Structural Damage Detection Using Digital Video Imaging and Wavelet Transformation

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ABSTRACT

Non-contact measurement offers convenient and less expensive setup. Sometimes, they may also be the only resort because the structure is too small to be instrumented with contact sensors and/or the additional mass of the sensor is too large to use. In other cases, the structure is inaccessible due to obstruction for the case of tall/large structures such as bridges or towers. The digital imaging techniques offer an alternative to contact measurement. In this paper, use of digital video imaging is proposed for detecting damage in structures. The theory of measuring structural vibration using high-resolution images based on sub-pixel edge detection technique is demonstrated with extraction of displacement time series from video images. From displacement time series, characteristic dynamic properties, i.e., natural frequency, damping and mode shapes, are established. Mode shape difference function is introduced and derived for damage detection purpose. These mode shape difference functions are subjected to continuous wavelet transformation for determining their singularity (discontinuity) at locations of damage. A laboratory test program was carried out to implement the concepts using a high-speed digital video camera. Results show that the proposed approach of data acquisition using digital video camera is effective to provide spatially intensive time-series data for non-contact structural testing. Furthermore, it is shown that the wavelet transformation of mode shape difference functions to isolate locations of singularity is able to identify the damage and their locations.

INTRODUCTION

Many structures in the civil infrastructure system need constant monitoring for deciding repair, maintenance, and rehabilitation. Research efforts have been reported in the past for monitoring these structures using their dynamic characteristics (Doebling et al \cite{24}), such as the natural frequencies and mode shapes. Previously proposed approaches overwhelmingly used accelerometers for measurement, attached to the structure at a limited number of points. The effectiveness of these approaches still needs to be demonstrated because the resolution of the acquired data is limited. Such approaches also require access to the structure, which may be costly or impractical. To circumvent these difficulties, dynamic digital imaging is proposed here for structural health monitoring. Aspects related to this approach are briefly reviewed next, before it is fully presented later.

Damage and alteration to structures change their behaviors. If these changes are accurately measured, they can be used to identify and locate the structural damage and alteration. This process of identification is referred to as damage detection. Such detection should cover at least two key aspects: 1) detecting presence of damage, and 2) identifying the damage locations or neighborhoods. Doebling et al \cite{24} presented a review for damage detection of civil structures using vibration measurement, and Dimarogonas \cite{1} for vibration based methods for detecting cracks in particular. A number of previously proposed methods require a comprehensive dynamic analysis of the structure including a finite element analysis. This analysis is meant to establish a reference to be compared with measured results for damage identification. There are two main issues associated with this approach. 1) It is not always cost-effective to conduct dynamic analysis of a structure. 2) It is always very costly, if not impossible, to obtain valid models of finite element analysis for as-built civil structures. Without physical testing, the validity of these models cannot be confirmed.

During the last two decades, structural dynamic system parameters such as natural frequencies, damping, and mode shapes have been investigated for their possible use in structural damage identification and localization. While changes in natural frequencies may be used to detect the existence of damage, the mode shapes are more important indices for damage location
identification. However, measurements using traditional sensors, such as accelerometers, offer low spatial resolution for mode shapes. Hence, digital imaging as an alternative to traditional measurement sensors is proposed in this research.

Edge detection algorithms at sub-pixel level use information from neighboring pixels to determine the edge location within a pixel (Lan and Mohr [26]). Typical approaches of sub-pixel edge detection are as follows. 1) Fitting a predetermined edge model to that of image data (Aghajan et al [9]). 2) Convoluting image data with predefined filters having sub-pixel accuracy (Huertas et al [3]). 3) Non-linearly interpolating the image data (Jensen and Anastassiou [14]). 4) Equalizing spatial moments of a selected edge model and the image data to solve for the edge parameters in the model (Lyvers et al [8]). However, these previously proposed approaches still do not meet the resolution requirement for civil structural health monitoring and damage detection. The work reported in this paper attempts to address this issue, by developing a new edge detection algorithm at sub-pixel level for high accuracy.

For damage presence detection and their location determination, previous research efforts have been reported on using the difference between the mode shape’s derivatives of intact and damaged states of the structure, which result in spikes at the location of damage (Gentile and Messina [2], Pandey et al [5], Chance et al [12], Yuen [18]). It is inferred that lower modes can be more useful than higher ones in such applications (Wahab and Roeck [17]). However, for using the mode shape’s derivatives, the measured data need to have high spatial resolution and low noise for reliable estimation of damage location. Hence, the difference in mode shapes for the damaged and reference states is to be used here in damage location identification, because mode shapes are less sensitive to noise.

Recently, wavelet transformation has found wider application in decomposing data to localize and zoom in on their local characteristics (Daubechies [10], Mallat [22]). A review is provided by Peng and Chu [27] of available wavelet transformation methods and their application to machine condition monitoring. Liew and Wang [15] found that crack location could be indicated by the variation of some wavelet coefficients along the length of a structural component. Furthermore, Wang and Deng [21] and Quek et al [23] demonstrated the potential of Haar Wavelet transformation for damage detection. The magnitude of Lipschitz exponent was used as an indicator of damage extent. Both numerical and experimental verification were shown utilizing continuous Mexican Hat Wavelet transformation having two vanishing moments for determining the Lipschitz exponent. It was concluded that the first mode shape is more useful than higher ones in the exponent estimation and thus in detecting damage. Lu and Hsu [13] presented a wavelet transform-based method for the detection of structural damage, by comparison of the discrete wavelet transforms of the signals before and after damage in the spatial domain.

THEORETICAL DEVELOPMENT

A. Sub-pixel Edge Detection

In digital images, traditionally an edge is defined as the location of sharp change in gray value. On the other hand, sudden changes of gray level as step function are rarely seen in real life. Instead, the change usually occurs over several neighboring pixels. Depending on the characteristics of the edge, several models have been proposed, including step edges, roof edges, and ramp edges. Edges convey information on the images objects and scene content, which is used extensively in visual recognition. Thus, edge detection is important for feature detection, segmentation, and motion analysis. To realistically model edges, it is critical to understand the process of image formation. This process involves with sub-pixel edge detection as discussed in length and can be found in the paper [25]. Two dimensional light intensity variation model around edge is modeled as:

\[
W(x, y) = h + k \Phi_a \left( y - (Px^2 + Qx + R) \right)
\]

(1)

where,

\[
\begin{align*}
2P & = \text{Edge's curvature at } x = 0 \\
R & = \text{Edge's y-interception (edge location at } x = 0) \\
Q & = \text{Edge's slope at } x = 0 \\
a & = \text{Blurring factor of edge} \\
h & = \text{Gray value of background} \\
k & = \text{Difference of gray value between dark and bright area (contrast)} \\
\Phi_a(\cdot) & = \text{Cumulative Gaussian probability function}
\end{align*}
\]

Accordingly, the gray value \( G(i, j) \) is modeled as the result of photon accumulation within the pixel of image sensor as:

\[
G(i, j) = \int_{i-0.5}^{i+0.5} \int_{j-0.5}^{j+0.5} W(x, y) \, dx \, dy \\
\quad (-1 \leq i \leq I, -J \leq j \leq J)
\]

(2)

Where \( i \) and \( j \) are identification indices of pixels.
In order to solve for the six parameters to identify the edge location, slope, and curvature, a least-square curve-fitting algorithm is developed as an optimization process. An error function is defined as follows to quantify the difference between edge model and the real image gray value data.

\[
\Delta = \sum_{i} \sum_{j} \left( G(i,j) - \tilde{G}(i,j) \right)^2 \quad \text{for } -I \leq i \leq I \& -J \leq j \leq J
\]

where \( G(i,j) \) is the gray value according to the edge model (Eq [2]) with the six parameters to be identified and \( \tilde{G}(i,j) \) is the gray level in the image. Detailed parameters extraction process is discussed in author’s earlier work [25].

**B. Time Series Extraction From Video Images**

Displacement time series extraction from sequence of high-resolution digital video images are as follows:

1. Acquire images of scale object (with known dimensions) along with the object whose motion is to be measured. Extract edge information (location, slope, and curvature) in the obtained images for the scale object and determine the scaling factor (in length per pixel).
2. Acquire baseline images of the interested object not in motion, and extract edge information from them.
3. Take images when the interested object is in motion, and extract edge information from the motion images.
4. Find the object’s relative motion by subtracting the baseline edge information (Step 2) from the motion edge information (Step 3). Repeat this step for all images.
5. Multiply the scaling factor to result of Step 4 to obtain motion time series in proper dimensional unit.

**C. Mode Shape Difference Function**

The mode shapes of a structure describe the relative motion relationship of various points of the structure. When damage occurs at one or more of these points, this relative relationship will change. This change is used here for damage detection, focusing on the difference of the mode shapes before and after structural damage.

For ease of demonstration, a simply supported prismatic beam is considered here, using its intact and damaged states. It is assumed that the beam’s mass remains constant after damage and only the structure’s stiffness is changed due to the damage. Let \( m \) be the constant mass. The flexural stiffness of the beam is a constant \( EI_0 \) when the beam is intact, and \( EI_0(x) \) when it has a crack. The stiffness of the damaged beam is modeled as follows, including a crack:

\[
EI(x) = EI_0(1 - C \left[ \frac{x-a}{\sigma} \right]^2)
\]

where \( \alpha \), \( C \), and \( \sigma \) are respectively the crack’s location, depth ratio, and characteristic width, and \( x \) is measured from the left support. \( E \) and \( I_0 \) are the Young's modulus and the intact moment of inertia of the beam. A schematic diagram of Eq. (4) is shown in Fig [1] for a damaged beam. The purpose of using the continuous function in Eq. (4) to model a supposedly discontinuous crack is to allow a closed form solution. For very small cracks, the solution may be used with the characteristic width approaching to zero. The equation of free vibration motion for the beam is written as follows considering only the flexural effect:

\[
\frac{d^2}{dt^2} \left[ EI(x) \frac{d^2v(x,t)}{dx^2} \right] + m \frac{d^2v(x,t)}{dt^2} = 0
\]

where \( v(x,t) \) is the displacement of the beam perpendicular to its axis, at location \( x \) for time \( t \). Separating the temporal and spatial variables as follows can help readily solve this equation:

\[
v(x,t) = \phi(x)Y(t)
\]

where \( Y(t) \) is a function of time, and \( \phi(x) \) is the mode shape function. The latter has the form in Eq. (7) for the intact state and Eq. (8) for the two segments of the damaged beam separated by the crack, respectively:

\[
\phi(x) = A_1 \cos \lambda x + A_2 \cosh \lambda x + A_3 \sin \lambda x + A_4 \sinh \lambda x
\]
\[ \phi_1(x) = A_{11} \cos \lambda x + A_{12} \cosh \lambda x + A_{13} \sin \lambda x + A_{14} \sinh \lambda x \quad \text{for} \quad 0 \leq x \leq \alpha \]
\[ \phi_2(x) = A_{21} \cos \lambda x + A_{22} \cosh \lambda x + A_{23} \sin \lambda x + A_{24} \sinh \lambda x \quad \text{for} \quad \alpha \leq x \leq L \]

(8)

where \( L \) is length of the beam. The mode shapes are required to satisfy the following boundary conditions for the two states respectively,

\[ \phi(0) = 0; \quad \phi(L) = 0; \quad \frac{\partial^2 \phi(x)}{\partial x^2} \bigg|_{x=0} = 0; \quad \frac{\partial^2 \phi(x)}{\partial x^2} \bigg|_{x=L} = 0 \]

(9)

\[ \phi_1(0) = 0; \quad \phi_2(L) = 0; \quad \frac{\partial^2 \phi_1(x)}{\partial x^2} \bigg|_{x=0} = 0; \quad \frac{\partial^2 \phi_1(x)}{\partial x^2} \bigg|_{x=L} = 0; \]
\[ \phi_1(\alpha) - \phi_2(\alpha) = 0; \quad \frac{\partial^3 \phi_1(x)}{\partial x^3} \bigg|_{x=\alpha} = 0; \quad \frac{\partial^3 \phi_2(x)}{\partial x^3} \bigg|_{x=\alpha} = 0; \]

(10)

where \( \lambda^4 = \frac{\omega^2 - \frac{m}{EI}}{2} \), with \( \omega^2 \) being the natural frequency of the beam. Coefficients \( A_{ij} \) and \( A_{ij} \) (\( i = 1, \ldots, 4 \)) in Eq. (8) can be solved using the boundary conditions in Eq. (10). This solution process results in a set of eight homogeneous linear algebraic equations for the eight unknown coefficients. Subsequently, the following natural frequency equation is obtained to have non-trivial solutions to the homogenous equations:

\[ \sin \alpha \lambda L \sinh \lambda L + \frac{\sigma CL\lambda}{2(1-C)} \left( \sin \alpha \lambda \sin(\alpha - L) \lambda \sinh \lambda L - \sin L \lambda \sin \alpha \lambda \sinh(\alpha - L) \lambda \right) = 0 \]

(11)

As shown, the natural frequencies are functions of characteristic crack width \( \sigma \), crack location \( \alpha \), and crack depth ratio \( C \). Furthermore, from Eqs. (8), (10), and (11) mode shape of cracked beam is obtained as:

\[ \phi_1(x) = A_{11} \cos \lambda x + \frac{\sin \lambda L \sinh \lambda (L - \alpha) \sinh \lambda x}{\sin \lambda (L - \alpha) \sinh \lambda L} \quad \text{for} \quad 0 \leq x \leq \alpha \]
\[ \phi_2(x) = A_{11} \sin \lambda L \left[ \cosh \lambda (\alpha - L - x) - \cos \lambda (\alpha - L + x) \right] + \sinh \lambda L \left[ \cos \lambda (\alpha - L + x) - \cos \lambda (\alpha + L - x) \right] \quad \text{for} \quad \alpha \leq x \leq L \]

(12)

Once the mode shapes for both the intact and damaged beams are known, their difference can be readily calculated as:
This mode shape difference function has a maximum magnitude at the location of damage or discontinuity. A numerical example of this function is shown in Fig [2] for the first mode of a 100 cm long beam having \( C = 10\% \) at the locations indicated.

### D. Wavelet Transformation and Its Application for Damage Detection

The classical Fourier transformation portrays a signal record as superposition of infinite sinusoidal waveforms of assorted frequencies representing an orthogonal basis. Due to the fixed frequency basis used, the Fourier decomposition is suitable for signal records having relatively stationary frequency characteristics throughout their entire length. On the other hand, the wavelet transformation uses special sets of basis that are localized both in the original (e.g., the time) and the transformed (e.g., the frequency) domains (Louis et al [4], Chui et al [6], Daubechies [10], Debnath [16], Wojtaszczyk [20]). Hence, the wavelet transformation is more suitable for analyzing non-stationery signal records, such as those with discontinuities or sudden (i.e., local) changes. The mode shape difference function defined in Eq. (13) is seen to have such local changes at the damage locations. They can be more effectively identified using wavelet transformation.

Continuous wavelet transformation of any square integrable function \( f(t) \) with respect to mother wavelet \( \psi(t) \) is given by:

\[
W(s,c) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{|s|}} \psi^{*} \left( \frac{t-c}{s} \right) dt
\] (14)

where \( s \), scale and \( c \), translation parameters are real and \( \psi^{*}(.) \) denotes conjugate of mother wavelet function. Furthermore, orthonormal basis for wavelet transformation can be expressed as:

\[
\psi_{s,c}(t) = \frac{1}{\sqrt{|s|}} \psi \left( \frac{t-c}{s} \right)
\] (15)

Hence, from Eq. (14) and (15)

\[
W(s,c) = \int_{-\infty}^{\infty} f(t) \psi^{*}_{s,c}(t) dt
\] (16)

where \( \int_{-\infty}^{\infty} |\psi(t)| dt = 0 \), and \( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\psi(t,\omega)|^2 dt<\infty \) are the properties satisfied by mother wavelet function, latter expression represents that the energy of mother wavelet function is confined in finite duration. The frequency content of the mother wavelet as well as \( W(s,c) \), wavelet coefficients, are given as:

\[
\psi(\omega) = \int_{-\infty}^{\infty} \psi(t) e^{-j\omega t} dt; \quad W(s,c) = \int_{-\infty}^{\infty} W(s,c) e^{-j\omega c} dc
\] (17)

Eqs. (14) to (17) for the function in time domain can be readily extended for function in spatial domain to represent their respective wavelet transformation, and the frequency content with in the function itself and that of transformed wavelet coefficients at particular scale \( s \). Hence, wavelet transformation in spatial domain for function \( f(x) \) is represented as:

\[
W(s,c) = \int_{0}^{L} f(x) \frac{1}{\sqrt{|s|}} \psi^{*} \left( \frac{x-c}{s} \right) dx; \quad \psi_{s,c}(x) = \frac{1}{\sqrt{|s|}} \psi \left( \frac{x-c}{s} \right) \quad \text{for} \quad 0 < x < L
\] (18)

\[
W(s,c) = \int_{0}^{L} f(x) \psi^{*}_{s,c}(x) dx
\] (19)

For detecting local changes in spatial function, \( f(x) \), it is important to choose an appropriate mother wavelet function \( \psi \) that is able to portray the changes in continuity of function. In this study, second order complex Gaussian wavelet function is choosen as mother wavelet function and is given as:

\[
f(x) = \phi_1(x) - \phi(x) \quad \text{for} \quad 0 \leq x \leq \alpha
\] 

\[
f(x) = \phi_2(x) - \phi(x) \quad \text{for} \quad \alpha \leq x \leq L
\] (13)
\[ (20) \]

\[ \psi_{comp,G}(x) = D_n e^{\frac{-i\pi n^2}{2\pi}} \quad ; \quad \psi_{comp,G}(x) = \frac{1}{\sqrt{3}} \left( \frac{2}{\pi} \right)^{\frac{1}{4}} (4x^2 + 4ix - 3)e^{-\frac{-x^2}{2}} / 4 \]

where \( i = \sqrt{-1}, \) \( n \) indicates the order of the wavelet, and \( D_n \) is a constant to make the second norm (\( L_2 \)) of the \( n \)th derivative of the wavelet function unity. Let \( \omega \) be the radian/length (spatial frequency) of wavelet function. Then the mother wavelet in spatial domain can be readily written in frequency domain as:

\[ (21) \]

\[ \psi_{comp,G}(\omega) = \frac{(2\pi)^{\frac{1}{4}}}{\sqrt{3}} \omega^{2} e^{-\omega^{2}(2+\omega)^{\frac{1}{4}}} \]

Furthermore, spatial resolution (spatial spread) and frequency resolution (frequency spread) of mother wavelet are represented as:

\[ (22) \]

\[ \Delta_{s,c}^2 = \frac{s^2 \Delta_s^2}{s^2} ; \Delta_{w,c}^2 = \frac{1}{s^2} \Delta_w^2 \]

where

\[ (23) \]

\[ \Delta_s^2 = \frac{\int \{ (x - \langle x \rangle)^2 \} \psi(x)^2 \, dx}{\int \psi(x)^2 \, dx} ; \quad \Delta_{w,c} = \frac{\int \omega \psi(\omega)^2 \, d\omega}{\int \psi(\omega)^2 \, d\omega} \]

From above relations it is clear that wavelets have good frequency resolution for larger values of scale \( s \) and good spatial resolution for smaller values of scale \( s \).

Furthermore, from Eqs. (14) and (16) it is seen that the continuous wavelet transform at \( (s,c) \) contains information from the location interval \( [c + s \langle x \rangle - s\Delta \langle \omega \rangle + s\Delta \langle \omega \rangle] \) and frequency interval \( [s \Delta \langle \omega \rangle / s, (s \Delta \langle \omega \rangle / s) + s \Delta \langle \omega \rangle / s] \). This frequency interval at particular scale is referred as the frequency bandwidth at that scale.

![Fig. 3 Energy Variation With Respect to Effective Window Size](image1)

![Fig. 4 Edge Effect Window for Finite Length Function](image2)

Moreover, when wavelet transform of finite length signal \( f(x) \) is computed, there is edge effect arising at the function boundary, due to the fact that at particular scale \( s \), half of analyzing window extend in the past location and half extend in future location. Hence, certain data lengths at the boundary cannot represent the total energy of the analyzing wavelet function. Thus, considering the energy variation of mother wavelet function with respect to effective window size at particular scale \( s \), effective window size for analysis can be established with respect to the energy content. Knowing the effective window size for analysis, final result from the affected part is omitted from final result interpretation. Fig. [3] shows the energy variation of second order complex Gaussian mother wavelet with respect to effective window size. Hence from there, for example, for function \( f(x) \) having 1131 data points in spatial domain, for scales \( s \) varying from 1-512, useful wavelet coefficients for result interpretation is shown in Fig [4]. Interpretation of Fig [4] is that if the wavelet coefficients \( W(s,c) \) (Eq. (19)) lies inside the curve, there is no edge effect otherwise those coefficients should not be used for further analysis.
While employing wavelet transformation for analyzing spatial signal $f(x)$ for its discontinuity, care should be taken for the presence of noise in the signal. If the signal is free from noise, wavelet coefficients for scale $s = 1$ are able to portray the discontinuity with sudden spike at such location. However, if there is the presence of noise then there will be such spike everywhere due to the noise. Hence, wavelet coefficients for certain lower scales should be omitted from further analysis procedure to mitigate the false identification of discontinuity arising from noise. This process is purely data dependent and will be discussed in experimental data analysis section.

**ANALYTICAL AND EXPERIMENTAL RESULTS**

Complex continuous wavelet transformation of analytical mode shape difference function discussed in Fig [2], ($C = 10\%$, $\sigma = 0.001$, and $\alpha = 30$ cm), is obtained using second order complex Gaussian wavelet as mother wavelet function and is shown in Fig [5]. From the figure, it can be seen at the damage location that there is sudden change in coefficients for angle and the modulus of the coefficients have maximum amplitude. When scale becomes larger, there is edge effect as discussed above, hence those coefficients are not effective in determining the damage location as seen in Fig [5(i), (j)]. Since the analytical signal donot have any noise in the data, coefficients for scales ($s = 200$) can be used for damage diagnostic purpose. However, experimental data without noise is rare, hence the lowest level of scale one can use for damage diagnostic depends entirely on the noise characteristic in experimental data.

The proposed concept of structural damage detection using high-resolution images was experimented in the laboratory. A simply supported prismatic steel beam was used as a structural model, to provide vibration measurement data for its intact and damaged states. The beam is 15.24 cm wide, 0.635 cm thick, and 110.5 cm long. Two progressive damage states were realized by saw cut with depth of 2.5 cm and 2.2 cm ($C = 16.40\%$ and $14.40\%$ in Eq. (7)), respectively at locations $\alpha = 34.3$ cm and 74.0 cm from the left support of the beam as defined in Fig [1]. The two damage states are referred to as D1 and D2 respectively. Note that the progressive damage term defined earlier means that D2 is additional damage inflicted in addition to D1, since the former damage state was not revoked when the latter damage state was introduced. The beam’s free vibration was recorded using a video camera, excited by imposing an initial displacement at 45.0 cm from the left support. The digital motion images were captured using a Nac Memrecam fx K3 digital video camera. The camera has a CMOS sensor with a 1280 x 1024 resolution and a 10-bit A/D converter. The camera is equipped with a 1.3 GB onboard memory and its frame rate ranges from 100 to 2000 frames per second.

The test setup is shown in Fig [6]. The procedure discussed in Section B of theoretical development section is followed for image acquisition and processing. First, the baseline (not in motion) images along with the scale object were taken. In the vertical direction, intended to be the direction of measured vibration displacement, a scale object (with two white marks) was used to allow acquisition of the scaling factor. These images were used to determine the scaling factor as 0.0714502 cm per pixel. Then, the intact beam was excited and its free vibration motion was captured at a rate of 1000 frames per second for a one-second duration. These images were processed using a personal computer for edge detection and displacement identification as presented earlier. The obtained time series of the beam in motion was used to extract dynamic properties for the intact beam. Then the beam was damaged using saw cut to its backside, so that the damage was not seen on the imaged front edge in edge identification. The same procedure was followed to obtain the time series of the beam with damage. A total of sixteen sets of video were recorded each consisting of 1001 images. Eight sets were for the intact state and four for each of the two damage states.

The images were analyzed using the proposed sub-pixel edge identification procedure using a window size of $I = 50$ and $J = 7$ pixels for analyzing each available data point (pixel) along the beam length. The edge location, slope, and curvature of the beam were determined. Each image took about five minutes of processing time using a PIV IBM compatible PC with a 3.06
GHz microprocessor and a 1024 MB onboard memory. Fig [7] shows a typical set of vibration time series for the intact beam. There are 1001 data points (i.e., images) in the time domain and 1131 data points in the spatial domain along the beam length. This time series shows that high-resolution video cameras are able to provide spatially intensive measurements, more cost-effectively than traditional sensors, such as dial gages, linear variable differential transformers (LVDT), and accelerometers.

The time series data were then processed using the MEscope Ves software [19] to obtain the natural frequencies, modal damping ratios, and mode shapes. Table 1 shows the first natural frequency and associated damping ratio for the intact and damaged states. Fig [8] displays the mode shape difference function for the two damage cases, referring to different states as given in the legend.

Table 1. First Natural Frequency and Damping of Intact and Damaged Beams

<table>
<thead>
<tr>
<th>ID</th>
<th>Undamaged State</th>
<th>Damaged State</th>
<th>Damage State</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. (Hz)</td>
<td>Damping (%)</td>
<td>Freq. (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>11.738</td>
<td>0.32</td>
<td>11.575</td>
</tr>
<tr>
<td>2</td>
<td>11.755</td>
<td>0.41</td>
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</table>

The mode shape difference functions were then subjected to the second order complex Gaussian wavelet transformation defined in Eq. (19) with $n = 2$, with scale $s$ ranging from 1 to 512 pixels. The results are shown in Figs [9] to [11]. Smaller values of scale correspond to higher spatial frequencies (which are able to portray a higher number of discontinuity in the spatial domain along the beam length). In contrast, larger values of scale $s$ correspond to lower spatial frequencies, useful for identifying a smaller number of changes or damage. Nevertheless, when the scale becomes large the edge influence also increases, as discussed earlier. The effective window for the mother wavelet function in Eq. (23) is about $\pm 1.37s$ pixels for representing 95% of the energy as shown in Fig [3]. Hence, depending on the location of damage, edge effect influence will be different as shown in Fig [4]. For example, if the damage is with in 30-40 cm range, then the maximum level of scale one can use without edge influence, from Fig [4], is $s = 150$. However, if the damage location is somewhere around 45-55 cm range, then maximum level of scale one can use become $s = 275$.

Fig [9] shows result for damage state D1 ($C = 16.40\%$ at $\alpha = 34.3$ cm). While analyzing the wavelet coefficients for scales $s \approx 132-150$, damage presence and their location is detected at the vicinity of 34 cm as shown in Fig [9 (e) and (f)]. Similarly, Fig [10] shows result for damage state D2 ($C = 16.40\%$ at $\alpha = 34.3$ cm and $C = 14.4\%$ at $\alpha = 74.0$ cm) referring to the intact state. The damage locations are identified at 30 cm and 78 cm where modulus maximum and phase angle sign change occur.
Fig [11] shows the result for state D2 Vs D1. Fig [11(e) to (h)] shows the additional damage beyond state D1 at 78.75 cm, where both modulus maximum and phase angle sign change are observed. From above figures, one can identify the damage location using judgment based on the matching local maxima of modulus of wavelet coefficients along with sudden phase angle change.

CONCLUSIONS

Using spatially intensive data, mode shape difference function is obtained. Complex continuous wavelet transformation of mode shape difference function for different scales results in coefficients, hence analyzing the variation of these coefficients along the beam length, damage presence and their location are identified for both analytical and experimental data sets. Hence damage presence detection and their location identification is successfully achieved.
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