac{1}{2\pi}
\end{align*}
\] (21)

and by imposing recursively an integer value for \( n_0 \), it is possible to calculate \( n_1 \) and \( n_2 \) until these values are as much as possible near to integers. When iterations are completed and \( n_i \) have been calculated for each pixel it is easy to unwrap the phase maps using equation (18). The final unwrapped phase maps of each projector are shown in figure 7.

<table>
<thead>
<tr>
<th>projector A</th>
<th>projector B</th>
<th>projector C</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Phase map" /></td>
<td><img src="image" alt="Phase map" /></td>
<td><img src="image" alt="Phase map" /></td>
</tr>
</tbody>
</table>

Figure 8 – Phase map after unwrapping of each projector

5. EXAMPLE OF 3D SHAPE ACQUISITION

In this section an example of 3D shape acquisition is given using the method previously proposed. The object under examination is a cylinder.

With the new technique, utilizing three projectors and three cameras disposed with intervals of 120° (figure 1), all surfaces, both vertical and horizontal on the top, can be acquired simultaneously. In figure 9 the superimposed fringe pattern generated by the projectors all over the cylinder is shown. In this application periods of fringes are \( P_0 = 5 \text{ px}, P_1 = 11 \text{ px}, P_2 = 21 \text{ px} \), so that their least common multiple is 1155 px, larger than the maximum dimension of the projection (1024 px). After the procedure of phase extraction the result will be a series of phase maps shown in figure 10 that will be then processed with the previously described unwrap algorithm. After unwrap we have three unwrapped phase maps for each projector, each one obtained with a different period of fringes. To proceed with the elaboration only three maps of the same fringes period would be sufficient. Nevertheless it may be useful to utilize all available maps, in order to mediate as much as possible random errors of the acquired signal.

Fig 9 – Fringe pattern on the surface of a cylinder
To this purpose all the maps have been converted by equation (19) to the common fringes period of 5 px and a mean value of them was considered for further elaborations. In a previous step phase maps were acquired also for a reference plane at \( Z = 0 \) and for a parallel plane at a fixed value \( Z = Z_0 \), so that \( Z \) coordinate for each acquired pixel can be calculated by equation (2). In figure 11a a map of \( Z \) values of the pixels acquired by one camera is shown, while other two maps will be available, each one for the other two cameras. To estimate also \( X \) and \( Y \) coordinates a procedure of 3D camera calibration is required. This operation has been performed using the Zhang algorithm [11]. Utilizing a number \( n \) of cameras (three in our acquisition) the result will be \( n \) point clouds, one for each camera. Joining them we’ll obtain a unique point cloud representing all the surface under examination (figure 11b).

REFERENCES


