ABSTRACT: In this paper, the use of the Genetic Algorithm (GA) optimization method for measuring displacement fields with the Digital Image Correlation (DIC) technique is investigated. The conventional kinematic description of deformations within a subset are restricted to constant displacements along with constant first or second order displacement gradients throughout the subset, which effectively acts as a smoothing filter on higher order displacement fields. Using the GA approach, displacements can be described independently, and therefore discontinuously, for each pixel (i.e., pointwise) within the subset. The resulting technique is called Pointwise Digital Image Correlation, and it eliminates the smoothing attributed to the conventional kinematic description. The effects of varying the parameters for the GA optimization method on the optimized correlation function value are determined for this new technique. Bilinear interpolation is used to obtain subpixel resolution of the displacement fields. The effects of different constraint conditions on the admissible displacements for adjacent pixels are also studied. It is shown that an optimum correlation function value, an order of magnitude smaller than obtained with the conventional DIC technique, can be achieved by choosing a proper combination of GA parameters and constraint conditions. However, there are random variations in the associated displacement fields that may be eliminated with a smoothing filter in order to obtain physically meaningful results when measuring discrete material or structural phenomena that produce rapidly varying and/or discontinuous deformations.

I. Introduction
Digital Image Correlation (DIC) is a full-field deformation measurement technique that has become an accepted method for measuring displacement and displacement gradient fields on the surface of objects. It was originally proposed for analyzing two-dimensional deformation using a correlation algorithm consisting of a coarse-fine search method to optimize the correlation function and a bilinear interpolation scheme to obtain subpixel accuracy, which effectively increases the resolution of the digital image for correlation purposes by a factor of 400 or greater [1-4]. The computational speed and subpixel accuracy corresponding to this analysis was later enhanced using the Newton-Raphson optimization method and bicubic spline interpolation, before being extended to the measurement of three-dimensional displacement fields [5,6]. In a further modification of the technique, Vondroux and Knauss, proposed using a least squares correlation function instead of a cross-correlation function in order to develop an approximation for the Hessian matrix, leading to a 25 percent improvement in both speed and convergence robustness for the correlation algorithm. Also, the finite deformation formulation was used to determine the deformation fields associated with large rigid body rotations. Lu and Cary [10] also refined the DIC method through the addition of second-order displacement gradients for measuring deformations when higher order displacement gradients occur. The errors associated with measuring higher-order deformation fields using undermatched shape functions have also been quantified [11].

DIC is performed by correlating the gray-level intensity values for a subset of integer pixel locations from a reference configuration with their corresponding positions in the deformed configuration assuming a kinematic description of the subset's deformation field. In most of the previous work, a zero-order (constant) or first-order (linear) approximation of the deformation field is assumed to calculate the subpixel displacements for each pixel in the subset. This approximation acts as a smoothing filter on the displacements within the subset, and is reasonably accurate when the subset size is small and deformation gradients are small. However, a small subset may not contain enough unique gray-level intensity information to produce an accurate deformation measurement using the correlation algorithm. While increasing the size of the subset results in more unique gray-level intensities, the smoothing effects also increase, reducing the accuracy that results from using a linear approximation. Higher-order deformation fields can be used to enhance the accuracy, but at the expense of computational speed. When discrete material or structural phenomena produce deformations that vary rapidly and/or discontinuously (e.g., cracks, the boundaries of grains or secondary phases, twinning), it becomes a challenge to resolve these deformations using...
any single continuous kinematic description over the subset of interest. The purpose of this investigation is to understand the effects of describing the displacements for each pixel in a subset independently as opposed to the more conventional zero-order or first-order variation. The resulting technique is called Pointwise Digital Image Correlation, and it eliminates the smoothing effects attributed to the conventional kinematic description.

By using a pointwise description of the deformation field, the number of independent variables for which the correlation function must be optimized increases dramatically. A variety of optimization methods have been previously employed for the correlation of digital images, such as coarse-fine search [1-3], Newton-Raphson method of partial differential correction [5], Hessian matrix approximation [8] and Genetic Algorithms (GAs) [12-13]. Of all these methods, only the GA has not yet been applied to the study of deformation fields. GAs are based on evolutionary principles, and are mathematically similar to the coarse-fine search methods. The evolutionary principles are used to restrict the search in the correlation function space, and are more ideally suited than gradient-based methods for determining optimal solutions to problems described by a large number of independent variables. Therefore, GAs were considered the most computationally efficient optimization method to be employed in this investigation for determining the effects of independent kinematic descriptions on the subset deformations measured using the Pointwise DIC technique.

II. Digital Image Correlation Technique

DIC is a pattern recognition technique that uses random patterns of gray or laser speckles on the surface of a specimen to measure deformation fields via mathematical comparison of the digital images before and after the specimen is deformed. A standard CCD video camera is used to obtain the digital images. The gray-level intensity of each pixel is represented by an integer, which for an 8-bit digital image will range from 0 to 255, where 0 represents black and 255 represents white. An example of a digital image for a random speckle pattern is shown in Figure 1.

After obtaining the digital images, it then becomes possible to compare subsets of pixels from the undeformed image with subsets from the deformed image. The cross-correlation function $S(x_{i,j},y_{i,j},u_{i,j},v_{i,j})$ is commonly used to determine how well the subsets match. This function is given as follows:

$$S(x_{i,j},y_{i,j},u_{i,j},v_{i,j}) = 1 - \frac{\sum [F(x_{i,j},y_{i,j}) * G(x^*,y^*)]}{\left(\sum (x_{i,j},y_{i,j})^2 \sum (x^*,y^*)^2\right)^{1/2}}$$  \hspace{1cm} (1)

where $F(x_{i,j},y_{i,j})$ is the gray-level intensity value for subset at coordinate $(x_{i,j}, y_{i,j})$ in the reference image, $G(x^*,y^*)$ is the gray-level intensity value at coordinate $(x^*, y^*)$ in the deformed image. The subscripts $(i,j)$ are used to describe the relative locations of the pixel within each subset. The coordinates $(x_{i,j}, y_{i,j})$ and $(x^*, y^*)$ can be related by the displacements $(u_{i,j}, v_{i,j})$ for pixel $(i,j)$ through the following zero-order kinematic description:

$$x^*_{i,j} = x_{i,j} + u_{i,j}$$
$$y^*_{i,j} = y_{i,j} + v_{i,j}$$  \hspace{1cm} (2)

In all of the previous DIC work, the deformation of the subsets in the undeformed image were described using displacement $u$ and $v$, displacement gradient $u_x, u_y, v_x, v_y$, for the first order approximation and $u_{xx}, u_{yy}, v_{xx}, v_{yy}, u_{xy}, v_{xy}$ for the second order approximation. In this paper, each pixel is given an independent displacement $(u_{i,j}, v_{i,j})$. A genetic algorithm is...
then used to find the value of \((u_{i,j}, v_{i,j})\) to minimize the objective function, which for this case is the cross-correlation function, \(S(x_{i,j}, y_{i,j}, u_{i,j}, v_{i,j})\).

III. Bilinear Interpolation Scheme
Due to the resolution limitation for digital images, there are only a finite number of discrete pixels that comprise the images, therefore no intensity value is obtained between pixels. In this case, the displacements for each pixel can only be integer values, and therefore the precision can only be at the pixel level. For small subsets, this will limit the strain resolution to values that are as large as 10%. In order to increase the level of resolution, subpixel displacements must be employed. Therefore, mathematical interpolation techniques are needed to obtain the intensity value between pixels, resulting in subpixel displacement accuracies of 0.05 pixels or better, and effectively increasing the resolution of the digital image by a factor of 400 or greater.

In this investigation, bilinear interpolation is used to reconstruct the deformed images with subpixel accuracy because it provides a reasonable combination of accuracy and computational speed. Bilinear interpolation approximates the intensity at point \((x^*_{i,j}, y^*_{i,j})\), which is located among pixels numbered \((i, j), (i+1, j), (i, j+1), (i+1, j+1)\) as the following,

\[
G(x^*_{i,j}, y^*_{i,j}) = a_{00} + (a_{10} - a_{00})(\Delta x) + (a_{01} - a_{00})(\Delta y) + (a_{11} - a_{00} - a_{01} - a_{10})(\Delta x)(\Delta y)
\]

Where
\[
\begin{align*}
\Delta x &= \text{distance of the point from pixel } (i, j) \\
\Delta y &= \text{distance of the point from pixel } (i, j)
\end{align*}
\]

and \(\Delta x\) and \(\Delta y\) are the distances of the point from pixel \((i, j)\). Ordinarily, bilinear interpolation can cause bias near integer displacement values when using gradient-based optimization methods due to the \(C^0\) discontinuity present at integer pixel locations. Schreier et al have shown that these systematic errors can be reduced through the use of higher order interpolation methods at the expense of computational speed [14]. However, the random search approach employed in the GA optimization method is insensitive to the \(C^0\) discontinuity problem present with linear interpolation, and is therefore limited only by the systematic errors due to reconstructing the deformed image using the bilinear interpolation scheme.

IV. Genetic Algorithm Optimization Method
The GA determines the set of independent variables that optimize a prescribed objective function using Darwinian principles of survival and reproduction of the fittest [15]. The basic steps in executing the genetic algorithm are as follows:

1. Construct the initial population (generation 0) consisting of an array of random individual members (called Chromosomes) each comprised of unique values for the set of independent variables (called genes) in the objective function. The total number of individual members is kept constant over all generations.

2. Iteratively generate new generations of the population until the criteria for termination (e.g., a critical value of the objective function) has been satisfied using the following steps:
   a. Calculate the fitness (i.e., objective function value) for each individual member of the population to determine the fittest members in the generation.
   b. Create a new population by applying the following genetic operations on the fittest members:
      i. Reproduction: Copy fittest members into the new generation.
      ii. Crossover: Create new population members by recombining subsets of genes from fittest members at a randomly chosen crossover point.
      iii. Mutation: Create new population members by randomly mutating the individuals at a given percentage.

There are many different forms of the GA optimization method. For this investigation, a Differential Evolution (DE) strategy is employed because of its superior efficiency [16]. In this strategy, random members are mutated through the addition of a weighted difference between two other random members in order to generate population members for the next generation.

V. Evaluation of Genetic Algorithm Parameters for DIC
According to the previous description of the GA, a number of parameters will be prescribed for the optimization method. Depending on the problem description, different sets of parameters may be chosen to yield the best solution within a prescribed number of iterations.

The parameters that are the focus for this investigation will be: Number of variables for the objective function (Chromosome Length, the number of genes present on the chromosome), Number of individual population member (the number of chromosomes in the population), Crossover probability (CR) and Mutation rate (MU).
1. In this investigation, standard 20×20 subsets were chosen from the full digital image. Each pixel has two variables \( u_{i,j}, v_{i,j} \) to define their deformation. Therefore, there are 800 independent variables for the objective function, which translates into 800 independent genes for each chromosome.

2. Larger population sizes increase the amount of variation present in the population at the expense of requiring more objective function evaluations. The best population size is dependent upon both the application and the length of the chromosome. Considering a tradeoff of computation time for the performance of the genetic algorithm, a population size of 20 was chosen.

3. Crossover probability (CR) is an important parameter affecting the convergence of objective function, and can range from values of 0 to 1. For this parametric study, CRs of 0.8, 0.1, 0.05 and 0.01 were chosen.

4. Mutation is employed to generate new genes for the population. It also prevents the population from becoming saturated with chromosomes that are genetically identical, which causes premature convergence. Large mutation rates (MUs) will increase the probability of destroying good chromosomes, but help prevent premature convergence. The best mutation rate is application dependent and related to both the length of the chromosome and the size of the population. In our work, the DE strategy is employed for the mutation process. Initially, MU values of 0.8 and 0.4 were tested.

VI. Specifying Constraints to Generate Admissible Displacement Fields

By implementing a pointwise description of the displacement field, it is possible to match the gray-level intensity value of any pixel location in the undeformed image with any subpixel location in the deformed image that has the same gray-level intensity value. Consequently, a variety of displacement fields can be generated that perfectly optimize the correlation function. To generate physically realistic displacement fields (i.e., admissible), it is necessary to constrain the relationship between displacements for neighboring pixel locations in the undeformed image. In this investigation, six types of constraints were proposed and analyzed. These constraints are described using the coordinate system shown in Figure 2.

![Figure 1 Coordinate system used to describe the pixel locations within a subset for each constraint case](image)

The six different constraint types and their corresponding results are as follows:

1. The first pixel in the subset can displace anywhere within the full deformed image. The next pixel in the first column and first row is then constrained to displace no more than one pixel from the first pixel. This constrains the corresponding strain to less than 100 percent, but eliminates the possibility of locating any subpixel location within the deformed image that has the same gray-level intensity value. The remaining pixels are likewise constrained from the preceding pixels in the same row to a maximum of one pixel difference for the \( u \) displacement, and from the previous pixels in the same column to a maximum one pixel difference for the \( v \) displacement. The resulting displacement fields for the four different CR values using this constraint are shown in Figure 3, along with the displacement field that was measured using the conventional DIC technique for comparison. The correlation function values corresponding to the CR values were: 0.00475, 0.00135, 0.00678, 0.00533. It is obvious from these results that different CR values have a significant effect on the value of the correlation function. The case for CR=0.1 produced the best objective function value, which was also lower than the value of 0.00310 for the conventional DIC results, and was therefore used in all subsequent analyses. Varying MU had no obvious effect on the convergence rate or the optimal correlation function value. Alternatively, a mutation rate, \( MU(i,j) \), was proposed that varies as a function of subpixel location as follows

\[
MU(i,j) = \left| \frac{G(x_{i,j}, y_{i,j}) - F(x_{i,j}, y_{i,j})}{256} \right|
\]

(4)
Where $F(x^{0}_{i,j}, y^{0}_{i,j})$ is the intensity value of point $(i,j)$ in the reference subset, $G(x_{i,j}, y_{i,j})$ is the intensity value in the deformed image with position $(x_{i,j}, y_{i,j})$, which is obtained from GA. The maximum intensity value is 256, therefore $MU(i,j)$ will always assume a value between 0 and 1 depending on the difference between the undeformed and deformed gray-level intensity values for each subpixel location. The idea behind employing this variable mutation rate was to prevent mutations from occurring on displacements at individual subset locations where there was already a good match between gray-level intensities in the undeformed and deformed images. However, it was determined that there appeared to be no obvious effect from using the variable mutation rate, therefore a constant MU value of 0.8 was used in all subsequent analyses.

a) CR=0.8

![Graphs for CR=0.8](image1)

b) CR=0.1

![Graphs for CR=0.1](image2)

c) CR=0.05

![Graphs for CR=0.05](image3)
2. From the results shown in Figure 3, it was determined that the displacement fields can exhibit large variations relative to the conventional DIC results. Therefore, it was proposed to enforce stricter constraints to reduce the level of these variations. Constraint type 2 extends the applied constraints to pixels in both directions. The first pixel and pixels in the first row and first column were constrained as in the first constraint type. The inside pixel \((i, j)\) is constrained to vary by more than one pixel of difference from both from the \((i-1, j)\) and \((i, j-1)\) pixels. The optimized correlation function value is 0.00602, which was larger than for constraint type 1, and the displacement field still exhibited large variations as seen in Figure 4.

3. In order to limit the variation in the displacement fields relative to the conventional DIC case, a third constraint type was proposed which uses the displacements determined from the conventional DIC case as the initial population members, and then allow the GA to create new population members by limiting varyiations in these values to no more than one pixel. The optimized correlation value for this type was 0.00042, and significantly less variation was observed for the displacement fields, seen in Figure 5, than for the previous two constraint types.
4. Starting from the displacement data obtained by constraint type 1, allow a maximum of one pixel difference in
displacement between adjacent pixels, as was enforced in constraint type 2. The correlation function value for this
case was 0.000226, which was less than that for constraint type 3. The displacement results for this constraint type
are shown in Figure 6.

5. To produce a fitter set of initial population members, constraint type 1 was run twenty times. The fittest members from
each run were then combined to form a new population. This new population was then used to generate the fittest
member under constraint type 1. The correlation value and displacement field for this new constraint type is almost
identical to constraint type 1, which was obtained in a single run. Therefore, the fitness of the initial population is not a
factor that affects the results in this investigation.

6. From the results in constraint types 1 and 4, the displacement fields had large deviation from the conventional DIC
values at boundary locations of the subset. In order to minimize this effect, an averaging method was used for
adjacent displacements along the boundary. The same constraint is applied as in type 1. Averaging displacements of
adjacent pixels for a corresponding pixel resulted in smoother displacements, as seen in Figure 7, but large variations
in displacements are still observed and the optimized correlation function values are similar to constraint type 1.

The optimal correlation function values for the six constraint types, along with the result obtained using conventional DIC,
are summarized in Table 1. For every constraint type except the second, the optimal correlation function value is less than
the conventional DIC result, with the best result obtained for constraint type 4 at a level that was an order of magnitude
less than conventional DIC. However, the GA results exhibited greater variations in the displacement filed, making it more
difficult to interpret the physical meaning. These variations are probably due to random errors that are inherent to the DIC
technique (e.g., interpolation, image acquisition, etc.), and should be smoothed out using an appropriate filter, such as
Generalized Cross Validation, to enhance the physical interpretation of the displacement fields.

Table 1. Optimal correlation function values for different constraint types

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<thead>
<tr>
<th>Constraint Type</th>
<th>Correlation Value</th>
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<th>Correlation Value</th>
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<tbody>
<tr>
<td>1</td>
<td>0.00135</td>
<td>5</td>
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<tr>
<td>2</td>
<td>0.00993</td>
<td>6</td>
<td>0.00190</td>
</tr>
<tr>
<td>3</td>
<td>0.00042</td>
<td>Conventional DIC</td>
<td>0.00310</td>
</tr>
<tr>
<td>4</td>
<td>0.000226</td>
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VII. Conclusions
It has been demonstrated that independent descriptions of subset displacement fields can be obtained with Pointwise DIC technique using the Genetic Algorithm (GA) optimization method. This is in contrast to the conventional kinematic description of the displacement field over a whole subset using a constant zero or higher-order approximation. From the study of GA parameters, it is shown that the crossover probability (CR) most significantly affected the optimal correlation function value and corresponding displacement field, with the best correlation function value being achieved for a CR value of 0.1. Both a constant and variable mutation rate were also investigated, but they had little impact on the correlation results.

To implement the Pointwise DIC technique for obtaining admissible displacement field, it was necessary to specify constraints on the relative displacements between pixels within a subset. From the study of six proposed constraint types, it was clearly evident that different constraint conditions will directly affect the correlated displacement fields. For nearly all constraint types, the optimized correlation function value was up to an order of magnitude less than the conventional DIC results. However, the variation in the displacement fields attributed to inherent random errors in the DIC technique indicated that a smoothing filter may be necessary to obtain physically meaningful results. This investigation demonstrates the potential of using Pointwise DIC to measure displacement fields that are rapidly varying and/or discontinuous due to discrete material or structural phenomena.

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