IDENTIFICATION OF MATERIAL DAMPING FROM BROADBAND MODAL ANALYSIS EXPERIMENTS

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ABSTRACT

The stress-strain relationship of linear visco-elastic materials is characterized by a complex-valued, frequency dependent elastic modulus \(E(j\omega)\) (Young’s modulus). Using system identification techniques it is shown in this paper how \(E(j\omega)\) can be measured accurately in a broad frequency band from forced flexural (transverse) and longitudinal vibration experiments on a beam under free-free boundary conditions. The advantages of the proposed method are (i) it takes into account the disturbing noise and the nonlinear distortions, (ii) \(E(j\omega)\) is delivered with an uncertainty bound, (iii) the low sensitivity to non-idealities of the experimental set up, and (iv) the ability to measure lowly damped materials. The approach is illustrated on plexiglass and copper.

1. INTRODUCTION

The complex modulus \(E(j\omega)\) is a widely used and powerful tool for describing the linear dynamic elastic and damping properties of visco-elastic materials. For example, knowledge of \(E(j\omega)\) in the audio frequency range is needed by acousticians when developing and characterizing new materials for noise and vibration control. The idea of measuring the complex moduli (Young’s, shear, ) from vibration tests (flexural, longitudinal, and torsional) is not new and dates from the early sixties (see the references in [1]). From those early days till now a continuing attention has been paid to this measurement problem in the literature: see [1-6] and the references therein for the longitudinal vibration tests, and [1], [7-10] and the references therein for the flexural vibration tests. The longitudinal (wave) experiments assume that the material is homogeneous and isotropic [8-10], while the flexural (transverse or bending) experiments can also handle layered structures [8, 10], and orthotropic materials [11]. In the wave experiments the beam is sometimes loaded by an end mass (e.g. for measuring rubber, plastic foams and mineral wools) [3, 4], while in the bending experiments the beam is often clamped at one [8-10] or two [7] sides. These boundary conditions are always non-ideal and should be avoided especially when measuring stiff and lowly damped materials.

According to the way Young’s modulus is obtained from the measured frequency response function (FRF), the different approaches can be split into two classes: the first class uses the poles (resonant frequencies) of the FRF and the partial differential equation (PDE) model only [1, 4, 8, 10, 11], while the second class uses the whole FRF and PDE model [5, 6, 9]. It is clear that the second class will be much more sensitive to non-idealities of the experimental set up such as misalignment creating other vibration modes than those described by the PDE model. These errors are easily eliminated in the first class by selecting the appropriate resonances (see also Section 3). The advantage of the second class is the much higher frequency resolution of the \(E(j\omega)\) measurement. On the other hand, except for highly damped materials, the resonant frequencies (poles) of the FRF contain most information about the material damping. In the first class one can also distinguish between the methods that calculate the poles (real and imaginary parts) of the PDE analytically [1, 4, 10], and those that obtain the resonant frequencies (imaginary part poles only) by numerical integration of the PDE [8, 11].

Except [6], none of the existing methods take into account the disturbing noise when estimating Young’s modulus, and no uncertainty bound is given. In [6], Young’s modulus is determined using strain measurements from longitudinal impulse response experiments (hammer and air gun) on a beam under free-free boundary conditions. Uncertainty bounds on \(E(j\omega)\) are calculated assuming that the disturbing noise is spatially and temporarily white.
It is well known that visco-elastic materials behave linearly for sufficiently small strain values. The nonlinear behaviour can be detected by repeating the measurements at different excitation levels \[12\] and verifying whether the resonance frequencies and damping coefficients vary significantly with the excitation amplitude (note that uncertainty bounds are needed for this purpose). However, none of the existing methods estimates the level of the nonlinear distortions on the FRF measurement and takes this information into account in the calculation of Young’s modulus.

This paper uses frequency domain system identification techniques to measure the elastic modulus \( E(j\omega) \) of homogenous isotropic materials in a broad frequency band from forced flexural and longitudinal vibration experiments on a beam under free-free boundary conditions. Special designed broadband periodic excitation signals are used for the measurement procedure. It allows to calculate simultaneously the frequency response function (FRF), the noise level, and the level of the nonlinear distortions. This information is used to estimate the poles (and their uncertainty) of the measured FRF. The final result is Young’s modulus together with its uncertainty due to the disturbing noise and/or the nonlinear distortions. The presented method takes into account the disturbing noise on the output and the input, the noise colouring, the correlation between the input and output disturbances, and the non-linear distortions.

The outline of the paper is as follows. First the theoretical resonances of the longitudinal (Love model) and the flexural (Euler-Bernoulli and Timoshenko models) vibration experiments are calculated (Section 2). Next the identification procedure for estimating Young’s modulus is explained step by step and illustrated on a wave experiment (Section 3). Finally some practical aspects of the experimental setup are discussed and a comparison between the wave and the transverse experiments is made (Section 4).

The full version of the paper including the experiment design and other experimental results (PVC and brass) can be found as a technical report at: http://elecftp.vub.ac.be/Papers/rpintelon.htm.

2. THEORETICAL RESONANCE FREQUENCIES

2.1. Longitudinal (wave) vibrations
Consider a longitudinal vibration experiment on a uniform (along its length) beam composed of a linear, homogeneous and isotropic visco-elastic material (see Fig. 1). According to the Love model \[2\] the poles \( s_k \) of the force \( u(t) \) to longitudinal displacement \( y(t, x) \) transfer function are related to Young’s modulus \( E \) by

\[
E(s_k) = -\rho \left( \frac{L s_k}{k \pi} \right)^2 \left[ 1 + \left( \frac{k \pi v(s_k) J}{L} \right)^2 \right] \tag{1}
\]

with \( k = \pm 1, \pm 2, \ldots, L \) is the length of the beam, \( \rho \) the density of the material, \( v(s_k) \) Poisson’s coefficient, and \( J \) the radius of gyration of the cross section of the beam about the \( x \)-axis \( (J = r / \sqrt{2} \) for a circle with radius \( r \); and \( J = (h_y^2 + h_z^2) / 12 \) for a rectangle with sides \( h_y \) and \( h_z \) [13]).

2.2. Flexural (transverse or bending) vibrations
Consider a flexural vibration experiment on a uniform (along its length) beam composed of a linear, homogeneous and isotropic visco-elastic material without axial loads (see Fig. 2). According to the Euler-Bernoulli model \[14\] the poles of the force \( u(t) \) to the transverse displacement \( y(t, x) \) transfer function are related to Young’s modulus \( E \) by
\[ E(s_k) = -\frac{\rho A L^4 s_k^2}{I \zeta_k^2} \]  

with \( k = \pm 1, \pm 2, \ldots \). \( I \) is the moment of inertia of the cross-section of the beam about the \( z \)-axis (axis perpendicular to the \( x \)- and \( y \)-axes); \( I = \pi r^4/4 \) for a circle with radius \( r \), and \( I = h_z h_y^3/12 \) for a rectangle with sides \( h_z \) and \( h_y \) in the \( z \)- and \( y \)-directions respectively [13]), and \( \zeta_k \) the wave number of the \( k \)th resonance frequency

\[ \cosh(\zeta_k) \cos(\zeta_k) = 1 \]  

Numerical values of \( \zeta_k \) are tabulated in the literature (see, for example, Table 6.4 of [14]): \( \zeta_1 = 4.730041 \), \( \zeta_2 = 7.853205 \), \( \zeta_3 = 10.995608 \), and \( \zeta_k = (2k + 1)\pi/2 \) for \( k \geq 4 \) within a relative error smaller than \( 1.2 \times 10^{-7} \).

The Euler-Bernoulli theory leading to (2) ignores the effects of shear deformation and rotary inertia, and is accurate for thin beams only [14]. In practice (2) can be used to model the few resonance frequencies corresponding to the simple mode shapes. Timoshenko’s theory [14] considers the effects of shear deformation and rotary inertia, giving the following implicit relationship between the poles \( s_k \) and Young’s modulus \( E \)

\[ \cosh(b_1(s_k)L)\cos(b_2(s_k)L) - 1 + \frac{b_2^2(s_k) - b_1^2(s_k)}{2b_1(s_k)b_2(s_k)} \sinh(b_1(s_k)L)\sin(b_2(s_k)L) = 0 \]  

where

\[ b_i^2(s) = (-1)^i c(s) + \sqrt{c_1^2(s) + a(s)} \quad \text{with} \quad c(s) = \frac{\rho}{2E(s)}(1 + \gamma(s))s^2 \quad \text{and} \quad a(s) = -\left( \frac{\rho A}{E(s)}s^2 + \frac{\rho^2 \gamma(s)}{E^2(s)}s^4 \right) \]  

\( A \) is the cross section area of the beam, and \( \gamma(s) \) depends on the shape of the cross section and Poisson’s coefficient \( \nu(s) \) (\( \gamma(s) = (12 + 11 \nu(s))/5 \) for a rectangle, and \( \gamma(s) = (7 + 6 \nu(s))/3 \) for a circle [15]).

2.3. Discussion

Equations (1), (2) and (4) establish a relationship between the poles \( s_k \) of the longitudinal and flexural vibration experiments and Young’s modulus \( E(s_k) \) when the beam \( (L, A, I, J) \) and the material \( (\rho, \nu(s)) \) properties are known. Fortunately, the cross section dimensions of the beam can always be chosen such that the terms in (1) and (4) depending on Poisson’s coefficient \( \nu(s) \) are correction terms. Hence, only a rough guess of \( \nu(s) \) is required. This is the basic idea used for identifying Young’s modulus \( E(s) \) in Section 3.

3. THE IDENTIFICATION PROCEDURE

The procedure consists of the follow main steps. (i) Choice of the broadband periodic excitation signals and measurement of the frequency response function (result: FRF with its uncertainty). (ii) Approximation of the measured FRF by a rational form in the Laplace variable \( s \) (result: poles and their uncertainty). (iii) Selection of the poles of the rational form corresponding to the longitudinal or flexural vibration modes and calculation of Young’s modulus \( E(s_k) \) via eq. (1), (2) or (4) (result: Young’s modulus \( E(s_k) \) and its uncertainty at the values of the poles \( s_k \)). (iv) Approximation of \( E(s_k) \) by a rational form in the Laplace variable \( s \) (result: Young’s modulus \( E(j \omega) \) and its uncertainty in the frequency band of interest). These four main steps are explained in detail in the sequel of this section, and each step is illustrated on the longitudinal vibrations of a plexiglass beam.

3.1. Measurement of the frequency response function

The frequency response function (FRF) of the system is measured using random phase multisines [16, 17]. These are periodic signals consisting of the sum of harmonically related sine waves with user defined amplitudes and random phases. The measurement procedure of [18] is followed to estimate the FRF and its uncertainty. It consists of the following basic steps:

1. Choose the amplitude spectrum and the frequency resolution of the random phase multisine.
2. Make a random choice of the phases of the random phase multisine, and calculate the corresponding time signal.
3. Apply the excitation to the system and measure \( P \geq 2 \) periods of the steady state response \( u(t), y(t) \).
4. Repeat steps 2 and 3 \( M \geq 6 \) times.
5 Calculate the DFT spectra of the input $u(t)$, and output $y(t)$ signals for each period of each experiment at the excited DFT frequencies.

From these $M \times P$ sets of noisy input/output spectra, one can calculate for each experiment the average FRF $\hat{G}_{[m]}$ over the $P$ periods and its sample variance $\hat{\sigma}_{G_{[m]}}^2$ ($m = 1, \ldots, M$). An additional averaging over $m$ gives the final FRF $\hat{G}$ of the whole measurement procedure

$$
\hat{G} = \frac{1}{M} \sum_{m=1}^{M} \hat{G}_{[m]}, \quad \hat{\sigma}_{G}^2 = \frac{1}{M(M-1)} \sum_{m=1}^{M} \left[ \hat{G}_{[m]} - \hat{G} \right]^2
$$

(6)

together with its sample variance $\hat{\sigma}_{G}^2$. If the system is linear, then $\hat{\sigma}_{G}^2$ should be equal to the mean value of $\hat{\sigma}_{G_{[m]}}^2$ divided by $M$

$$
\hat{\sigma}_{G}^2 = \frac{1}{M^2} \sum_{m=1}^{M} \hat{\sigma}_{G_{[m]}}^2
$$

(7)

(the extra factor $M$ copes with the averaging over the $M$ experiments). Indeed, in the linear case the variability of the FRF measurement over the $M$ experiments with different excitation signals can be explained by the disturbing input/output noise only. In that case $\hat{\sigma}_{G}^2$ (6) and $\hat{\sigma}_{G_{[m]}}^2$ (7) are both a measure of the influence of the disturbing input/output noise on the FRF. If $\hat{\sigma}_{G}^2$ (6) is larger than $\hat{\sigma}_{G_{[m]}}^2$ (7), then this is an indication that the systems behaves nonlinearly. Indeed, the difference

$$
\hat{\sigma}_{G_{[m]}}^2 = \hat{\sigma}_{G}^2 - \hat{\sigma}_{G_{[m]}}^2
$$

(8)

quantifies the contribution of the stochastic non-linear distortions to the FRF measurement; while the non-linear bias contributions $G_B$ are bounded by

$$
\sqrt{M} \hat{\sigma}_{G_{[m]}} \leq |G_B| \leq \sqrt{2M} \hat{\sigma}_{G}
$$

(9)

(see [17, 19 and 20] for the details).

The measurement procedure is illustrated in Figures 3 and 4. For each measurement $P = 10$ and $M = 25$. It can be seen that the stochastic nonlinear distortions are mostly dominant, and that the signal-to-noise ratio $|\hat{G}|/\hat{\sigma}_{G}$ of the FRF measurement (6) varies between 40 to 60 dB.

3.2. Approximation of the FRF by a rational form

According to the Mittag-Leffler theorem [21] the infinite dimensional transfer functions shown in Figures 3 and 4 can be approximated arbitrary well by a rational form of finite order in a particular frequency band.

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Fig. 3: Force-to-acceleration FRF (bold black line) and its standard deviation (thin black line: (7) noise errors only, and gray line: (6) noise errors + nonlinear distortions) of the longitudinal vibration experiments. Left: plexiglass ($L = 1.983 \text{ m}$), right: copper ($L = 2.209 \text{ m}$).

Fig. 4: Force-to-velocity FRF (bold black line) and its standard deviation (6) (gray line) of the flexural vibration experiments. Left: plexiglass ($L = 1.372 \text{ m}$), right: copper ($L = 2.209 \text{ m}$).
The parameters of the rational form are found by minimizing the sample maximum likelihood cost function. The result is an estimate together with its covariance matrix. Finally, from and the poles and their uncertainty are calculated (see [17] for the theoretical background and [22] for the software implementation).

Fig. 5 illustrates the procedure on the longitudinal vibration experiment of a plexiglass beam. Using a rational form (10) of order 34, the approximation error is about at the level of the uncertainty of the FRF measurement (between -50 dB and -60 dB). It shows that in a given frequency band the infinite dimensional transfer function and the nonlinear bias contribution in (9) can be approximated very well by a rational form. Note that seventeen poles are needed to model the FRF accurately, which is much larger than the number of resonance peaks (seven) in the FRF. The additional ten poles model the nearest poles above 4 kHz, the in band flexural vibration modes due to the non-perfect alignment of the set up, and possibly also G_B.

3.3. Calculation of Young’s modulus at the poles

The first step consists in selecting the poles s_k of G(s, \hat{\theta}) (10) corresponding to the longitudinal or flexural vibration modes. This can easily be done by comparing |s_k|/(2\pi) to the resonance frequencies of the FRF. Next the index k of each observed resonance peak (pole) is determined. This is done by comparing the first few peaks of the measured FRF to the resonance frequencies predicted using eq. (1), (2) or (4), the beam parameters, and a rough guess of Young’s modulus and Poisson’s coefficient. Finally Young’s modulus is calculated at the values of the poles s_k via eq. (1), (2) or (4), where (1) and (4) need a rough guess of \nu(s_k) (\nu = 0.33 for plexiglass and \nu = 0.3 for copper). By an appropriate choice of the beam cross section dimensions, the correction term in (1) depending on \nu(s_k) can often be neglected. Equation (4) is a nonlinear algebraic equation in E(s_k), and is solved via the Newton-Raphson root finding algorithm [23]. As starting value we use the Euler solution (2) or the previous solution E(s_{k-1}) of (4). Uncertainty bounds on E(s_k) are obtained from the uncertainty of the poles through a first order sensitivity analysis of eq. (1), (2) and (4).

Fig. 6 gives the result for the longitudinal vibration experiments. The following standard deviations are found

\begin{align*}
\text{plexiglas: } \text{std}(E(s_k)) &= 1 \times 10^5 \text{ N/m}^2 \\
\text{copper: } \text{std}(E(s_k)) &= 1 \times 10^6 \text{ N/m}^2
\end{align*}

3.4. Modelling of Young’s modulus

In Section 3.3 Young’s modulus E and its variance var(E) are obtained at the value of the poles of the longitudinal or flexural vibration modes. In practise Young’s modulus should be known along the j\omega-axis, which requires an additional modelling step. Based on the spring-dashpot representations of linear visco-elastic materials [24] Young’s modulus is modelled as a rational from E(s, \theta) (10). For metals Young’s modulus is modelled as a complex, frequency independent constant

\begin{equation}
E(s, \theta) = R + Ij
\end{equation}

The parameters \theta in (10) and (12) are found by minimizing the following cost function w.r.t. \theta.
The procedure is illustrated in Fig. 6 on the longitudinal vibration experiments shown in Fig. 3. For plexiglass a rational form (10) with \( n_\varphi = n_h = 2 \) is sufficient to model the elastic modulus. The poles and zeroes of the estimated second order model have multiplicity one, lie on the negative axis, and are alternating. This proves that \( E(\zeta, \theta) \) can be represented by an RC-network [25] or an equivalent spring-dashpot scheme [24]. Although models (10) and (12) are quite accurate (model errors less than 0.05 dB = 5‰ for plexiglass, and less than 0.008 dB = 0.8‰ for copper), the model errors are significantly larger than the uncertainty of \( E(s_k) \) (compare eq. (11) and Fig. 6). It can be due to the non-idealities of the experimental set up and the test specimen (non-homogeneity of the material, non-uniform cross section beam, and/or the fact that the true material behaviour cannot be described perfectly by (10) or (12).

4. EXPERIMENTAL RESULTS

The following test beams are used for the experiments: plexiglass \( (\rho = 1200 \text{ kg/m}^3 \), rectangular cross section with sides \( h_y = 10.3 \text{ mm} \) and \( h_z = 20.0 \text{ mm} \), \( L = 1.372 \text{ m} \) and \( L = 1.983 \text{ m} \)), and copper \( (\rho = 8900 \text{ kg/m}^3 \), rectangular cross section with sides \( h_y = 10.0 \text{ mm} \) and \( h_z = 30.0 \text{ mm} \), \( L = 2.209 \text{ m} \)). The beams are hung by two or three nylon threads perpendicular to the excitation direction. They are excited in the longitudinal or transverse direction by a mini-shaker. A stinger section with sides \( 20.0 \text{ mm} \times 30.0 \text{ mm} \). The beams are hung by two or three nylon threads parallel to the test specimen (distance less than 7 mm). The flexural resonances of the stinger rod cause non-axial forces on the impedance head, and hence disturb the axial force measurement. To reduce these errors the stinger rod is kept as short as possible (12 mm).

The force and acceleration at the excitation point are measured with an impedance head (B&K 8001). These signals are amplified (charge amplifier B&K 2635) and buffered \( (Z_{\text{in}} = 5 \text{ M} \Omega \), \( Z_{\text{out}} = 50 \text{ \Omega} \)) before being applied to the acquisition channels (HPE 1430A, \( Z_{\text{in}} = 50 \text{ \Omega} \)) of the VXI measurement device. The periodic excitation signals are generated by an arbitrary function generator (HPE 1445A, \( Z_{\text{out}} = 50 \text{ \Omega} \)) at a sampling frequency \( f_s = 10 \text{ MHz}/2^{10} \approx 9.77 \text{ kHz} \). The output of the arbitrary function generator is lowpass filtered \( (7\text{th order inverse Chebyshev filter with a cut off frequency of } 4 \text{ kHz}) \) before being applied to the mini-shaker. To reduce the effect of the inductive impedance of the shaker on the generator unit, an 18 \( \Omega / 5 \text{ W} \) resistance is put in series with its input.

The measurement procedure of Section 3.1 is followed using random phase multisines with a flat amplitude spectrum. Only the odd harmonics \( (2k + 1)f_s/N \) in the band \([100 \text{ Hz}, 4 \text{ kHz}] \) are excited, where \( f_s = 10 \text{ MHz}/2^{10} \approx 9.77 \text{ kHz} \) is the sampling frequency and \( N \) the number of points per period. Since the signal does not excite the even harmonics \( 2k f_s/N \), the influence of the even degree nonlinear distortions on the FRF measurement is eliminated [17, 19]. For the longitudinal \( (N = 8192) \) and flexural \( (N = 16384) \) vibration experiments the frequency resolution of the FRF measurement is respectively 2.38 Hz and 1.19 Hz. All measurements were performed at room temperature. Figures 3 and 4 show the measured FRF’s of the longitudinal and flexural vibration experiments. As rough guess for Poisson’s ratio \( \nu(s_k) = 0.33 \) is taken for
plexiglass and $\nu(s_k) = 0.3$ for copper. For these values the correction terms in (1) are smaller than $6\epsilon - 4$ and $3\epsilon - 4$ respectively.

The results of the longitudinal and flexural vibration experiments are compared in Fig. 7. It can be seen that the amplitude of Young’s modulus obtained from the Timoshenko (4) and Love (1) models coincide (errors of less than 0.1 dB = 1%), while that of the Euler model (2) is accurate for the first few (five) values only. The phase of Young’s modulus matches very well for the three models. This can be explained by the fact that Timoshenko’s solution (4) does almost not change the phase of the poles. Note, however, that the differences between the Timoshenko and Love models are larger than the uncertainty of the $E(s_k)$ measurement (compare eq. (11) with Fig. 7). For plexiglass this can be due to the variability of the material properties over the two test beams, and/or a temperature difference between the two experiments; while for the copper beam it can be due to non-isotropy caused by the rolling process (the side walls are stiffer).

To verify the influence of the connection between the impedance head and the beam on the identified material damping, a contact free flexural vibration experiment on the copper beam is performed. The beam is excited using a loudspeaker, the sound pressure is measured with a microphone (prepolarized condenser microphone, B&K 4155), and the velocity is measured at ten positions using a scanning laser Doppler vibrometer (Polytech PSV 300). The poles are identified from the sound-pressure-to-velocity FRF, and Young’s modulus is estimated using Timoshenko’s model (4). Fig. 8 compares the elastic modulus obtained from the contact free experiment ($E_x$) (loudspeaker/laser vibrometer set up) with that obtained with the mini-shaker/impedance head set up. Note the very good agreement between both measurements (amplitude errors of less than 0.02 dB = 2‰, and phase errors of less than 0.01°). The very small increase in the amplitude of $E_x$ w.r.t. $E_x$ (2‰) can mainly be explained by the lower temperature at which the contact free experiment was performed (about 3 °C) and partially by the slightly lower mass of the loudspeaker/laser vibrometer set up (the total mass of the brass screw, the two nuts, and a part of the impedance head is about 5 g). Indeed, both effects result in a slight increase of the observed resonance frequencies, and hence in a small increase of the calculated elastic modulus (see, for example, eq. (2)): about 1‰ for the temperature difference (the temperature sensitivity of Young’s modulus is about $4\epsilon 7$ N/(m² °C) for copper [26]), and 0.8‰ for the mass difference.

5. CONCLUSION

A system identification approach for measuring the elastic modulus of homogeneous visco-elastic materials from longitudinal and flexural vibration experiments has been presented. The final result is a parametric model for Young’s modulus over a broad frequency band together with an uncertainty bound. The uncertainty bound takes into account the disturbing input/output (measurement) noise and the stochastic nonlinear distortions. As such it is a measure for the repeatability of the experiment over the class of random excitation signals with the same power spectrum. It can be used to study the influence (non-idealities) of the measurement set up, the temperature, the nonlinear material behaviour, the variability of the material over different test specimen, the value of Poisson’s coefficient.

Note that the same ideas could be used to measure the shear modulus $G(j\omega)$ from torsional vibration experiments.

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