A New Generation of Frequency-domain System Identification Methods for the Practicing Mechanical Engineer

P. Verboven, B. Cauberghe, S. Vanlanduit, E. Parloo and P. Guillaume

Vrije Universiteit Brussel (VUB)
Department of Mechanical Engineering
Acoustics and Vibration Research Group (AVRG)
Pleinlaan 2.B-1050 Brussels, Belgium
Peter.Verboven@vub.ac.be - www.avrg.vub.ac.be

ABSTRACT

This paper gives an overview of recently developed frequency-domain system identification methods, which can significantly extend the modelling power of the practicing engineer when starting from experimental data. The goal of this paper is to present recently developed state-of-the-art frequency-domain identification schemes that combine accuracy, computation speed and suitability for an automation of the identification process. Two types of models, a common-denominator transfer function and a state space model, are considered in a deterministic, stochastic and combined identification framework. The practicing engineer’s judgment in distinguishing between so-called physical and mathematical modal parameters is classically done using a stabilization diagram. It is shown that this validation step is significantly improved by the presented identification schemes. The presented techniques are demonstrated for two practical cases related to structural dynamics modelling for applications in the aerospace and automotive industry.

1 INTRODUCTION

Today, Experimental Modal Analysis (EMA) has become a common tool for the practicing engineer to study the dynamical behaviour of mechanical and civil structures. Traditionally, modal analysis starts from experimental data that is acquired under well-controlled laboratory conditions and preprocessed using a nonparametric Frequency Response Function (FRF) estimator such as $H_1$ or $H_v$ [35; 42]. Next, the modal parameters are derived by means of system identification and extensive research efforts have been devoted to the development of methods that are specifically adapted for modal analysis purposes. Although a wide variety of methods has been described during the last decades, only a small number has become well integrated in and commonly-used for modal analysis practice. The Ibrahim Time-Domain (ITD) method [28], the Least Squares Complex Exponential (LSCE) method [3] and the Eigensystem Realization Algorithm (ERA) [30; 31] have become the most important time-domain methods today. Frequency-domain counterparts are the Frequency-domain ERA algorithm [32], the Direct Parameter Identification (FDPI) [34], the Least Squares Frequency-Domain (LSFD) algorithm and the Rational Fraction Polynomial based methods (RFP, OPOL) [15; 44; 47]. The recently developed frequency-domain LSCF and polyreference LSCF algorithms, presented in [5; 20; 49], were introduced as a new modal parameter estimator standard for industrial applications [40].

However, it is important to notice that all of these EMA methods are based on a deterministic identification framework that is purely based on the deterministic relation between the measured input and output. Therefore, the contribution in the structure’s response resulting from any unmeasured source of excitation is considered as undesirable, and is therefore considered as noise disturbing the measurements. The use of so-called consistent Input/Output (I/O) or FRF-based frequency-domain stochastic estimators can avoid model errors (bias) due to this measurement noise by including a model for the measurement noise. In [17; 20; 51; 52; 54; 56], the use of the I/O and FRF-based Maximum Likelihood and Generalized Total Least Squares estimators (GTLS, BTLS, ...) is demonstrated.
for data such as flight flutter data. Obviously, this requires that a model for the noise is a priori known, e.g. through means of nonparametric processing \cite{5,17,57,58}.

More recently developed Operational Modal Analysis (OMA) methods start from output-only measurements. Given the lack of measured inputs, the knowledge of the input signals is replaced by the assumption that the the structure’s responses now results from a stochastic process with unknown white noise as input. Identifying modal parameters from output-only data is known as stochastic system identification and during the last decade, some of the traditional EMA methods, such as the LSCE, have been adapted for OMA \cite{24} while in addition time-domain stochastic subspace methods \cite{25,26} and Maximum Likelihood methods \cite{1,18,23} were extensively elaborated for OMA purposes. In \cite{37-39} OMA was extended in order to obtain scaled mode shapes from output-only measurements by means of a so-called "mass-controlled" experiment.

However, considering a modal test set-up in a more general context, one can always consider the response of a structure under test as the result from both measurable (deterministic part) and unmeasurable (stochastic part) forces. This concept of a combined deterministic-stochastic identification for control and electrical engineering \cite{46}. This OMAX concept was further investigated for modal parameter estimation, which required the optimization of algorithms for large (and sometimes noisy) data sets characterized by a high modal density. Important recent results for modal parameter estimation based on both Rational Fraction Polynomial and State Space models are presented in \cite{5,9,10}.

This paper gives an overview of frequency-domain identification methods for the three different identification frameworks: i.e. deterministic, stochastic and combined starting from both a common-denominator rational fraction polynomial and state space model formulation. The different approaches are compared by means of simulated and experimental data for the modelling of an automotive engine subframe and for in-flight aircraft flutter analysis.

2 FREQUENCY-DOMAIN PARAMETRIC MODELS

In this section, two different parametric frequency-domain models, i.e. a common denominator and state space model are discussed in relation with a modal model formulation. These models will be considered in the next section for structural dynamics modelling starting from frequency-domain data.

A first model to describe the vibration behaviour of a mechanical structure with \(N_m\) degrees of freedom is given in the time-domain by Newton’s equation of motion:

\[
M \ddot{x}(t) + C_1 \dot{x}(t) + Kx(t) = f(t)
\]

with \(M\), \(C_1\) and \(K\) respectively the mass, damping and stiffness matrices, and \(f(t)\) and \(x(t)\) the applied force and structural response vectors. Using the Laplace transform, the frequency-domain equivalent is given as

\[
G(s)X(s) = F(s)
\]

with \(G(s) = Ms^2 + C_1 s + K\) the dynamic stiffness.

A matrix inversion of (2) yields

\[
F(s)H(s) = X(s)
\]

with \(H(s)\) the transfer function matrix. From a generalized eigenvalue problem, the transfer function matrix can be formulated in its modal form \cite{27}

\[
H(s) = \phi [sI_{N_m} - \Lambda]^{-1} L^* + \phi^*[sI_{N_m} - \Lambda^*]^{-1} L^*
\]

where the modal parameters \(\lambda_r\), \(\phi_{[r]}\), and \(L_{[r]}\) are respectively the pole, mode shape and modal participation factor of mode \(r\) \((r = 1, \ldots, N_m)\). The diagonal matrix \(\Lambda\) is given by

\[
\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{N_m})
\]

with \(N_m\) the number of modes. The poles \(\lambda_r = \sigma_r + \iota \omega_r\) contain the natural frequencies \(f_r = \omega_r / (2\pi)\) and the damping ratios \(\zeta_r = -\sigma_r / (\sigma_r^2 + \omega_r^2)\). In real-life applications, the number of modes \(N_m\) generally differs from the number of measured output degrees of freedom \(N_o\) and the number of input forces \(N_i\).
2.1 Common-Denominator Model

A common-denominator model (CDM), or also called a scalar matrix fraction description, expresses the relation between output \( o \) and input \( i \) as a rational fraction of two polynomials, where the denominator polynomial is common for all input-output relations. The transfer function matrix \( H(s) \) can be expressed as

\[
H(s) = (G(s))^{-1} = \frac{G_{adj}(s)}{|G(s)|}
\]

(6)

where \( G_{adj}(s) \) the adjoint matrix is a \((N_m \times N_m)\) matrix containing polynomials in \( s \) of order \( 2(N_m - 1) \). The common-denominator is then given by the characteristic equation \(|G(s)|\), a polynomial in \( s \) of order \( 2N_m \), which roots are the poles of the studied structure. In general, the common denominator model (6) can be written as

\[
H(s) = \frac{B(s)}{A(s)} = \begin{bmatrix} B_{1,1}(s) & \cdots & B_{1,N_i}(s) \\ \vdots & \ddots & \vdots \\ B_{N_o,1}(s) & \cdots & B_{N_o,N_i}(s) \end{bmatrix}
\]

(7)

The relation between the CDM and the modal model is found by considering the frequency response function between output \( o \) and input \( i \)

\[
H(s) = \frac{B_{o,i}(s)}{A(s)} = \sum_{r=1}^{N_m} \left( \frac{\phi_{[o,r]} L_{i,[r]}(s)}{s - \lambda_r} + \frac{\phi_{[o,r]}^T L_{i,[r]}^T(s)}{s - \lambda_r^2} \right)
\]

(8)

From this equality it follows that the structural poles are given by the roots of the denominator \( A(s) \), while the mode shapes and participation factors are obtained from a SVD decomposition of the residue matrix \( \tau_r \in \mathbb{C}^{N_m \times N_i} \) as

\[
\tau_r = \phi_{[i,r]} L_{i,[r]}^T
\]

(9)

with the elements of the residue matrix \( \tau_r \) given by

\[
\tau_{o,i,r} = \lim_{s \rightarrow \lambda_r} (s - \lambda_r) H_{o,i}(s) = \phi_{[o,r]} L_{i,[r]}
\]

(10)

From modal analysis theory it follows that this residue matrix \( \tau_r \) is of rank 1. However, the common-denominator model does not force rank 1 residue matrices on the measured FRF data. As a result, common-denominator based algorithms, such as the frequency-domain LSCF and Maximum Likelihood \([20,40]\), loose quality by converting the common-denominator model to the modal model by reducing the residues to a rank-one matrix using an SVD. Other examples \([40,46]\) proof this fact. It seems that the loss of quality by transferring common denominator models into modal models tends to be more problematic for highly-damped cases (damping ratios > 2\%). By using a Right Matrix Fraction Polynomial (RMFD) model, such as in the case for the polymode implementations p-LSCF (also called PolyMax) and p-MLFD \([11,22]\), a rank-one residue matrix is inherent to the model and as a result less quality is lost by converting the RMFD model into a modal model.

The polynomial model (7) is expressed in the continuous time-domain. However, in practice, typical modal model identification is characterized by a high modal density resulting in a poor numerical condition when using a continuous time-domain model. Therefore, the use of orthogonal polynomials, such as Forsythe or Chebyshev, is inevitable in order to preserve numerical stability. A common-denominator model (CDM), or also called a scalar matrix fraction description, expresses the relation between output \( o \) and input \( i \) as a rational fraction of two polynomials, where the denominator polynomial is common for all input-output relations. The transfer function matrix \( H(s) \) can be expressed as

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H(s) = (G(s))^{-1} = \frac{G_{adj}(s)}{|G(s)|}
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Instead, the use of discrete time-domain models with a uniform frequency grid ensures numerical stability for high modal orders \((N_m = 100 \text{ or more})\), since the corresponding basis functions \( z_n^k \) are orthogonal with respect to the unity circle \([55]\). Moreover, the use of a discrete time-frequency domain common-denominator model allows a fast and memory efficient frequency-domain Least Squares or Maximum Likelihood algorithm implementation based on Toeplitz structured submatrices and the use of the FFT algorithm for the computation of their entries in the matrix formulations of the identification problem \([20,49,54,56]\). The use of a discrete time-domain model by mapping the frequencies in the frequency band of interest between 0Hz and 1Hz introduces model errors, which requires an higher model order than the exact one to obtain a good modal fit. Although this results in some mathematical poles, the use of a discrete time-domain model in combination with a good selection for the Least Squares parameter constraint generally results in a clear stabilization diagram as is demonstrated in \([12]\).
2.2 State Space Model

Another type of model is the so-called state space model that introduces the concept of states of a dynamical system. Reformulating Eq. (2) as

\[
\begin{bmatrix}
    sX(s) \\
    s^2X(s)
\end{bmatrix} = \begin{bmatrix}
    0 & I \\
    -M^{-1}K & -M^{-1}C_1
\end{bmatrix} \begin{bmatrix}
    X(s) \\
    sX(s)
\end{bmatrix} + \begin{bmatrix}
    0 \\
    M^{-1}
\end{bmatrix} F(s)
\]

and defining respectively the state vector \(Y(s)\) and the system matrices as

\[
Y(s) = \begin{bmatrix}
    X(s) \\
    sX(s)
\end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix}
    0 & I \\
    -M^{-1}K & -M^{-1}C_1
\end{bmatrix}, \quad B = \begin{bmatrix}
    0 \\
    M^{-1}
\end{bmatrix}, \quad C = [I \ 0] \quad \text{and} \quad D = [0]
\]

a first order differential expression is obtained as the state-space description of the system’s equations (2) given by

\[
\begin{align*}
    sY(s) &= AY(s) + BF(s) \\
    X(s) &= CY(s) + DF(s)
\end{align*}
\]

with the matrices \(A, B\) respectively the (dynamical) system and input matrix, which together with \(C\) and \(D\) represent the system’s realization describing the system’s response to a known input signal.

The transfer function matrix between the outputs and inputs is then given by \([29]\)

\[
H(s) = C[sI - A]^{-1}B + D
\]

Considering the eigenvalue problem for the \((2N_m \times 2N_m)\) system matrix \(A\) yields the system poles, i.e. \(AV = AV\) (with \(V\) right hand eigenvectors of \(A\)), while the mode shape and modal participation factor vectors are respectively found from the matrices \(C\) and \(B\). This also allows to transform the state space equations (13) to their modal form

\[
H(s) = C[sI - A]^{-1}B + D = [\phi \ \phi^*] \begin{bmatrix}
    \Lambda & 0 \\
    0 & \Lambda^*
\end{bmatrix}^{-1} \begin{bmatrix}
    L \\
    L^*
\end{bmatrix}
\]

Since in practice the time signals \(f(t)\) and \(x(t)\) are only known for discrete time instants \(n\Delta t\), the discrete Fourier transformed input and output signals \(F(s)\) and \(X(s)\) in the state space model (13) are only available for discrete values \(s_k\) of \(s\)

\[
X(s_k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N} x(n\Delta t)e^{-2\pi nk/N} \quad \text{and} \quad F(s_k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N} f(n\Delta t)e^{-2\pi nk/N}
\]

with \(s_k = e^{-2\pi k/(N\Delta t)}\) and \(N\) the number of time samples. Discrete time-domain models give the relation between discrete time signals, e.g. \(x_n = x(n\Delta t)\) of continuous time signals. Assuming the sample period to be constant during the measurements results in a signal that is piecewise constant over the sample period \(\Delta t\) for a Zero-Order Hold (ZOH) assumption. The state space formulation in the discrete time-domain is then given as

\[
\begin{align*}
    z_kY_k &= A_dY_k + B_dF_k \\
    X_k &= C_dY_k + D_dF_k
\end{align*}
\]

with \(z_k = e^{s_k\Delta t} = e^{2\pi k/N}, \quad X_k = X(s_k), \quad F_k = F(s_k)\) and with \(A_d = e^{\Delta t}, \quad B_d = \int_0^{\Delta t} e^{A\xi}d\xi B, \quad C_d = C\) and \(D_d = D\)

3 FREQUENCY-DOMAIN IDENTIFICATION FRAMEWORK

In this section, the two different parametric models, i.e. the common-denominator and state space model, will be considered for structural dynamics modelling starting from frequency-domain data in 3 different identification frameworks, i.e. deterministic, stochastic or combined. At the same time, attention is paid to the type of primary data to start from, i.e. Average Based Spectral functions, such as Frequency Response Functions (FRFs) or Power Spectral Densities (PSDs), or directly from the Input/Output (I/O) Fourier data. Figure 1 shows a general representation of a typical measurement set-up used for testing of mechanical structures. One can distinguish between 2 types of testing, whether artificial excitation forces are applied or not.

This paper discusses the identification of the frequency-domain common-denominator polynomial and subspace models for which recently optimized algorithms were developed. For details about the algorithm implementations, such as matrix formulations and computational efficiency, sufficient literature references have been cited throughout this Section.
3.1 Deterministic Identification

Traditionally, Experimental Modal Analysis (EMA) is based on data that describes the relation between artificially applied forces $F_1, F_2, \ldots, F_N$ and the resulting structural responses $X_1, X_2, \ldots, X_N$, where both the forces and responses are measured during a well-controlled experiment. In this case, the setup is described by a deterministic framework, where, in order to preserve a high degree of correlation between inputs and outputs, the test conditions are optimized in order to minimize effects from extraneous (unmeasured) noise sources.

A wide variety of modal parameter estimators has been developed in a deterministic framework during the last 3 decades. These estimators are based on system identification methods that originate from electrical engineering but have become generally applicable to many different fields of modelling research for e.g. econometrics, control, chemical, biomedical, civil and mechanical engineering. The application for modal analysis requires special efforts for improving the performance of the identification schemes since they must be able to deal with large data sets from MIMO testing (typically $N_i = 3$, $N_o = 300$), high modal density with closely-spaced poles and very low and high damping.

In the field of system identification most methods start from the measured input and output time histories in the case of time-domain identification or from the Fourier spectra in the case of frequency-domain identification. Although, a typical modal test results in a large amount of data, such that some preprocessing of the data is highly recommended to reduce both the size of the data set as well as the noise levels. Therefore, it is common practice in EMA applications to use Frequency Response Functions (FRFs) as primary data to start the analysis from instead of using raw Fourier spectra, where the FRFs are estimated in a nonparametric preprocessing step using typically the so-called $H_1, H_v$ [35, 42] or the more advanced $H_{ev}$ [19] or $H_{iv}$ [49, 58] FRF estimators. These averaging-based estimators divide the force and response signals in different data records to compute the FRFs from.

However, in the cases that only a limited amount of data is available, it is preferable to start directly from the raw Fourier force and response spectra (I/O Fourier data), since averaging will then reduce this resolution below the critical level that is required in order to preserve an acceptable data quality for modelling systems with a high modal density.

- For the deterministic case, the discrete time common-denominator model starting from the I/O Fourier data is given by

$$X_o(f_k) = \sum_{i=1}^{N_i} \frac{B_{oi}(z_k)}{A(z_k)} F_i(f_k)$$

(18)

Based on this model formulation, frequency-domain (Total) Least Squares [56] and Maximum Likelihood [55] algorithms have been developed for the specific application of modal parameter estimation. Starting from FRFs as primary data, the model becomes

$$H_{oi}(f_k) = \frac{B_{oi}(z_k)}{A(z_k)}$$

(19)

with $H_{oi}(f_k)$ the FRF between output $o$ and input $i$. Details on the algorithms for both (Total) Least Squares and Maximum Likelihood identification are available in [20, 46, 51, 52].
The discrete time state space model for I/O data is given by

\[ z_k Y_k = A_d Y_k + B_d F(f_k) \]
\[ X(f_k) = C_d Y_k + D_d F(f_k) \]  \hspace{1cm} (20)

with \( Y_k \in \mathbb{C}^{n \times 1} \), \( X(f_k) \in \mathbb{C}^{N_o \times 1} \) and \( F(f_k) \in \mathbb{C}^{N_i \times 1} \). The FRF based version is given by

\[ z_k Y_k = A_d Y_k + B_d H(f_k) \]
\[ H(f_k) = C_d Y_k + D_d \]  \hspace{1cm} (21)

with \( Y_k \in \mathbb{C}^{n \times N_i} \), \( H(f_k) \in \mathbb{C}^{N_o \times N_i} \). More details about a computational efficient frequency-domain subspace algorithm implementation for both I/O and FRF-based deterministic state space model identification is discussed in \([7, 8]\).

### 3.2 Output-Only identification

However, in the case that one cannot apply any artificial excitation or one is interested to model a structure under its operating conditions, the structure’s response is the result of ambient and often unmeasurable forces \( E_1, \ldots, E_N \) (cf. Figure 1). A typical and important example is the modelling of civil structures such as a bridge, where the ambient excitation results from wind, traffic, seismic activity (micro-earthquakes), which are extremely difficult to measure. Moreover, turning off these ambient excitation sources is often impossible and applying an artificial measurable force, which exceeds the ambient excitation is expensive and practically complicated. Other examples are in-flight testing for aircraft flutter analysis, automotive road testing for noise and vibration harshness analysis, gas turbine testing for torsional critical modes, etc.

The identification of the modal parameters is referred to as Output-only or Operational Modal Analysis (OMA), which uses techniques from the field of stochastic identification. One can distinguish between two categories, i.e.

- **Data-driven stochastic identification** which directly starts from the output time signals or output Fourier spectra
- **Correlation or Auto/Cross Power Spectral Density-driven stochastic identification** which first estimates covariances or Power Spectral Densities (PSDs) between the outputs and certain reference output DOFs. The auto and cross PSDs are nothing else than the columns of the covariance matrix of all output signals corresponding to the reference output DOFs.

Based on the models discussed in Section 2, the model formulations for the stochastic case (i.e. for \( F(f_k) = 0 \)) now become:

- the discrete time common-denominator model for Output-only PSD data is given by
  \[ G_{X_o X_r}(f_k) = \frac{B_o(z_k)}{A(z_k)} \]  \hspace{1cm} (22)

with \( G_{X_o X_r}(f_k) \) the PSD matrix between output \( o \) and reference output \( r \) at spectral line \( k \). Notice that the common-denominator model now requires a doubled model order in order to cope with 4-quadrant symmetry related to the PSD data. In \([18, 38]\), a detailed discussion is given on the formulation of a dedicated algorithm for modal parameter estimation using a Least Squares and Maximum Likelihood approach. The 4-quadrant symmetry can be avoided by the use of **Positive Power Spectral Densities** which as well enforces the estimated model to represent a stable system \([9]\).

- The discrete time state space model for Output-only Fourier spectra is given by
  \[ z_k Y_k = A_d Y_k + W_k \]
  \[ X(f_k) = C_d Y_k + V_k \]  \hspace{1cm} (23)

with \( Y_k \in \mathbb{C}^{n \times 1} \), \( X(f_k) \in \mathbb{C}^{N_o \times 1} \) and with \( W_k \in \mathbb{C}^{N_i \times 1} \) and \( V_k \in \mathbb{C}^{N_o \times 1} \) respectively representing the process noise and output measurement noise. An Output-only PSD version only makes sense when considering a combined framework as is explained in the next paragraph.
3.3 Combined identification

As discussed in the previous two sections, the difference between EMA and OMA is based on the fact whether the input forces are known or unknown. EMA only considers the deterministic relation between the measured (applied) forces and therefore the contribution from unmeasurable forces in the structure’s response is considered as measurement noise, reducing the input/output correlation which is considered as highly undesirable. This is contradictory to OMA, which considers the structure’s response as a result from a stochastic process where the modal parameters are obtained from output-only data that in general results in a higher uncertainty on the final parameter estimates [50].

In reality, a structure’s response generally results from a combination of a deterministic - stochastic process where this response is a sum of both the deterministic contribution from measurable inputs and a stochastic contribution from all unmeasurable forces. Therefore, the concept of a combined EMA-OMA approach facilitates an optimal data exploitation since the dynamical response can be completely described by a combined parametric model. This concept of a combined approach, also known as an Operational Modal Analysis with eXogenous inputs (OMAX).

A typical example that fits this OMAX framework is flight flutter analysis, where an aircraft is excited by both an artificial measurable force (rotating vanes, random perturbation of ailerons, etc.) and the unmeasurable turbulence excitation. Other examples are civil structures simultaneously excited by drop-mass impacts or hammers and unmeasurable ambient forces (wind, traffic, seismic activity, etc.) as well as noise and vibration harshness (NVH) testing of cars that are simultaneously excited by the measurable input force (rotating vanes, random perturbation of ailerons, etc.) and the unmeasurable turbulence excitation. More general one can state that every modal analysis fits the OMAX framework. Although in well-controlled laboratory conditions the influences from unmeasurable forces in the structure’s response are considered as measurement noise, reducing the input/output correlation which is considered as highly undesirable. This is contradictory to OMA, which considers the structure’s response as a result from a stochastic process where the modal parameters are obtained from output-only data that in general results in a higher uncertainty on the final parameter estimates [50].

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- For the combined deterministic-stochastic case, the discrete time common-denominator model for I/O data is given by

$$X_o(f_k) = \sum_{i=1}^{N_o} \frac{B_o(z_k)}{A(z_k)} F_i(f_k) + \frac{C_o(z_k)}{A(z_k)} E_o(f_k)$$

with $C_o = \sum_{j=0}^{N_o} c_{o,j} z_k^j$ modelling the stochastic part of the response $X_o(f_k)$ excited by the unknown force $E_o(f_k)$. Although in this case a consistent Least Squares identification of this model is possible, this LS method results, similarly to the Maximum Likelihood approach, in an iterative optimization problem requiring a Gauss-Newton approach. The FRF-based version is described in detail in [5].

- The discrete time state space model for I/O data is given by

$$z_k Y_k = A_d Y_k + B_d F(f_k) + W_k$$
$$X(f_k) = C_d Y_k + D_d F(f_k) + V_k$$

with $Y_k \in \mathbb{C}^{n\times 1}$, $X(f_k) \in \mathbb{C}^{N_o\times 1}$ and $F(f_k) \in \mathbb{C}^{N_o\times 1}$ and with $W_k \in \mathbb{C}^{n\times 1}$ and $V_k \in \mathbb{C}^{N_o\times 1}$ respectively representing the process noise and output measurement noise. The FRF based version is given by

$$z_k Y_k = A_d Y_k + B_d + W_k$$
$$H(f_k) = C_d Y_k + D_d + V_k$$

with $Y_k \in \mathbb{C}^{n\times N_i}$, $H(f_k) \in \mathbb{C}^{N_o\times N_r}$ and with $W_k \in \mathbb{C}^{n\times N_i}$ and $V_k \in \mathbb{C}^{N_o\times N_r}$. It should be noticed that for Output-only PSD data, the model (26) can be used again, now with $B_d$, $D_d$ equaling zero and $H(f_k)$ replaced by $G_{X,X,e}(f_k) \in \mathbb{C}^{N_o\times N_{ref}}$ ($N_{ref}$ the number of reference output DOFs) the Power spectral density matrix. Notice that in the case of the I/O-based version, the combined model formulation has a physical interpretation since the response is considered as a result of a part due to artificially applied and measured excitation and a part related with unmeasured stochastic (noise) excitation sources.

It should be noted that compared to combined common-denominator algorithm [7], the combined subspace algorithm as presented in [9] has the following advantages:

- it forces rank 1 residues on the measurements
- it is a 1-step procedure, not requiring an optimization procedure
it facilitates a fast construction of stabilization diagram \[5, 12, 13]\n
estimates both the amplitude and phase of the modes that are only excited by the (unmeasured) noise excitation, while the combined CDM based nonlinear LS approach (24) only can determine the amplitude (no phase) information for those modes.

This, together with the fact that the same efficient algorithm implementation \[9\] can be used for the three different identification frameworks, makes the combined frequency-domain subspace algorithm in the general case superior to the common-denominator based combined algorithm.

3.4 Parametric Modelling of Transient Phenomena

The effects of transient phenomena due to a non-steady state response of the structure, and thus errors introduced by leakage, can lead to important errors on the final modal parameter estimates. A common way to reduce these errors is to use time windows, such as a Hanning window, during the nonparametric preprocessing when computing the Averaging-based FRFs and PSDs or the I/O Fourier data. However, the use of time windows as well distorts the frequency-domain data while the effects of transients can only be reduced but not perfectly eliminated.

Therefore, another approach, based on the concept presented in \[43\], can be used to compensate for the effects of transient phenomena by extending the parametric model in order to describe these effects in a parametric way. Parametric modelling of transient effects improves especially the damping estimates and the overall quality of the estimated modal model. This makes it possible to identify, under the same assumptions as in the time-domain, frequency-domain models from data that is measured using arbitrary signals (non-periodic) or periodic signals that are corrupted by transients.

For the deterministic identification approach, the use of transient polynomials in combination with a common-denominator polynomial to estimate modal parameters starting from I/O data using optimized frequency-domain (Generalized) Total Least Squares and Maximum Likelihood identification schemes \[54, 56\].

For the combined deterministic-stochastic case, the extended discrete time common-denominator model for I/O data is given by

\[
X_o(k) = \sum_{i=1}^{N} B_{oi}(z_k) F_i(f_k) + T_o(z_k) + C_{o}(z_k) E_o(f_k)
\]

with \(T_o(z_k) = \sum_{j=0}^{N_m} t_{o; j} z_k^j\). The FRF based version is described in \[14\].

The extended discrete time state space model for I/O data is given by

\[
z_k Y_k = A_d Y_k + B_d F(f_k) + B_T z_k + W_k
\]

\[
X(f_k) = C_d Y_k + D_d F(f_k) + V_k
\]

with \(B_T \in \mathbb{C}^{N \times 1}\) an additional parameter vector that models the influence of the initial and final conditions, i.e. \(B_T = \frac{z_k}{z_k^{(N-1)}} (N \text{ the number of frequency lines})\). These additional parameters model the non-steady state behaviour of the system making the frequency-domain state space model robust for leakage and transients in the case that no window (i.e. a rectangular time window) is used for the calculation of the Fourier spectra. This additional term in the state space model can also be interpreted as an additional input \(U_k = \frac{1}{N} z_k\). More details about the use of this approach for the case of I/O or ABS-driven frequency-domain subspace identification can be found in \[9, 7\].

4 APPLICATIONS

The applicability of the presented methods was studied for the 2 applications, i.e. the modelling of an automotive engine subframe and for aircraft flight flutter analysis.
4.1 MIMO Modal Analysis of an Automotive Engine Subframe

A modal analysis is performed on an engine subframe. The accelerations were measured at 23 response locations and two electro-dynamic shakers excite the structure with random forces.

(Extended) Deterministic Subspace Identification

Starting from 256K time samples of both the acceleration and force measurements, reference FRFs are estimated, by dividing the time records in 8 blocks of 64K resulting in a frequency domain resolution of 0.0156Hz (i.e. the observation window length of \(64 \times 10^4\) s). These FRFs can be assumed to be free of leakage errors and serve as a reference for validation purposes. Next only the first 8K data samples are used and divided in 4 equal blocks of each 2K samples resulting in a frequency resolution of 1Hz. First the FRFs are calculated by the \(H_1\) method with a Hanning window. From these FRFs the classic state space model (\(A, B, C\) and \(D\) matrices) is estimated with the deterministic frequency-domain subspace (basic projection) method. Secondly the same 4 blocks of data are used to obtain the FRFs with a rectangular window. These FRFs are processed to estimate the extended state-space model for FRFs (including the terms \(T_1, \ldots, T_N\)).

Table 1 shows a good agreement between the natural frequencies and damping ratios obtained from the reference FRFs and the estimates derived from the short data sequences when the extended state-space model (\(Edet\)) approach. The use of a Hanning window and a classic deterministic state space identification (\(Cdet\)) clearly results in poor estimates, especially for the damping estimates.

![Stabilization Diagram](image)

**Figure 2:** Stabilization diagram for both the classic (\(Cdet\)) and extended (\(Edet\)) state-space model approach. Although the \(Edet\) approach does not use a Hanning window and as a result the FRF looks much noisier, all the poles clearly appear in the stabilization diagram. The \(Cdet\) approach suffers from estimating double poles, where in reality only one physical pole is present e.g. around 129Hz, 205Hz, 286Hz and 380 Hz. This is typically caused by leakage phenomena and makes the stabilization diagram confusing for the end-user. Finally, Figure 3 compares the synthesized FRFs of both approaches with the reference FRF from the leakage free measurement. One concludes that for the \(Edet\) approach, although starting from only 8K data samples averaged in 4 blocks of 2K samples, still a very good agreement is found between the model and the reference FRFs, while a \(Cdet\) approach clearly fails due to errors introduced by leakage.

**Stochastic Subspace Identification**

Frequency-domain stochastic subspace identification was also applied to fit a model through the measurements on the engine subframe starting only from the 23 response (acceleration) spectra that were measured at a sampling rate of 2048Hz during 10s, while the structure was still excited by two shakers with (unmeasured) random forces. Before the actual modelling task starts, the frequency band of interest between 190Hz and 390Hz was uniformly re-scaled to the unity circle.

<table>
<thead>
<tr>
<th>(f_{ref})</th>
<th>(f_{Edet})</th>
<th>(f_{Cdet})</th>
<th>(d_{ref})</th>
<th>(d_{Edet})</th>
<th>(d_{Cdet})</th>
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<td>0.110</td>
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<td>0.110</td>
<td>0.160</td>
</tr>
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<td>0.091</td>
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</table>

Table 1: Natural frequencies and damping ratios estimated for the subframe using "Extended" (\(Edet\)) and "Classical" (\(Cdet\)) deterministic frequency-domain subspace identification.
The identification starts from output-only spectra, which can be modelled by both a stable and an unstable system (both resulting in the same fit of the power spectra). Therefore, the frequency-domain stochastic subspace approach proposed in [6] was used since this algorithm always forces a model with stable poles and the solution is consistent for \( N \to \infty \) (\( N \) the number of frequency lines) for a stable system. This is clearly illustrated by the stabilization diagram in Figure 4, which only shows stable poles.

Figure 5 compares the estimated power spectral density functions with the measured output spectra, while Table 2 shows a good agreement between the natural frequencies \( f_O \) and damping ratios \( d_O \) obtained from the output-only analysis and the natural frequencies \( f_{IO} \) and damping \( d_{IO} \) ratios obtained from extended deterministic frequency-domain subspace algorithm (Edet) starting from input/output measurements (i.e. the 23 responses and the 2 electrical generator signals) serving as a validation data set.

Combined Frequency-domain Subspace Identification

The combined deterministic-stochastic frequency-domain algorithm is now illustrated starting from ABS data (i.e. both FRFs and positive power spectra) as primary data in order to estimate the system matrices and modal parameters. It is now shown that this combined approach is certainly applicable to analyse laboratory measurements with the advantages of a frequency band selection and capability to handle a large number of outputs, several inputs and a high modal density.

A frequency band of interest between 210 Hz and 410 Hz and a model order of 30 modes was chosen. Both the models identified by the classical deterministic (Cdet) and the combined frequency-domain subspace algorithm, are validated with a high quality validation data set by means of the MSRE over all frequencies per FRF. Figure 6 shows the MSRE for the 46 FRFs for both identification approaches. The combined approach resulted in an average MSRE over all FRFs of 0.2, while the classical method results in a mean MSRE of 5.4. Figure 7 compares some identified FRFs for both identification approaches with again the same validation data set (as used for the deterministic case study), clearly showing the large bias errors introduced by the classical method.

4.2 Flight Flutter Analysis

Aircraft and winged-launch vehicles must be free from aerodynamic instabilities such as flutter to ensure safe operation. Flutter is a dynamic instability that involves coupling of aerodynamic, elastic and inertial forces of the structure. During flight the structure extracts energy from the airstream. At speeds larger than the critical airspeed, the energy dissipated is less than the available structural damping and the motion is divergent. The airworthiness regulation requires that a full-scale aircraft is demonstrated free from flutter by a flight flutter test. In these tests natural frequencies and modal damping ratios are estimated for different flight conditions. Most common approaches track the damping ratios of the different flight conditions, which are then extrapolated in...
Figure 3: Comparison between the reference FRF (full line) and FRF synthesized from the Classic (○) and Extended deterministic state-space model (×).

Figure 4: Stabilization chart for stochastic frequency-domain subspace algorithm. Notice that all identified poles are stable (positive damping ratio); (+ stable pole).

Figure 5: Comparison between the measured output spectra (×) and the power spectra synthesized from the estimated stochastic state space model (solid line) with respectively zoom (bottom).
<table>
<thead>
<tr>
<th>$f_{IO}$ (Hz)</th>
<th>$d_{IO}$ (%)</th>
<th>$f_{O}$ (Hz)</th>
<th>$d_{O}$ (%)</th>
</tr>
</thead>
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<tr>
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<td>0.15</td>
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<td>359.9</td>
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</tr>
<tr>
<td>380.0</td>
<td>0.31</td>
<td>380.3</td>
<td>0.26</td>
</tr>
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</table>

Table 2: Natural frequencies and damping ratios of the subframe using the Extended deterministic (IO) and the stochastic (O) frequency-domain subspace method.

order to determine whether it is safe to proceed to the next flight test point. Flutter can occur when one of the damping values tends to become negative. The speed at which such an instability takes place is called the flutter speed and is one of the most important design parameters for an aircraft wing. Before starting the flight tests, ground vibration tests as well as numerical simulations and wind tunnel tests are used to get some prior insight into the problem.

There are numerous system identification methods that allow the estimation of the modal parameters from a vibrating system, but not all of them can deal with flight flutter data. In-flight test data is typically characterized by short time records and noisy measurements due to the unmeasured atmospheric turbulent forces. Many traditional methods used in modal testing work well with clean data, but, as the noise increases and available measurement time decreases, the accuracy of the flutter parameter determination rapidly degrades, especially for the damping ratio estimates. Increasing the level of the artificial applied force to increase the signal-to-noise ratio is no option, since the response level is limited for structural integrity and comfort reasons. Furthermore, non-linear effects will appear and the measured working point will then differ from the operational flight conditions. However, the use of specially designed broadband signals as multisines and chirp signals can improve the quality of the measurements [4], [21].

Recently, extensive research was focused on operational modal analysis, where the structure is excited by ambient noise excitation e.g. atmospheric turbulence [1, 23, 25, 41]. Although these output-only methods have been used with some success, there are several disadvantages. The turbulence is often not intense enough and excites usually only the lower modal modes. Furthermore output-only methods need long data records to obtain accurate parameter estimates and expensive flight time is lost looking for sufficient turbulence levels [13]. The state-of-the-art of flight flutter testing techniques in aircraft industry has been reviewed in [16], [36] and [8].

Until today, traditional flight flutter analysis start from averaged data like frequency response functions (FRFs) or impulse response functions (IRFs), since usually there is an attempt to reduce the influence of the noise level on the outputs by collecting many data blocks. However, this increases the required flight time at a fixed flight condition, which is adversely to the desired procedures. The applicability of the different identification approaches, discussed in Sections 3, for flight flutter analysis is now studied by means of both simulations and in-flight vibration data.

4.2.1 FLIGHT FLUTTER SIMULATIONS

In order to obtain a profound comparison of the different identification approaches, Monte Carlo simulations were done to evaluate with respect to bias errors and estimator efficiency. In order to obtain a representative study, simulations were based on a modal model that was extracted from a ground vibration test (GVT) of an actual aircraft, shown in Figure 8(a), with the grid used for the
Figure 7: Comparison between validation FRFs (×) and the synthesized FRFs for the Classical deterministic (top) and the combined frequency-domain subspace algorithm (bottom).

GVT in Figure 8(b). Data was simulated in the continuous time-domain with a state space model based on the first six modes of the GVT test, as visualized in Figure 9. During simulations, one random noise force \( f(t) \) was considered to act on a discrete point perpendicular to the surface of the left wing. A sample frequency of 256Hz was used and 16K samples, corresponding to a total measurement time of 64s, are simulated for each test.

To simulate the influence of the atmospheric turbulence, spatially correlated white noise sources \( e_o(t) \) acting on the nose, wings and tail of the aircraft. The level of atmospheric turbulence is set as the (percentage) ratio between the standard deviation of the unmeasured and measured time responses, i.e.

\[
\frac{\text{std}(G_o(q,e_o(t)))}{\text{std}(H_o(q,f(t)))}
\]

Simulations were performed for increasing levels of atmospheric turbulence of 7, 14, 21, 28, 35 and 42%.

For each of the 6 increasing turbulence levels 10 different runs are simulated and processed by three different common-denominator based estimators, i.e. (1) the Maximum Likelihood estimation of the deterministic FRF-based CDM (ML FRF), (2) the Maximum Likelihood estimation Extended deterministic I/O-based CDM (ML IO) and (3) the Non-linear Least Squares estimation of the combined I/O-based CDM (CLSF IO). Figure 10 shows the 68% confidence bars of the damping ratio estimates of each mode for an increasing level of the turbulence. Since only a limited amount of data is considered the FRFs estimates are very noisy and biased due to leakage effects. Since a classical LSCF FRF estimator [46] failed the classic ML estimator starting from FRFs is used. This ML FRF method suffers from a bias on the damping ratios of the low frequent modes due to the leakage effects in the nonparametric FRF estimates. Furthermore, the variance on the ML FRF estimates is large even in the case of low turbulence levels. From these results one can conclude that the FRF based methods are not recommended in the case of short time records.

Since the extended I/O data driven methods easily model the transient effects they do not suffer from bias errors in the case of low turbulence levels. Compared to the ML I/O data driven estimator, the CLSF IO method is also consistent, under the assumption
Figure 8: Aircraft and the grid used for the GVT

Figure 9: Different mode shapes (from (a) to (f) for increasing natural frequency).
of white output noise. Although the ML IO estimator is consistent under the assumption of only output noise, the results show a bias for increasing noise levels. This can be clarified by local minima and an insufficient number of iterations to reach convergence. The CLSF IO resulted in unbiased estimates with the smallest uncertainty for all modes even in presence of high levels of turbulent forces. Interesting to notice is that the CLSF IO method estimates the damping ratios of the 3rd, 4th and 5th mode very accurate, although these modes are only weakly excited by the artificial force, while well excited by the turbulent forces (these modes are mainly vibrating at the tail, nose and cabin as can be seen in Figure 9).

To show the applicability of the combined deterministic-stochastic subspace algorithm the same in-flight simulations were also processed to compare the I/O based combined common-denominator (CLSF IO) estimator with both the deterministic and combined deterministic-stochastic frequency-domain subspace estimators. For each of the 6 increasing turbulence levels 10 different runs are simulated and processed. Figure 11 illustrates the mean values and standard deviations of the estimated damping ratios by the different algorithms for the 6 turbulence levels. The classic deterministic projection algorithm is less accurate than both combined algorithms. For the first two wing modes (cf. Figure 9), the larger uncertainty and bias errors can be explained by the influence of leakage. Both combined algorithms included an additional polynomial $T(z)$ or state space term $T$ to model the initial and final conditions, while this was not the case for the classic deterministic subspace algorithm. Furthermore, it is clear that the damping ratios of the tail modes 3 and 4, which are well excited by both the artificial applied force on the wing and by the unknown turbulent forces, can only be accurate identified by the combined deterministic-stochastic algorithms. Both the combined common-denominator algorithm and the combined subspace algorithm result in a comparable accuracy of the estimated damping ratios.

### 4.2.2 FLIGHT FLUTTER MEASUREMENTS

In-flight vibration data was obtained on a military aircraft by measuring the accelerations at 12 DOFs without the use of an additional excitation device (such as e.g. rotating vanes or impulse rockets). Although, the airplane could be excited by the natural turbulence only, extensive research from the NASA organization reports that the natural excitation is often band limited \cite{2} and many time is lost during flight to search for sufficient turbulent force levels. Therefore, the use of artificial excitation is still often used and nowadays fly-by-wire systems allow to apply perturbation signals by means of the control loops of the flap mechanisms of the wings.
in order to excite the structure. The same fly-by-wire approach was used in order to increase the structural vibration levels during the in-flight measurements that are presented in this section. In this case, the rotation angle of the flaps was as well measured.

Stochastic Frequency-domain Subspace Identification

Since the dynamical response is also partially included as a perturbation on the angle rotation measurement, one can discuss whether this rotational vibration can be considered as an input or output signal. Starting therefore from the acceleration (response) measurements only, both the turbulences and the artificial (fly-by-wire) excitation are to be considered as ambient excitation forces. Figure 14 illustrates the stabilization chart from the stochastic subspace analysis of 12 acceleration measurements.

Similar as for the subframe all identified poles are stable. To make a distinction between the physical and mathematical poles, one has to rely on both a priori known insights in the dynamical behavior (from simulations and GVT tests) and criteria based on both mathematics and physics. In the use of pole-zero criteria combined with the uncertainty levels on the estimated poles is presented to select the physical poles, while proposes an automated interpretation of the stabilization chart by the use of cluster algorithms and physical parameters like mode complexity and modal assurance criteria. Some different approaches based on a heuristic approach for an automatic interpretation of the stabilization chart is proposes in. Figure 12 illustrate the synthesized power spectra of 4 different acceleration measurements.

Combined Deterministic-Stochastic Frequency-domain Subspace Identification

To show the applicability of the combined deterministic-stochastic frequency-domain the same in-flight vibration measurements were analysed including now the measurement of the angle rotation of the flaps as a known input signal. The stochastic contribution in the responses is again caused by the turbulent forces acting on the airplane during flight. Starting from the input and output spectra in a frequency band from 3Hz to 11Hz, a state-space model was estimated by both the classic deterministic projection and combined deterministic-stochastic frequency-domain subspace algorithms. Figure 13 compares the stabilization charts for both the (deterministic) projection algorithm and the combined deterministic stochastic approach.
Based on the relative differences between the eigenfrequencies and damping ratios of the poles estimated for subsequent model orders, the poles are labelled in the stabilization diagram using the symbols $s$, $f$, $d$ and $o$, which respectively mean:

- $s$: Relative damping ratio difference $< 15\%$ and relative eigenfrequency difference $< 3\%$
- $f$: Relative damping ratio difference $\geq 15\%$ and relative eigenfrequency difference $< 3\%$
- $d$: Relative damping ratio difference $< 15\%$ and relative eigenfrequency difference $\geq 3\%$
- $o$: Relative damping ratio difference $\geq 15\%$ and relative eigenfrequency difference $\geq 3\%$

After applying these criteria it is clear from Figure 13 that the stabilization diagram of the combined algorithm results in a much easier interpretation than the deterministic algorithm.

Table 3 shows that some of the natural frequencies and damping ratios are not identified by the deterministic approach. Furthermore, large deviations in the estimated damping ratios can be observed between the deterministic and combined identified parameters. Based on the aircraft manufacturer’s results, it was confirmed that all 9 poles identified by the combined estimator are the only 9 poles in the analysis frequency band and that the damping ratios were in very close agreement with the manufacturer’s results.

Figure 15 compares the synthesized FRFs for both the deterministic and combined algorithm with the measured FRFs (i.e. $H_1$ between angle and accelerations), from which it is clear that the deterministic algorithm results in much larger (bias) errors than the combined algorithm.

---

**Figure 12:** Comparison between the power spectra synthesized from the estimated model (full line) and the output spectra (×).
Figure 13: Comparison between stabilization charts obtained by the deterministic (a),(c) and combined (b),(d) frequency-domain subspace algorithm: deterministic algorithm and combined algorithm.

5 CONCLUSIONS

This paper gives an overview of recent developments for frequency-domain common-denominator and state space model identification from which the modal parameters can be derived. For both model types, the formulation of the relation between the model and the data have been formulated in a deterministic, stochastic and combined identification framework. Next, the pros and cons of each formulation has been illustrated based on two different modal analysis applications, i.e. a MIMO modal analysis for the dynamical modelling of an automotive engine subframe and for modelling aircraft flight flutter behaviour. The use of a so-called combined identification approach yields promising results since an optimized data exploitation is possible by considering the structural response as the result from both measured (artificially applied) forces and unmeasured (stochastic) forces. This combined approach can be used to model either a common-denominator model which then results in an optimization algorithm or a state space model through techniques of non-iterative subspace identification.

ACKNOWLEDGEMENTS

The financial support of the Institute for the Promotion of Innovation by Science and Technology in Flanders (IWT), the EUREKA project E!2419 FLITE (Flight Test Easy), the Concerted Research Action "OPTIMech" of the Flemish Community and the Research Council (OZR) of the Vrije Universiteit Brussel (VUB) are gratefully acknowledged.
Table 3: Natural frequencies and damping ratios identified by the deterministic and combined deterministic-stochastic frequency-domain subspace algorithm.

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Figure 14: Stabilization chart for the stochastic frequency-domain subspace algorithm. Notice that all identified poles are stable (positive damping ratio): (+ stable pole).

Figure 15: Comparison between the $H_1$ FRFs (×) and the synthesized FRFs obtained by the Classic deterministic (left) and the combined frequency-domain subspace algorithm (right).
REFERENCES


