ABSTRACT

In the automation field is of significant economical value to accomplish accurate and fast positioning of handling devices. Examples are given by some robot manipulators, disk drive heads, pointing systems in space, etc. Many of them are made of flexible and light structures to manoeuvre as quickly as possible. This paper deals with the kinematic and dynamic control in the point-to-point motion of a deformable system. Prior to this work there are, by the same authors, a number of studies concerning with the problem of reducing the residual (final) vibrations in point-to-point movements. This article presents an important extension of the former works: not only the rest condition is considered as a desirable final position, but any (admissible) kinematic condition may be required. The adopted control technique is open–loop and it is concerned with the pre–shaping of the input law in the form of a limited number of a piece-wise acceleration/force/torque steps. In fact the so called “constant acceleration laws” proved themselves to be suitable for this purpose and showed many advantages over other pre–shaping techniques. This work also provides an extension of the previously mentioned approach in order to add good insensitivity to errors in modelling parameters without involving complex and expensive computations, giving in this way a deeper insight thought robustness. An experimental set with results is also described.

Introduction

Light and flexible mechanical systems are growing in number as they presents many energetic and transportability advantages over heavier systems. However the control of these deformable systems is more difficult and it represents one of the biggest challenge in the control field. When the mechanical components undergo deflection during operations, it may result difficult to track a desired trajectory or to reach a specified point. Furthermore, once the final position has been reached, the residual vibrations may degrade accuracy and may introduce delays in operations. It has to be underlined that, even in very stiff systems, closed–loop control can introduce a sort of flexibility. In all cases and with any kind of control, there must be a command generator which should provide a reference command, which translates the desired movement into the language of the actual controller–actuator.

This paper deals with the kinematic and dynamic control in the point–to–point motion of a deformable system. The control is performed through generating very particular “reference commands” in open–loop–control. This technique is well known in literature as pre–shaping of the input laws. Prior to this work there are, by the same authors, a number of studies concerning with the problem of reducing the residual (final) vibrations in point–to–point movements. Historically, two main approaches have been used to solve this problem: the feed–back loop control and the pre–shaping of the forcing law in open–loop control. Also mixed techniques are present where the pre–shaping of the input law could be required to reduce the complexity of the feed–back controllers and to obtain high performances even in the presence of uncertainties, non–linearity and changing of the inertial parameters.

This article specifically presents some extension of the former works together with robustness analysis. Most important, a series of lab experiments is presented. As mentioned above, the adopted control technique is open–loop and it is concerned with the pre–shaping of the input law in the form of a limited number of a piece-wise acceleration/force/torque steps. In fact the so called “constant acceleration laws” proved themselves to be suitable for this purpose and showed many advantages over other pre–shaping techniques. As a short acknowledgement of similar and preexisting theories we cite [1], [3], [4], [5], [6], [7], [8], [9], [10].
3 Mathematical Theory

The target of this theory is to obtain pre-shaping input laws that eliminate the residual vibration after a flexible device performs a desired movement. To obtain this result, we state that the final (residual) vibrational energy has to be kept to a minimum at the end of the actioning time in order to eliminate, or totally lead a zero, the final vibrations. As mentioned above, we consider piece-wise constant input laws \( F(t) \) (in levels \( F_i, i = 1, \ldots, N \)) over time intervals of the same length, \( \Delta t \), that should be contained in the total actioning time \( t_f \) as shown in figure 1. To attack the problem, let’s consider a simple \( n \) degree of freedom spring–mass system as a useful model for many flexible mechanical devices. For the sake of simplicity we do not consider damping effects in the theoretical development: if damping has a proper structure and real mode exists (that is damping is a linear combination of mass and stiffness of the system) there would be nothing different in theory but a little cumbersome response formula. Hence the equations which describe the motion of the system is:

\[
M\ddot{z} + Kz = GF(t),
\]

where the \( F(t) \) is projected onto the coordinate which identify the control input point by the vector \( G \). After working out the system modal matrix \( X \), we can express the equations of motion in the eigenspace as:

\[
\ddot{\eta}_r(t) + \omega_r^2 \eta_r(t) = \gamma_r F(t) \quad r = 0, \ldots, n.
\]

For \( r = 0 \), the equation (2) represents the rigid body mode. The scalar \( \gamma^T = X^T G \), taking into account how the representation of the input force changes in the eigenspace, allow us to consider the system response in the modal coordinates as a function of the only unknowns input force steps \( F_i \). Therefore, if the system is at rest at the instant \( t = 0 \), we will have for each mode:

\[
\eta_r(t_f) = \sum_{i=1}^{N} \left\{ \frac{\gamma_r}{\omega_r^2} \left[ -\cos(\omega_r(t_f - (i-1)\Delta t)) + \cos(\omega_r(t_f - i\Delta t)) \right] \right\} F_i,
\]

Obviously we can take other initial conditions considering the initial displacement and velocity in the convolution integral. Since, for a linear model, any given state of vibration is a superposing of vibrations each of which is characterised by a single frequency, we can write for the residual energy at the time \( t_f \):

\[
E_{\text{res}}(t_f) = \frac{1}{2} \sum_{r=1}^{n} \left[ \dot{\eta}_r^2(t_f) + \omega_r^2 \eta_r^2(t_f) \right].
\]

Because the system response in equation (3) as well as its first time derivative are linear with respect to the unknowns \( F_i \), the residual energy function in (4) is homogeneous quadratic function in the \( F_i \), \( E_{\text{res}} = \sum_{i,j=1}^{N} h_{ij} F_i F_j = (HF, F) \) and satisfies to the relation: \( (F, \nabla E_{\text{res}}(F)) = 2E_{\text{res}} \). Therefore the search for a minimum of the residual energy (4) with respect to \( F_i \) coincides with the search of the point \((F_1, \ldots, F_N)_0\) which brings that function to a zero. The aim of the control is not only to eliminate the residual vibration but also to track a desired trajectory Many applications, and in
particular the point–to–point motion, require that the system reaches a certain displacement at a certain velocity, therefore two of the most common kinematic constraints may be:

\[
S(F_1, F_2, \ldots, F_N)_{t_f} = H \\
V(F_1, F_2, \ldots, F_N)_{t_f} = K
\]

where \(S\) and \(V\) are respectively the total displacement and the final velocity at the time \(t_f\). In modal coordinates the two constraints are applied to the rigid mode and, thanks to the specific form of the input law adopted (figure 1) we can write equations in (5) as:

\[
\begin{align*}
g_1 &= \sum_{i=1}^{N} \gamma_0 F_i \Delta t - \dot{\eta}(t_f) = 0 \\
g_2 &= S = \sum_{i=1}^{N} \gamma_0 F_i \Delta t^2 + \gamma_0 F_i \Delta t^2(n - i) - \eta(t_f) = 0,
\end{align*}
\]

where \(\eta(t_f)\) and \(\dot{\eta}(t_f)\) are obtained by the transformation \(\eta(t_f) = X^T M z(t_f)\), \(\dot{\eta}(t_f) = X^T M \dot{z}(t_f)\). The above two constraints make explicit the requirements of final displacements and final velocity but other constraints (ex. other kinematics requirements or more particular bounds) can be added as auxiliary conditions in the mathematical formulation.

Our search for the stationary point of the function \(E_{res}\) is constrained; as a general example, we take the kinematics conditions in (6). To reduce the variation problem with auxiliary conditions to a free variation problem we use the Lagrange multipliers method: we build a new function \(\Upsilon = E_{res} + \lambda_1 g_1 + \lambda_2 g_2\) and we search for the stationary point of \(\Upsilon\) with respect to the variables \(F_1, \ldots, F_N, \lambda_1, \lambda_2\). The method let us to write the following \(N + 2\) relations in which the unknowns only appears in the vector \(F\):

\[
\begin{bmatrix}
H & U \\
U^T & 0
\end{bmatrix} \begin{bmatrix} F \lambda \end{bmatrix} = B,
\]

where \(U = u_{ij} = \frac{\partial g_i}{\partial F_j}\), \(F = [F_i, \lambda_j]^T\), \(B = [0, 0, \ldots, \dot{\eta}(t_f), \eta(t_f)]^T\), \(j = 1, 2; \ i = 1, \ldots, N\).

These equations are easily solvable and allows to generate pre–shaped input law that performs point–to–point movements and eliminate residual vibration also when a large number of flexible modes are assumed in the model. No relations are required between the total actioning time and the characteristic times of the system. The only restriction that we have to consider is that the auxiliary/ boundary conditions should be linearly dependent on the \(F_1, \ldots, F_N\) to easily fit into (7). As an example to show the effectiveness of the method we report in figure 2 a pre–shaped law designed to control the end point position of a flexible manipulator model together with the end point position and system response to the input law. Six flexible modes are retained in the model.

![Figure 2: Thirteen steps null velocity law, end–point position (control slew angle 20°), modes responses](image_url)

4 Auxiliary Conditions to add Robustness

In many real situations the system parameters are known within some tolerances, for example wrong estimations of the inertial/stiffness parameters of the system, changing in the system parameters during the operational conditions etc. This
may be a problem for a dynamic control by pre–shaped input laws alone because of its open–loop nature. The method, in fact, requires precise evaluation of the natural frequencies of the system. To overcome this problem sometimes it is possible to apply both open–loop and feed–back loop control, nevertheless, as a first stage, the reference command could be modified to add robustness. The sensitivity of the proposed method towards parameter errors is reduced by adding new auxiliary conditions in (7). These new constraints are obtained setting the first partial derivatives of the modes response and its first time derivative with respect to the correspondent natural frequencies equal to zero. We point out that these constraints fulfill the requirement to be linear with respect to \( F_1, \ldots, F_N \) hence they can be used in (7). Not considering the rigid body mode, we have the following \( 2(n-1) \) auxiliary conditions:

\[
gr_k = \frac{\partial \eta_k}{\partial \omega_k} = 0; \\
g^r_k = \frac{\partial \dot{\eta}_k}{\partial \omega_k} = 0 \quad k = 1, \ldots, n-1,
\]

where \( n \) is the number of degree of freedom of the mechanical model.

Adding the (8) to the procedure described in the previous section, the \( F_1, \ldots, F_N \) obtained should be a stationary point for the residual vibrational energy as a function of \( F_i \) and this energy should also be a flatter curve in a neighbourhood of the estimated natural frequencies when it is considered as function of the natural frequencies themselves. It seems natural to suppose that as the number of steps increase the less sensitive the system becomes to the lack of precise knowledge of the resonant frequencies.

5 Experimental Results.

5.1 Experimental Set–Up

Figure 3 describes the experimental set–up especially built to generate the pre–shaped input law that control some flexible devices in point–to–point motion. The experimental set–up was designed to evaluate the effects of the constant acceleration/force input law on system that can model many real cases of handling mechanical devices. This setup is comprehensive of the input signal generator, data logger, actuator and flexible mechanical system to be driven. As it is shown in figure 4, a double pendulum is sustained by a translating connection which is driven by a long step HIWIN® ball screw with reduced clearances and good efficiency. The driving motor is a Parker Hannifin® , SBC®, SMB-82 model with a nominal torque of 3 Nm and the motor controller is SBC®, SPD–2. There is also a positioning feedback, not used for the control, constituted by a resolver.

The motor controller is driven via a Sensoray® 626, PCI i/o computer card. All the software is written in “c” language and based on the card drivers. The computer used to get this all working is a Linux Box. This chain, i.e. the computer, the i/o card, the motor controller and the brushless motor itself, is capable of generating the “step laws” required from the theory. The system oscillations are measured via a micro laser sensor Matsushita Electric Works, Ltd, LM10.

![Figure 3: Experimental setup.](image-url)
5.2 Experimental Measurements.

The measurements of the vibration at the end of the actioning time are obtained evaluating the residual amplitude by the micro laser sensor. The laser is located 50 mm far from the final position point that the system will have to reach, capturing, in this way the residual vibrations. In evaluating the residual vibration amplitude we do not filter the output signal to eliminate errors due to rotations with respect of the vertical axis errors due to the changes of the measured point during oscillations because the very low order of the errors with respect to the distance measured. The experiments have confirmed that the method proposed is very satisfactorily in eliminating the residual vibrations. The first group of measures are worked out to evaluate the residual vibration reduction of many degree of freedom devices modelled as pendula as shown in figure 4. Many pre–shaped law with different actioning time have been calculated and accomplished to evaluate their effectiveness in reducing the residual vibration. In figure 5, some of the experimental results using unshaped and pre–shaped input law are reported. The figures refers to input laws to control double and triple pendula, the first with resonant modes at 0.43 Hz and 1.76 Hz, the second one with resonant modes at 0.44 Hz, 1.44 Hz, 2.59 Hz. The total displacement performed by the input law in figure 5 is 0.8 m.

As an example of the input law used to obtain the data in figure 5, the pre–shaped law n.1, n.2 and n.9 are shown in figure 6. Other experiments are carried out considering a thin steel beam with an end mass. In modelling the system we make the hypothesis that the cross sectional dimension of the beam is small compared with its length. The beam dimension are 0.05x0.05x1 m and the end mass is 0.03 kg of weight. Figure 7(a) shows a pre-shaped input law computed to perform 0.8 m in $t_f = 1.6$ s for the beam-mass system. The residual vibration amplitude of the tip mass using unshaped law and the pre–shaped are shown in figure 7(b). We can see that the pre–shaped command has demonstrated a marked improvement in eliminating the final location error. Let’s evaluate the robustness of the method by adding the auxiliary
conditions in (8) to the algorithm that design the pre–shaped law. The system performance is assessed with 10% error tolerance in natural frequencies. In figure 8(a) we can see the residual vibration amplitude versus the error in the estimated natural frequencies for a discrete model with resonant frequency at 0.59 Hz. In this figure the residual vibration amplitude obtained by a pre–shaped law with robustness are compared with the one obtained by pre–shaped law without robustness. The figure 8(b) shows the residual vibration amplitude when the system is controlled with and without robustness when the error in the natural frequency is $\omega/\omega_n = 0.95$. Figure 9 show the residual vibration amplitude for the three masses of a three degree of freedom system (triple pendulum) using pre–shaped law with robustness and pre–shaped law without robustness.

The figures reveal that the pre–shaped input laws with robustness are successful in attenuating the residual vibration of the system when parameter errors are present in the model.

6 Concluding Remarks

In this paper a series of experimental results about a technique suitable in reducing the residual vibration in a flexible positioning systems is given. A short description of the theory based on particular laws of motion “constant by steps” has been presented.

Experimental results have been worked out using a quite sophisticated experimental setup. This equipment, described and reported in pictures in the present work, is mainly constituted by a series of pendula, actioned by computer controlled servo-motors. The measures have been carried out via a laser vibrometer.

The reducing of vibrations is very effective Figure 5 shows the strong difference between non pre–shaped and pre–shaped laws; it also shows a substantial equivalence among different pre–shaped laws, which can lead to different times of movement, different speeds and so on. The continuous pendulum deserves special mention: figure 7 shows how a quite simple pre–shaped law almost erases the vibrations, leaving a tiny oscillation probably due to an incorrect estimating of the structural parameters.
Figure 8: One degree of freedom model. (a) Residual vibration amplitude versus the errors in the estimated natural frequencies. (b) Residual response when $\omega/\omega_n = 0.95$. Input law without robustness (pointed line) and input law with robustness (continuous line).

Figure 9: Three degree of freedom model. Residual vibration amplitude versus the errors in the estimated natural frequencies. Input law without robustness (pointed line) and input law with robustness (continuous line).

Robustness has been finally added. Figure 8 well explains how robustness conditions behaves. The curve representing the maximum vibration amplitude versus errors in parameters estimation is significantly flattened (figure 8(a)), and an oscillation plot for a non–correct parameter case is reported, with and without robustness (figure 8(b)). Further development of this work should encompass a wider range of actuators, time varying parameters of the system and possibly with weaker dynamic in order to simulate actual economical introduction of these devices.

References


